Self-similarity in wide-area network traffic

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Abstract

Recent statistical work on traffic measurements are showing traditional models to be poor descriptors of what really goes on for WAN traffic. Here we discuss briefly the shortcoming of those models and illustrate some theoretical features of possibly more realistic models.

Keywords

Stochastic processes, network traffic, modelling

1 INTRODUCTION

Stochastic models of traffic in telecommunications systems are an important application of the modern theory of probability and stochastic processes. In turn problems coming from telecommunications have been stimulating the development of new mathemics at least since the work of Erlang. Queuing theory plays a central role, but approximation, limit theorems and most parts of the theory of stochastic processes are relevant in modelling and understanding traffic phenomema.

In the following we mention briefly some of the classical tools used in modelling and analysis and point at their shortcomings in describing what goes on in systems such as optic networks.

2 THE CLASSICAL TOOLS

The three basic ingredients of the conceptual framework are the Poisson process, Brownian Motion and the $M/M/1$ queue. Most of our intuition about stochatic behaviour is built by looking at this examples.

The original version of this chapter was revised: The copyright line was incorrect. This has been corrected. The Erratum to this chapter is available at DOI: 10.1007/978-0-387-35398-2_19

G. de Marchis et al. (eds.), *Optical Networks: Design and Modelling*

2.1 The Poisson Process

It is very convenient to think that some system is described by either a renewal process or a Markov process. Both the renewal and the Markov property tell you that the system will not be remembering for very long what its past states were; therefore, to describe its future, you don't need to look too much at its past history.

The Poisson Process is the only process that enjoys both the renewal and the Markov property. Here is a few features that make it extremely attactive as a paradigm in building models:

- The interarrival times T_i with exponential distribution

$$P(T_i > t) = e^{-\lambda t}$$

- Number of arrivals with Poisson distribution

$$P(N(t)) = e^{-\lambda t}\frac{(\lambda t)^k}{k!}$$

- The exponential distribution is memoriless
- For the Poisson Process you can compute almost everything explicitly
- Most traditional models for sources (e.g. renewal type models) share the same qualitative features.

2.2 The $M/M/1$ queue

The $M/M/1$ queue is built with a couple of Poisson processes. One describes customers arriving at a shop and the other the owner of the shop taking care of the customers with a 'first come, first served' policy. Almost anything we want to know about the $M/M/1$ queue can be computed explicitly. (see e.g. (Cooper 1981))

2.3 Brownian Motion

The ubiquitous nature of Brownian Motion is due to the Central Limit Theorem. In most applications it is fair to say that, when suitably rescaled, *every process tends to Brownian Motion.* You only need some reasonable decay of correlations and marginal distributions satisfing very mild conditions. In particular almost everything we can build by toying with Poisson processes will be well approximated by Brownian Motion. A Brownian Motion X_t is characterized by the following properties:

- X_t has stationary indipendent increments
- X_t has gaussian distribution with mean 0 and variance $\sigma^2 t$
- X_t has continuos trajectories

2.4 Classical models

For several years we have been using a wide variety of stocastic models to describe traffic. All of this models share the basic qualitative features of at least one of the three basic examples mentioned above.

We may not be able to compute explicitly as many relevant quantities as we can get in our three basic examples, but we know that the qualitative behaviour is not radically different and our intuitions works well as a guide for getting quantitative estimates of what goes on.

We will not even try to list these models here, but refer the reader to the recent excellent review (Jagerman *et al.* 1997).

3 NOVELTIES

Recent traffic measurement seem to fit poorly with the clasical tools we described so far. They display unfamiliar statistical features. Here are the most apparent ones:

- Heavy tailed distributions (with infinite variance)
- Long Range Dependence
- Self-similarity

3.1 Heavy-tailed distributions

The random variable X is said to have a heavy-tailed distribution F if

$$1 - F(x) = P(X > x) \sim x^{-\alpha} L(x)$$

where $L(x)$ is slowly varying at infinity. (see (Samorodnitsky *et al.* 1994))

The expectation of a heavy-tailed random variable need not be finite: $E[X^\beta] < \infty$ if $\beta \leq \alpha$ and $E[X^\beta] = \infty$ if $\beta > \alpha$.

A case of special interest is $1 \leq \alpha < 2$, the so called Noah effect (finite mean, infinite variance, see (Mandelbrot 1982)). Teletraffic data presents statistical evidence of heavy tailed distributions (see (Resnick 1997)).

3.2 Long-range dependence

We say a process $X = (X_i\ ,\ i = 1, 2, \ldots)$ is covariance stationary if $\rho(i, j) = \mathrm{Cov}(X_i, X_j) = \rho(|i - j|)$.

A stationary process is said to exhibit long-range dependence (long memory, *Joseph effect*, persistence) if its covariance decays slowly

$$\rho(k) \sim k^{2H-2} \quad k \to \infty \quad \frac{1}{2} < H < 1$$

As a measure of long-range dependence we used the Hurst parameter H, with $\frac{1}{2} \leq H < 1$. Processes with long range dependance are notoriously hard to analyze, but there are effective techniques available ((Beran 1994), (Cox 1984)).

3.3 Self-similarity

We say a stationary process X is **exactly self-similar** (with self-similarity parameter H) if for all $m = 1, 2, \ldots$

$$X \approx m^{1-H} X(m) \quad \text{(in distribution)}$$

where for $k > 0$,

$$X^{(m)}(k) = m^{-1}(X_{km-m+1} + \ldots + X_{km})$$

X is **asymptotically self-similar** (with self-similarity parameter H) if

$$X \approx m^{1-H} X(m) \quad \text{as } m \to \infty$$

If we are restricting ourselves to a second order description of a process (which as you see may well not be sufficient!), we can similarly define exact/asymptotic second-order self-similarity:

$$\mathrm{Var}(X^{(m)}) = \sigma^2 m^{2H-2} \quad \text{for all } m \geq 1$$
$$\mathrm{Var}(X^{(m)}) \sim \sigma^2 m^{2H-2} \quad \text{as } m \to \infty$$

Self-similar processes exhibit fractal-like behaviour; their graph tends to look the same when you look at it from different distances. Therefore they capture well the idea that all time scales are relevant in the description of the process. The three figures in this tutorial are taken from ((Pagano 1998)) and

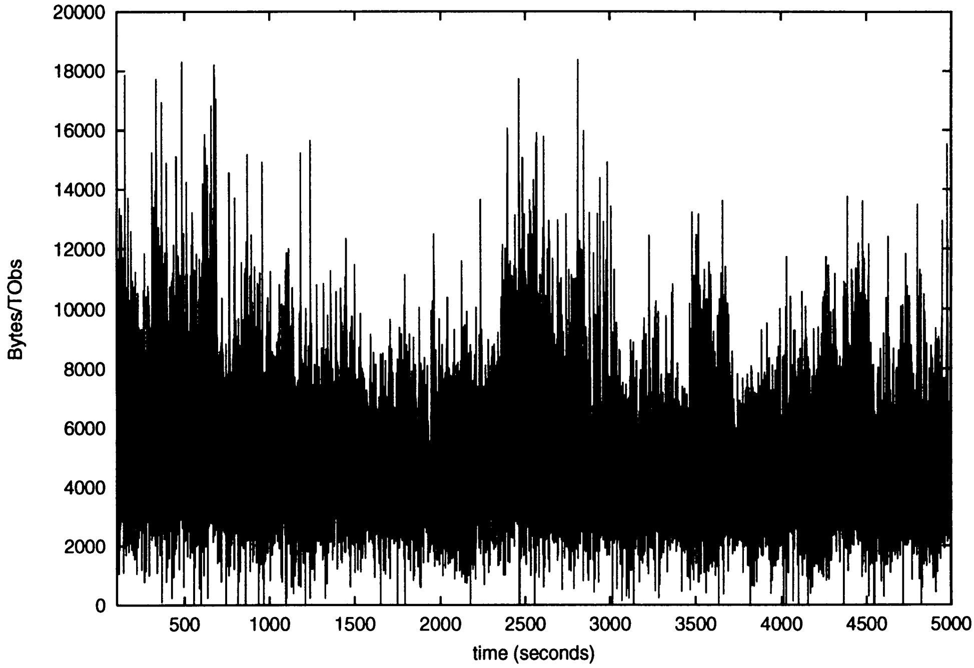

Figure 1 LAN traffic observed every 0.1 s. T_{obs}=1 s

show actual measures of LAN traffic on three different time scales. Anything which is well described by one of the traditional models should become flatter and flatter as you look at it on larger time scales (this is just a consequence of the law of large numbers).

While you can reproduce this data's qualitative and quantitative features with a parsimonious self-similar model, traditional models perform badly. You need a large number of parameters to fit the data and, as traces get longer, you find you need quickly more and more.

If we *assume* the process to be self-similar, then it is easy to extrapolate (a longer session is *not* simulated by glueing together shorter sessions) *and* interpolate.

3.4 Self-similarity in LAN and MAN/WAN traffic

There is increasing experimental evidence for self-similarity in LAN traffic (see figures 1, 2 and 3 , (Leland *et al.* 1994))

We can identify at least two causes for this. A simple fact is that single sources exhibit strong LRD. Less immediate, but of great importance is the fact that the Noah effect leads to the Joseph effect: the superposition of a

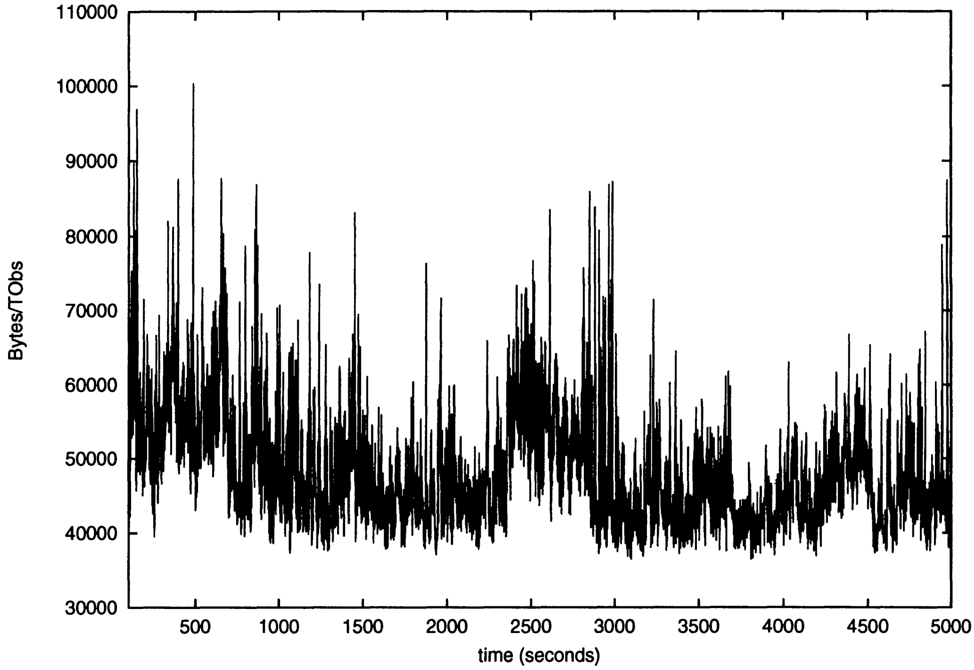

Figure 2 LAN traffic observed every 1 s. T_{obs}=1 s

large number of ON-OFF sources with heavy-tailed ON or OFF periods gives rise to Fractional Brownian Motion. (see (Willinger 1995))

More appropriate for WAN traffic modelling is Kurtz's theorem: consider a Poisson number of source activations with the holding time of each source heavy-tailed with infinite variance. Then, the normalized workload converges to FBM. As noted by Willinger and other authors one finds statistical evidence of heavy tails and self-similarity in single components of the Internet and other WANs.

- File system events are self-similar
- CPU time of a typical UNIX process is heavy-tailed with infinite variance
- File sizes on file servers and document sizes on web servers are heavy-tailed with infinite variance
- TCP and HTTP connections sizes/durations are heavy-tailed with infinite variance

It is not still clear what are all the consequences of self-similarity, but we can list a few here ((Erramilli *et al.* 1997)):

- Queue lenght distribution no longer exponential

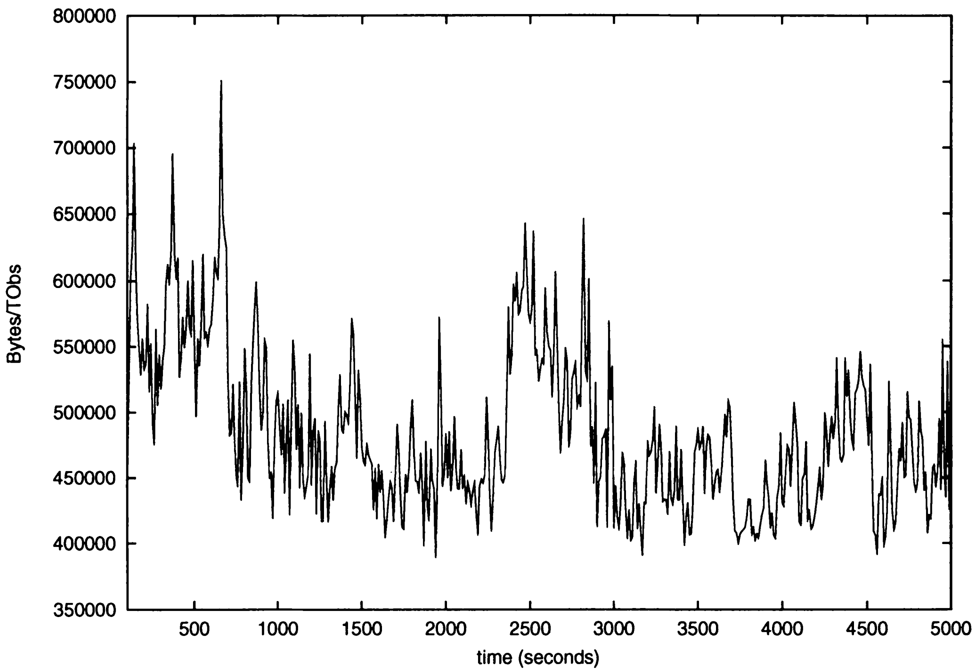

Figure 3 LAN traffic observed every 10 s. T_{obs}=10 s

- Very different (worst) heavy traffic behaviour
- No significant expected gain from within source multiplexing
- Potential for multiplexing gains across sources

4 CONLUSION AND SOME OPEN ISSUES

These new features cannot be ignored, but they do not mean necessarily bad news. Certainly they leave a number of open issues. The main one is of course how to exploit self-similarity. On a more particular level there is still a lot of work to do on fast generation of fractal traffic for simulation and the analysis of networks of queues. Lastly we notice that LRD and the fact that one expects no gains from within source multiplexing, mean that we need to rethink the structure of tariffs and devise a rational pricing scheme for self-similar or bursty traffic (see (Kelly 1997)).

In conclusion I can say that from the point of view of an applied mathematician these new finds in traffic data open are opening the way for a lot of fruitful work in the years to come.

REFERENCES

Beran, J. (1994) *Statistics for Long-Memory Processes.* Chapman & Hall, New York.

Çinlar, E. (1975) *Introduction to Stochastic Processes.* Prentice–Hall, Englewood Cliffs.

Cooper, R.B. (1981) *Introduction to Queuing Theories.* North Holland, New York.

Cox, D.R. (1984) Long–range dependence: A review, in *Statistics: An Appraisal* (ed. H.A. David and H.T. David). The Iowa State University Press, Ames.

Erramilli, A. , Narayan, O. and Willinger, W. (1997) Fractal queuing models, in *Frontiers in Queuing* (ed. J.H. Dshalalow). CRC Press, Boca Raton.

Jagerman, D.L. , Melamed, B. and Willinger, W. (1997) Stochastic modelling of traffic processes, in *Frontiers in Queuing* (ed. J.H. Dshalalow). CRC Press, Boca Raton.

Kelly, F.P. , (1997) Charging and accounting for bursty connections, in *Internet Economics* (ed. L.W. McKnight and J.P. Bailey,). MIT Press, Cambridge, MA.

Kurtz, T.G. (1996) Limit theorems for workload input models, in *Stochastic Networks: Theory and Applications* (ed. F.P. Kelly, S. Zachary and I. Ziedins). Claredon Pres, Oxford.

Leland, W.E. , Taqqu, M.S. , Willinger, W. and Wilson, D.V. , (1994) On the self-similar nature of network traffic (extended version). *IEEE/ACM Trans. on Networking,* **2**:1, 1–15.

Mandelbrot, B.B. (1982) *The Fractal Geometry of Nature.* W.H. Freeman and Co. , San Francisco.

Pagano, M. (1998) *Analisi Multiscala e Valutazione delle Prestazioni in Reti a Larga Banda.* Tesi di Dottorato, Università di Pisa.

Resnick, S.I. , (1997) Heavy tail modeling and teletraffic. *The Annals of Statistics,* **25**, 1805–1869.

Samorodnitsky, G. and Taqqu, M.S. (1994) *Stable non–Gaussian Random Processes: Stochastic Models with Infinite Variance.* Chapman & Hall, New York.

Willinger, W. , Taqqu, M.S. , Sherman, R. and Wilson, D.V. , (1995) Self-similarity through high-variability: statistical analysis of Ethernet LAN traffic at the source level (extended version). Preprint.

5 BIOGRAPHY

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