DENSITY SPATIAL MODEL FOR COMPUTER AIDED INDUSTRIAL DESIGN

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1. INTRODUCTION

In the early stages of industrial design, industrial designers draw many concept sketches in order to define the shape of the product. In this process, industrial designers create images of the product in order to be confirmed the objecture of the design and identify and develope the concept. We propose a new shape model called a density spatial model (DSM) to support this idea-developing phase of industrial design. Initial ideas, involving several basic shapes of the proposed product, become the foundation for developing many variations of the form of the product. Therefore, and shape model used in the early stages of industrial design must provide the ability to generate many variations for further discussion and proposal. In order to realize such a shape model, we modified the shape model traditionally used in the field of computer animation. Several shape models have been introduced for use in computer animation in order to model soft organic shapes. Such shape models include the Blobby Model by J. F. Blinn, and the Metaball by Nishimura, et al. The former models the equipotential surface (equi-energy surface) of hydrogen atoms and expresses the shapes by implicit surfaces, producing shapes using shape

F. Kimura (ed.), *Geometric Modelling*

primitives. We modified and extended the Blobby Model to develop the DSM, which is able to model soft organic shapes and rigid geometric shapes requiring fewer shape primitives than conventional organic shape modelers. Shapes are modeled using one consistent method in the DSM. The internal conditions (strengths and reducing factors of the point charge models) of the DSM are determined from the shape of the passing-points net constructed from the group of points defined by the defined shape. An equipotential surface is created by approximating the shape of the passing-points net. This procedure is called the inverse transformation of the DSM, and allows the conditions of the electric charge models to be determined rapidly by solving simultaneous equations involving only a few charge models. In conventional CAD systems, shape creation operations based on surface patchs are performed using the control points of the surface patch. In a similar way, the inverse transformation of the DSM is used in the shape creating operation with the passing-points net. An explicit function form of the equipotential surface is made by the DSM, representing the boundary of the shape and the space, and is called a equal density surface. The ability to observe and process the equal density surface directly is crucial to industrial design. Using the equal density surface speeds up display of the DSM. In conventional CAD systems, mean curvature and total curvature are used as evaluation criteria for surface manipulation. The mean curvature and total curvature are derived from the equal density surface of the DSM to evaluate the results of surface manipulation. As the DSM ia based on an exponential function, a complete connection with a plane can not be made. We attempt to create a smooth connection by applying a surface connecting function to the equal density surface to be connected to the plane. The same procedure is used to connect two equal density surfaces. In industrial design, the industrial designer first determines the basic structure of the product shape and later adds edges, corners, surfaces and fillets, etc. In the DSM, geometric primitives and density primitives are combined to form an equal density solid with uniform density. The equal density solid can then be combined with additional geometric primitives and density primitives to form a new equal density solid. Equal density surfaces are mapped on to an equal density solid by shape mapping. This kind of shape operation is intuitive and free from the many shape operating parameters that confuse industrial designer, and is therefore suitable for use in the idea-developing stages of industrial design, where trial-and-error is frequently used.

2. DENSITY SPATIAL MODEL

2.1 Shape Primitives of Density Spatial Model

The shape primitives used in the DSM are geometric and density primitives. Geometric primitives have uniform density values, and include such forms as rectangular solids, cylinders, spheres, ellipsoids, and one-leaf hyperboloid rotation bodies. Density primitives represent free-form shapes with density gradients, including plane-based primitives, pipe-based primitives, sphere-based primitives, and blobby shapes. The plane-, pipe- and sphere-based primitives are spetial electric charge models. An object is constructed by performing shape operations for a number of shape primitives. Shape primitives are given positive or negative density values, and two shapes can be added, subtracted, and multiplied together.

2.2 Basic Theory for Density Spatial Model

The potential at a point (x, y, z) in space is expressed as the linear combination of the potentials of the various point-wise electric charges defined in the space. In this study, the potentials are called density values. The range of density values taken by the electric charge models of the density primitives is referred to as the influencing range. Outside this influencing range, the density value taken by density primitives is considered to be 0. Shape primitives with density gradients sometimes produce unexpected result. For this reason, shape groups are introduced. A shape group is a classification scheme for shape primitives.

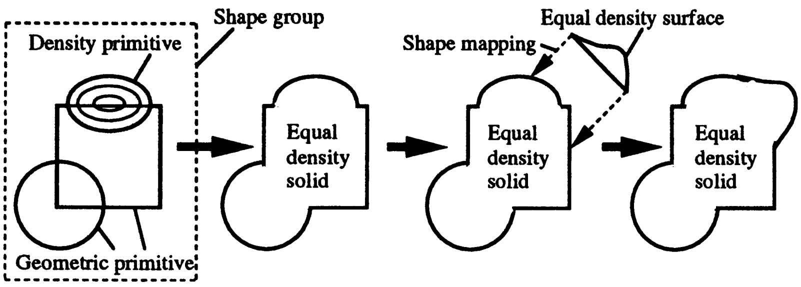

Figure 1. Density spatial model

The fusion of shape primitives is limited within the shape group. As shown in figure 1, density primitives and geometric primitives of the same shape group can be combined. The density values of the combined shape are changed to a uniform value. A combined shape that has a uniform density is called an equal density solid. The equal density solid, and density and geometric primitives can be repeatedly combined to form new equal density solid. As shown in formula (1), $F_{ik}(x,y,z)$ is the density value of the i-th density primitive in the k-th shape group.

$$F_{ik}(x,y,z) = D_{ik}(x,y,z)\exp(-a_{ik}r_{ik}) \qquad (1)$$

Here, a_{ik} is the reducing factor of the entire shape primitive, and r_{ik} is the distance of the shape primitive from the reference plane, line or point. As shown in figure 2, in the case of the plane-based primitive, r_{ik} is the length of the perpendicular segment from the point immediately above the primitive to the reference plane of the primitive. $D_{ik}(x,y,z)$ is the density distribution

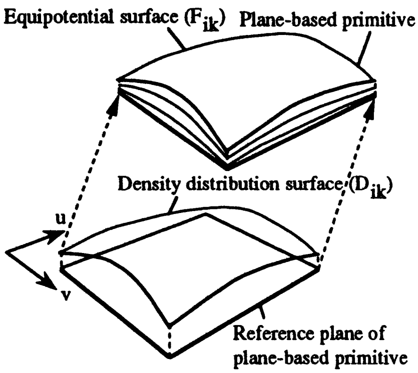

Figure 2. Plane-based primitive

surface. As shown in formula (2), $G_{jk}(x,y,z)$ is the uniform density value of the j-th geometric primitive in the k-th shape group.

$$G_{jk}(x,y,z)\begin{cases} = D_e & \text{(Interior of primitive shape)} \\ = 0 & \text{(Exterior of primitive shape)} \end{cases} \tag{2}$$

In the interior of the shape primitive, the uniform density value D_e (or 0) is given. As shown in formula (3), $R_k(x,y,z)$ is the sum of the total density value of the geometric primitives at point (x,y,z) and the uniform density value of the shape group to which the shape has belongs.

$$R_k(x,y,z) = \sum_{j=1}^{n} G_{jk}(x,y,z) + Q_1(x,y,z) \tag{3}$$

Here, $Q_1(x,y,z)$ is the uniform density value of the first equal density solid defined for a given shape group, and $G_{jk}(x,y,z)$ is the density value of the j-th geometric primitive in the k-th shape group. Once the shape of the shape group is fixed and the equal density solid is created, the equal density solid can be incorporated into other shape groups. If other shape primitive groups do not yet exist, or the defined shape group will not be incorporated into other shape groups, only the total of density value of the geometric primitives will be obtained using Formula (3). The equipotential surface representing the shape of the DSM is expressed as

$$E_k(x,y,z) = \sum_{i=1}^{m} F_{ik}(x,y,z) + H_k(x,y,z) - D_e \tag{4}$$

Here, $F_{ik}(x,y,z)$ is the density value of the i-th density primitive in the k-th shape group expressed in Formula (1). As shown in formula (5), $H_k(x,y,z)$ is the uniform density value of the equal density solid of the k-th defined shape group.

$$H_k(x,y,z)\begin{cases} = D_e & (R_k(x,y,z) \geq D_e) \\ = 0 & (R_k(x,y,z) < D_e) \end{cases} \tag{5}$$

Here, $R_k(x,y,z)$ is the sum of the total density value of the k-th shape group at point (x,y,z) and the uniform density value of the equal density solid of the defined shape group. Formula (6) describes the condition for the uniform density value of the equal density solid of the k-th shape group.

$$Q_k(x,y,z)\begin{cases} = D_e & (E_k(x,y,z) \geq 0) \\ = 0 & (E_k(x,y,z) < 0) \end{cases} \tag{6}$$

Here, $E_k(x,y,z)$ is as expressed in formula (4). Formula (7) defines the DSM equation expressing the shape of an object in terms of the equal density solids of shape groups. Domain S is the shape of the object, and $Q_k(x,y,z)$ is the uniform density value of the equal density solid of the k-th shape group.

$$S = \left\{ (x,y,z) \middle| \sum_{k=1}^{1} Q_k(x,y,z) \geq D_e \right\} \tag{7}$$

The shape of the object is represented by the domain with total density value exceeding the density value for the presentation. Hence the shape of the entire object is the sum of domains occupied by shape groups.

3. EQUAL DENSITY SURFACE AS A SURFACE PATCH

3.1 Inverse Transformation of Density Spatial Model

One way of forming a shape, is to define the points that the shape will pass through. Therefore, the DSM also needs this kind of shape operation procedure. This procedure is similar to the conventional parametric surface patch. As shown in figure 3, on the reference plane of a plane-based primitive, the set of points (passing-points net) through which the equipotential surface defining the shape passes through is defined. The inverse transformation of the DSM is a procedure to obtain, from the position of the passing-points net, information on the internal conditions (strength and reducing factor for electric charge model) of a group of point charge models distributed on the reference plane of a plane-based primitive. The point charge models can be distributed at equal intervals or at random; however, these models must be positioned directly below the nodes of the passing-points net. The number n of point charge models must be equal to the number of nodes, excluding peripheral nodes on the extreme edges that do not have a point charge model directly

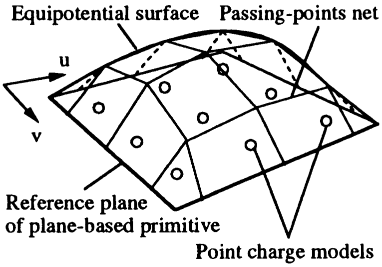

Figure 3. Plane-based primitive with passing-points net

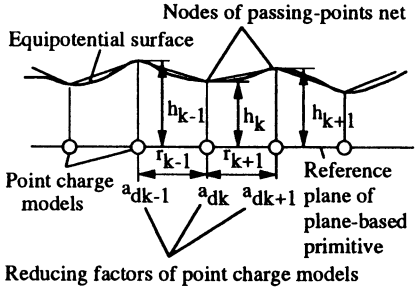

Figure 4. Reducing factors and Passing-points net

below them. As shown in figure 4, the reducing factor a_{dk} for each point charge model on the reference plane of the plane-based primitive can be obtained using the equation. The variables h_k, h_{mk} and r_{mk} denote the height of k-th node, the mean height of surrounding k-th node and the mean distance of surrounding k-th node respectively. And a_{min} is constant.

$$a_{dk} = \left| \frac{\log(h_k / h_{mk}) \exp(a_i (h_k - h_{mk}))}{r_{mk}^2} \right| + a_{min} \tag{8}$$

The strength D_{dk} of each point charge model on the reference plane of the plane-based primitive can be obtained by solving the set of simultaneous equations.

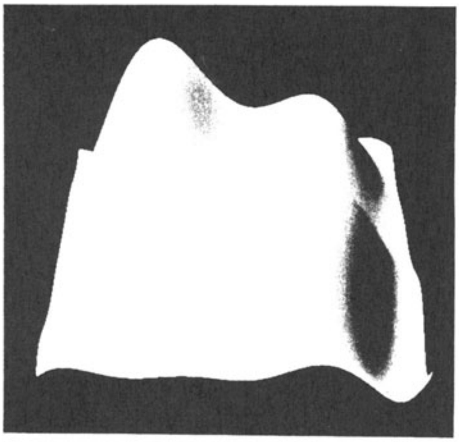

Figure 5. Equal density surface

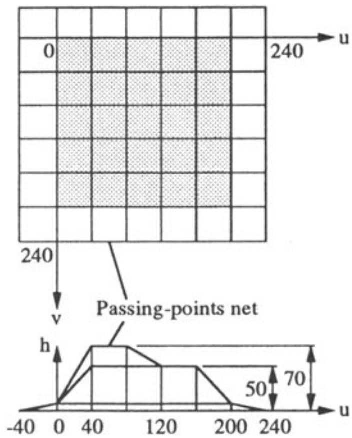

Figure 6. Example of passing-ponts net

$$\begin{cases} C_{11}X_1+C_{12}X_2+\cdots+C_{1n}X_n=D_e \\ C_{21}X_1+C_{22}X_2+\cdots+C_{2n}X_n=D_e \\ \cdots\cdots\cdots\cdots \\ C_{n1}X_1+C_{n2}X_2+\cdots+C_{nn}X_n=D_e \end{cases} \tag{9}$$

Here,

$$X_k=D_{dk} \tag{10}$$

$$C_{jk}=\exp\left(-a_{dk}\left(\left(u_j-u_k\right)^2+\left(v_j-v_k\right)^2\right)-a_i h_j\right) \tag{11}$$

The coordinate (u_j,v_j,h_j) is the position of j-th node of passing-points net. The coordinate (u_k,v_k) is the position of k-th point charge model. Then, the internal conditions of the plane-based primitive are determined. As shown in figure 5, the equipotential surface approximating the passing-points net (fig. 6) is formed.

3.2 Explicit Function Form of Equipotential Surface

The explicit function form of the equipotential surface defining the shape derived from formula (4) is shown in formula (12). Here, a_i is the reducing factor of the entire plane-based primitive. $D_i(u,v)$ is the density distribution surface given by formula (1).

$$h_i(u,v)=\frac{1}{a_i}\left\{\log\left(D_i(u,v)\right)-\log\left(D_e\right)\right\} \tag{12}$$

In industrial design, it is essential that it be possible to directly observe and

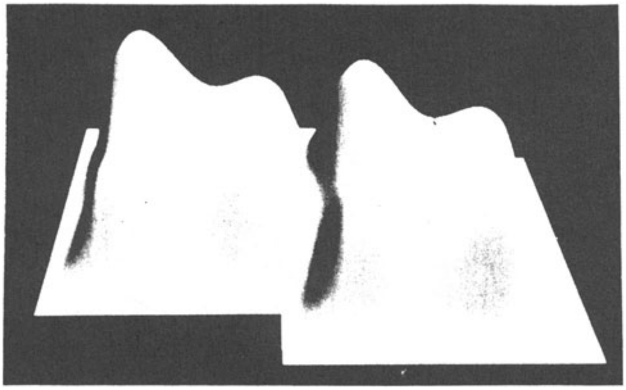

Figure 7. Connection of equal density surfaces

control the equal density surface. As shown in figure 5, the use of the equal density surface allows faster presentation of the DSM.

In conventional CAD systems, total curvature and mean curvature are used for evaluating the surface characteristics of a proposed product. The mean curvature and total curvature are derived from the equal density surface of the DSM to evaluate the results of surface manipulation. We control the shape of the equal density surface by varying the reducing factor of the point electric charge models distributed on the reference plane of the plane-based primitive.

3.3 Smooth Connection of Equal Density Surfaces

As the DSM is based on an exponential function, it is impossible to achieve complete connection with a plane, and even connection between two plane-based primitives has its difficulties. Figure 7 shows a smooth connection between two equal density surfaces on two plane-based primitives. As shown in formula (13), the surface connecting function $s_i(u,v)$ is applied to the equal density surface in the overlapping domain to achieve smooth connection.

$$h_i(u,v) = \frac{s_i(u,v)}{a_i}\left\{\log(D_i(u,v)) - \log(D_e)\right\} \tag{13}$$

4. SHAPE OPERATIONS BY DENSITY SPATIAL MODEL

4.1 Shape Operations

An object is constructed by set operations on a number of shape primitives. Shape primitives are given positive or negative density values, and two shapes can be added, subtracted, and multiplied together easily.

As shown in figure 8, if a pipe-based primitive (part of a pipe) with a negative density value is added to the rectangular geometric primitive around the edge, and a sphere-based primitive (part of a ball) with a negative density is added close to a vertex of the geometric primitive, a round corner is formed on the

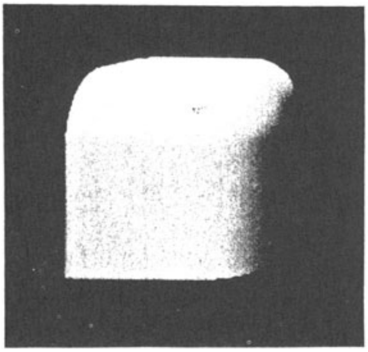

Figure 8. Round corner

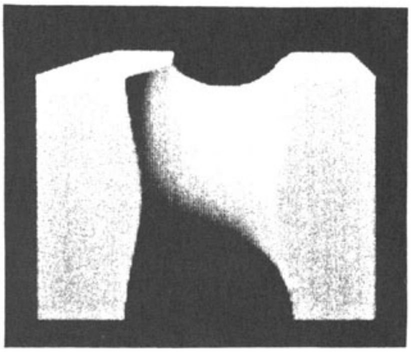

Figure 9. Smooth blending

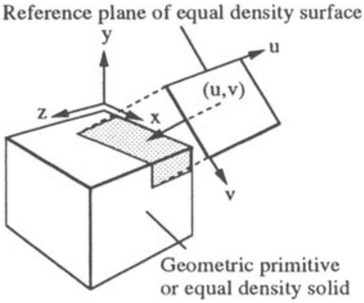

Figure 10. Shape mapping

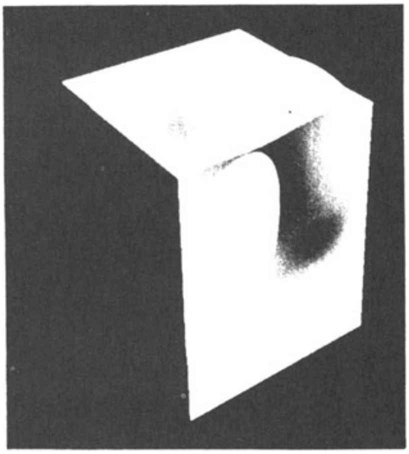

Figure 11. Example of shape mapping

edge and at the vertex of the geometric primitive. As shown in figure 9, two disjoint geometric primitives (rectangular) and a plane-based primitive with a uniform density distribution surface can be joined via a non-uniform reducing factor surface, forming a smooth connection between the two primitives. As shown in figure 10, shape mapping is the shape operation procedure in which an equal density surface formed on a plane-based primitive is mapped onto a geometric primitive. Figure 11 shows equal density surface mapped onto a rectangular primitive.

4.2 Example of Industrial Design using Density Spatial Model

Figure 12 shows an example of an application to industrial design. Figure 12

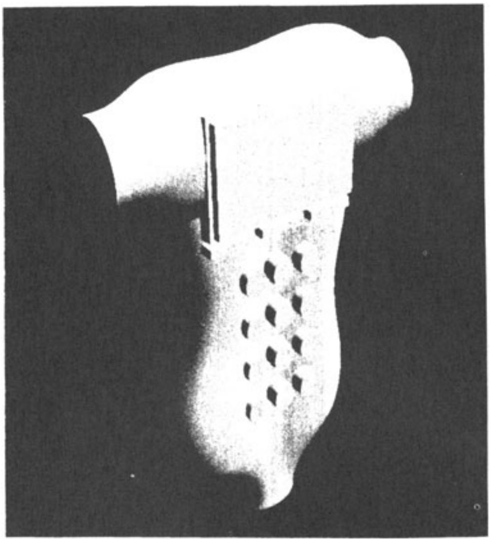

Figure 12. Example of industrial design

is a sample of product shape variation. Figure 12 is made by shape mapping onto the surfaces of two cylinders that form part of the basic structure of the proposed shape.

5. CONCLUSIONS

(1) A new shape model, the density spatial model, has been proposed to aid in the idea development process of industrial design.

(2) The inverse transformation of the DSM was also presented. This is a process to define the internal conditions (strength and reducing factor of electric charge model) from a passing-points net, defined as the points through where an equipotential surface of the shape passed.

(3) An explicit function form of the equipotential surface of the DSM, an equal density surface, was derived. The presentation of the DSM is made faster, making it possible to directly observe and control the design surface.

(4) As an evaluation criteria for shape operations based on the mean and total curvatures of the equal density surface, was derived. For smooth connection

of equal density surfaces, surface connecting function was introduced.

(5) Shape mapping, to map an equal density surface onto an equal density solid of the DSM, was proposed. The function was implemented in a computer, and shape operations were performed by shape mapping.

References

(1) Blinn J., A Generalization of Algebraic Surface Drawing. ACM Transaction on Graphics 1982; 1, 3:235-256

(2) Nishimura H., Hirai M., Kawai T., Kawata T., Shirakawa I,, Omura K., A Technique of Object Modeling and Image Generation using Distribution Functions. Transactions of Institute of Electronics and Communication Engineers of Japan 1985; J68-D, 4:718-725

(3) Sakita K., Igoshi M., Density Spatial Model for Computer Aided Industrial Design. Journal of Japan Society for Precision Engineering 1995; 61, 6:794-798

(4) Sakita K., Igoshi M., Shape Manipulation Using Inverse Transformation of Density Spatial Model for Computer Aided Industrial Design. Transactions of the Japan Society of Mechanical Engineers 1998; 64, 626(C):368-372

(5) Sakita K., Igoshi M., Surface Manipulation Using Density Spatial Model for Computer Aided Industrial Design. Transactions of the Japan Society of Mechanical Engineers 1999; 65, 639(C):309-314

(6) Sakita K., Igoshi M., Shape Mapping Method of Density Spatial Model for Computer Aided Industrial Design. Transactions of the Japan Society of Mechanical Engineers 1999; 65, 639(C):315-319

(7) Bloomenthal J., Introduction to Implicit Surfaces. Morgan Kaufman, 1997.

(8) Yamaguchi F., Shape Processing Engineering III. Nikkan Kogyo press, 1988.

(9) Sederberg T. W., Parry S. R., Free-form Deformation of Solid Geometric Models. Computer Graphics 1986; 20, 4:151-160