Point-based Geometric Modelling

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Abstract: Surface modelling techniques based on parametric polynomials offer simple interactive design and efficient interrogation techniques, and are the foundation of most current CADCAM systems. However, they are known to have limitations in respect of their shape defining capabilities and the efficacy with which users can make detailed adjustments to the model. A new point-based approach to surface modelling, using solely geometric constructions, is presented to overcome these problems. Techniques for surface interrogation, visualisation and manipulation are introduced and industrial case studies are presented

1. INTRODUCTION

In this paper we present a fresh approach to CADCAM: not a change in detail or method but a fundamental shift in philosophy. Instead of allowing a polynomial representation to occupy the central role as the determinant of shape, we return to the physical object and employ geometrically-based procedures that operate directly on points taken from it. In this respect, we address the problem originally posed by Ferguson[1] and seek to fit a fair surface through a grid of points.

The proposed surface interrogation and manipulation procedures rely on estimates of curvature and unit tangent that are computed from a grid of points that characterise the underlying surface. This willingness to estimate is the nub of the new approach. We observe that the total design/machining process is not exact but an approximation bounded by a tolerance. We allow some of the tolerance to be used within the modelling process. In doing this, we move away from the notion of an 'exact' mathematical representation, of which design drawings and the manufactured object are but derivations and approximations, to the position where the equivalence class of acceptable surfaces (i.e. those surfaces within tolerance of the nominal geometry) is the ultimate shape reference. In terms of accuracy, the only restriction is that the estimation techniques are sufficiently accurate that overall error lies within the specified tolerance.

Use of a purely geometric modelling process avoids some of the difficulties that have been observed in current CAD systems. Specifically:

- *Data exchange* The problem of accurate conversion between different surface representations is avoided since shape is captured directly from surface geometry.
- *Reverse Engineering* The proposed approach requires only sufficient data to characterise the underlying shape and avoids the 'data clouds' required by some reverse engineering techniques.
- *Ab Initio Design* The designer is no longer restricted to polynomial surfaces.

In section 2 we present the interrogation, manipulation and visualisation procedures that underpin the new approach. Two industrial case studies are be presented in section 3 to illustrate its accuracy and power.

The original version of this chapter was revised: The copyright line was incorrect. This has been corrected. The Erratum to this chapter is available at DOI: 10.1007/978-0-387-35392-0_40

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2. POINT-BASED MODELLING

2.1 Assumptions

In the following sections we shall describe techniques that allow a surface to be modelled using a grid of points. It is assumed that the surface is piecewise curvature continuous and that the points are sufficiently dense to characterise its shape. As a matter of practical convenience, it is also assumed that the grid has a rectangular topology. In order to ensure high accuracy, it is assumed that the points are sampled using a spacing that is regular and geometric in nature (e.g. arc length). Surfaces that have significant features are most accurately modelled when the grid runs parallel or perpendicular to the features. The latter is consistent with normal practice when digitising components.

2.2 General Outline

A point-based system should possess the functionality of a conventional CADCAM system. In particular, it should include procedures for surface interrogation, visualisation and manipulation of the model. This can be realised if points that lie between the given data and derivatives up to the second order can be estimated in a reliable and accurate manner.

We shall describe methods for estimating the principal curvature and unit tangent vectors at grid points in section 2.3. Using these results, an interpolation procedure for estimating curvature, tangent and normals between grid points will be presented in section 2.4. In section 2.5 we outline some visualisation techniques to assess the quality and appearance of the surface model. In section 2.6 we describe how they may be combined with the interpolation techniques to improve the quality of a surface model.

2.3 Principal Curvature and Unit Tangent Vectors at Grid Points

Consider a string of five consecutive points $\mathbf{p}_{i-2}$, ..., $\mathbf{p}_{i+2}$ that lie on the surface to be modelled. The curvature at the centre point, $\mathbf{p}_i$, may be estimated using a technique based on circular constructions. If the curvature vectors at $\mathbf{p}_i$ of the interpolating circles through $\{\mathbf{p}_{i-1}, \mathbf{p}_i, \mathbf{p}_{i+1}\}$ and $\{\mathbf{p}_{i-2}, \mathbf{p}_i, \mathbf{p}_{i+2}\}$ are denoted by $\mathbf{k}_i^1$ and $\mathbf{k}_i^2$ respectively, then it is reasonable to expect that, for well spaced data, $\mathbf{k}_i^1$ provides a better estimate for $\mathbf{k}_i$, the curvature vector at $\mathbf{p}_i$, than $\mathbf{k}_i^2$. The Richardson extrapolation principle[2], may be applied to $\mathbf{k}_i^1$ and $\mathbf{k}_i^2$ to obtain an improved estimate for $\mathbf{k}_i$:

$$\mathbf{k}_i = \frac{(s_{i-2} + s_{i-1})(s_i + s_{i+1})\mathbf{k}_i^1 - s_{i-1}s_i\mathbf{k}_i^2}{(s_{i-2} + s_{i-1})(s_i + s_{i+1}) - s_{i-1}s_i}$$

where s_i is the arc length between grid points $\mathbf{p}_{i-1}$ and $\mathbf{p}_i$. Arc lengths may be similarly estimated by a method based on circular interpolation. Two estimates of the arc length between $\mathbf{p}_i$ and $\mathbf{p}_{i+1}$ may be estimated by fitting circles through the four points $\{\mathbf{p}_{i-1}, \mathbf{p}_i, \mathbf{p}_{i+1}, \mathbf{p}_{i+2}\}$, with the final estimate of s_i being taken as the arithmetic mean to remove any bias.

It can be shown that $\mathbf{k}_i$ is a second order approximation to the principal curvature vector when the data is approximately regularly spaced. The approximation can still be used even if the data is irregularly spaced because it is well-defined and continuous for any combination of arc lengths. It is also invariant with respect to constant scaling and reversal of point ordering.

Using a similar procedure, the interpolating circles may be used to provide two estimates for the unit tangent, $\mathbf{t}_i^1$ and $\mathbf{t}_i^2$, at point $\mathbf{p}_i$ from which an improved value may be calculated. Details of the method, error estimates and sample calculations are given in the paper by Tookey and Ball[3].

2.4 Interpolation Between Grid Points

Once the principal curvature and unit tangent vectors have been computed at the grid points, a geometric construction, based on a generalised Cornu spiral (GCS)[4], may be used to estimate intermediate points and unit tangents. A particular benefit of using a GCS is that it not only matches end points and tangents exactly but has a curvature that varies monotonically as a rational linear function of the arc-length. This guarantees a high quality interpolating curve.

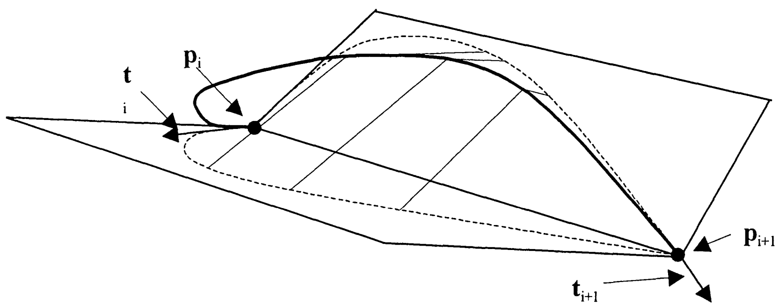

Figure 1. Construction of GCS from planar projections

Consider the general case where consecutive grid points, $\mathbf{p}_i$ and $\mathbf{p}_{i+1}$, and their associated unit tangents, $\mathbf{t}_i$ and $\mathbf{t}_{i+1}$, do not lie in a plane. Since the GCS construction procedure is planar, it is necessary to calculate two projections of the intermediate point and then combine them to obtain its three-dimensional position. See figure 1. Let Ω_0 and Ω_1 be the two planes that contain $\mathbf{p}_i$, $\mathbf{p}_{i+1}$, $\mathbf{t}_i$ and $\mathbf{p}_i$, $\mathbf{p}_{i+1}$, $\mathbf{t}_{i+1}$ respectively.

The GCS fitting algorithm requires the curvature and unit tangent at both end points and so $\mathbf{t}_{i+1}$ and $\mathbf{k}_{i+1}$ must be projected onto Ω_0, with similar projections onto Ω_1. The formulae to accomplish this may be found in Tookey[5].

The GCS fitting algorithm[6] employs a robust iteration scheme that ensures the interpolating curve exactly matches the end points and tangents whilst producing approximate values for the end curvatures and arc length. Once these values have been determined, it is a simple matter to evaluate positions and derivatives at intermediate points.

Assuming both planar GCSs are single-valued with respect to the chord, then the three-dimensional GCS is well-defined. It can be computed by intersecting the ruled surfaces formed by extruding the planar curves parallel to their respective normals, as shown in figure 1.

If u $(0 <= u <= 1)$ is a parametric distance along the chord and $\mathbf{q}_i^0(u)$ and $\mathbf{q}_i^1(u)$ are the projections of the intermediate point on planes Ω_0 and Ω_1, then the point's three-dimensional position $\mathbf{q}_i(u)$ may be obtained by taking the mean position of the points' projections, as described in Tookey[5]. There are a number of sub-cases (e.g. coplanar points and tangents) but procedures exist to deal with them. We shall discuss the accuracy of the interpolation method in section 3.

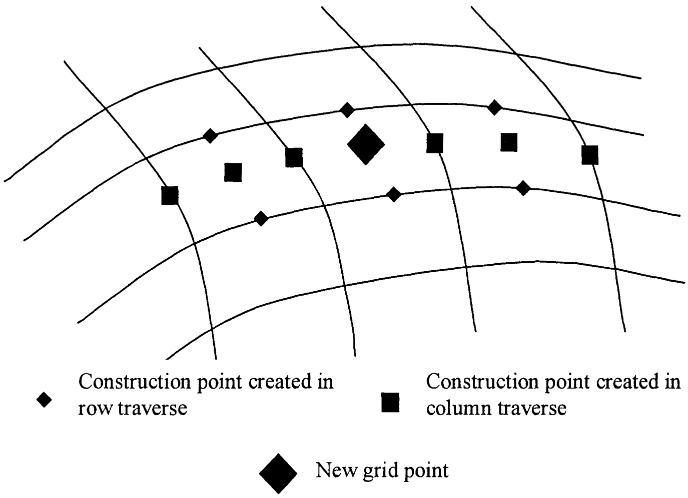

Figure 2. Insertion of new points by Interpolatory subdivision

The interpolation forms part of a point insertion procedure that may be used to create a denser grid of points lying on the surface. To achieve this, each row of the grid is traversed and the interpolation is applied to consecutive pairs of points so as to introduce columns of construction points, as shown in figure 2.

The revised grid, including the columns of construction points, is then traversed column-by-column, to produce an estimate $\mathbf{q}^{1}_{i,j}$ of each new grid point. To remove any bias, the complete procedure is repeated, but this time the grid is swept in 'first column then row' order so as to produce a second estimate for each new point, $\mathbf{q}^{2}_{i,j}$. Finally, the revised grid is defined to be the original points plus the arithmetic mean of the two sets of estimates.

The process may be applied recursively to generate the limit surface of the object, to within a specified tolerance. A similar process is used to compute unit tangents at the intermediate points. Experiment has shown the point insertion process to be stable, well behaved and accurate.

2.5 Surface Visualisation

Techniques for visualising and assessing surface quality have been developed to support point-based modelling. Being based directly on geometry enhances their power to diagnose surface quality[7]. It should be emphasised that the need for quality is not just a question of mathematical rigour or aesthetics. Even small surface imperfections can have serious and expensive consequences at the manufacturing stage[8].

A particularly useful method of assessing the quality of a point-based surface model is to plot geometric parameter (gp) curves. These are the discrete analogue of conventional isoparametric curves but are related to geometric invariants: in this case, arc length, tangent angle and normal angle. All three methods are founded on the discrete form of the generic reparametrisation :

$$g(u_i) = \frac{\int_0^{u_i} G(u)\,du}{\int_0^1 G(u)\,du}$$

where u $(0 <= u <= 1)$ is an underlying parametrisation and the function G(u) is chosen according to the required reparametrisation

Each of the strings of points bounding the grid can be reparametrised with respect to the chosen geometric invariant, so that points equally spaced in that invariant can be generated. Connecting corresponding parameter values with a linear interpolant will produce a simple but useful approximation to the true geometric surface curves. Procedures for computing geometric points in the interior of the grid have been developed to produce a more accurate visualisation of gp curves. We will provide examples of the output from such procedures in figures 6 and 7.

Procedures for plotting contours on the surface model are also available. The grid of points and a chosen height vector are rotated so that the height vector lies in the z direction. The grid is then recursively sub-divided until the distance between consecutive points is small enough for heights to determined, within a given tolerance, using linear interpolation. The sub-divided grid is then faceted with triangles. Intersecting the height vector with the faceted surface along lines of constant x and y provides sufficient points to generate the required contours. We note that this method is crude but it serves to illustrate how a point-based system can provide analogues of the facilities normally found in conventional CAD systems.

2.6 Surface Manipulation: Sanding and Filling

The ability to manipulate the surface model in a simple and intuitive manner is a key practical feature of the point-based approach. Shape may be adjusted by moving points, safe in the knowledge that the interrogation procedure will modify the surface to fit the new data exactly. This contrasts with conventional systems where modification is usually carried out by altering vertices in the characteristic polygon or by changing weights. Being non-geometrical in nature, it may be difficult for the designer to relate these controls back to the required geometry. The problem of 'tidying up' an imperfect polynomial surface so as to achieve quality of fit as well as quality of form has been discussed in the literature [9, 10].

3. CASE STUDIES

Figures 3 and 4 provide indications of the amount of data required to construct a useful surface model and the accuracy of the point insertion procedure. Figure 3 shows a grid of 6 by 26 points sampled from a five patch, biquintic Bezier CAD surface model of an Austin Rover car roof. In this example, the number of grid points is identical to the number of distinct vertices that were used to construct the original CAD model.

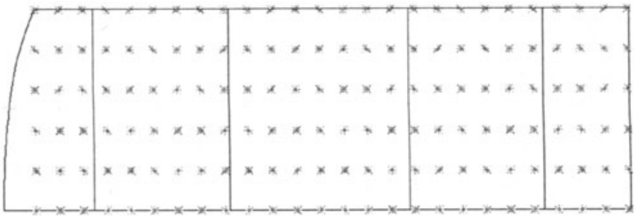

Figure 3. Grid of points used to model car roof

Figure 4 shows the original grid (marked by squares) and the new half points (marked by arrows) that were generated by the point insertion procedure. The inserted points were compared with the original CAD surface and discrepancies greater than 0.01mm are marked by black circles. All five circled points lie on the boundary between two patches and correspond to discontinuities in curvature (i.e. surface flaws) in the original CAD model. Similar comparisons have been made with other test cases and the same level of accuracy has been observed.

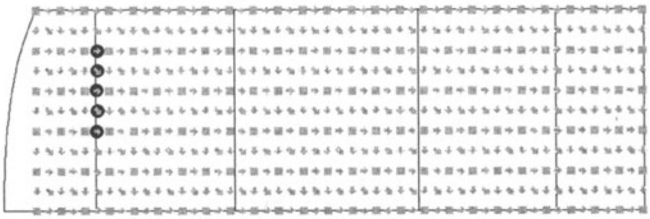

Figure 4. Roof model after insertion of half-points

Figures 5 to 7 illustrate the visualisation and sanding techniques introduced in sections 2.5 and 2.6. Figure 5 shows a grid of 8 by 25 regularly spaced points taken from a polynomial-based model of a part of an aircraft foreplane. Figure 6 shows a tangent parametrisation of the data which was generated using the visualisation techniques discussed a section 2.5. Irregularities in two areas to the left and right of centre suggest local surface imperfections.

Figure 7 shows the same surface but after the sanding procedure outlined in section 2.6 had been applied to three sets of points in the worst affected areas. The greater regularity of the constant tangent lines indicates a higher quality surface that is less likely to induce problems during manufacture. The improvement involved Euclidean point movements of not more than 0.005mm.

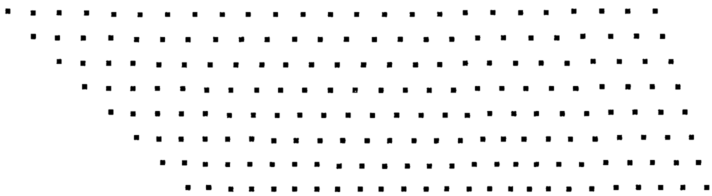

Figure 5. Point definition of a foreplane

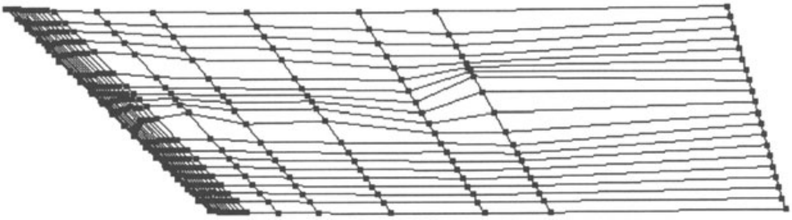

Figure 6. Tangent parameterisation of original foreplane

4. CONCLUSIONS

We have presented a point-based, purely geometric approach to modelling free-form surfaces. The approach has significant benefits including greater flexibility in terms of the geometries than can be modelled and diagnostic and manipulation tools that relate directly to the shape and quality of the surface. A high quality surface is assured since the method is based on a linear curvature segment. The data requirement and accuracy of the new method is comparable with current CAD techniques. Research and development is continuing with the active collaboration of industrial partners.

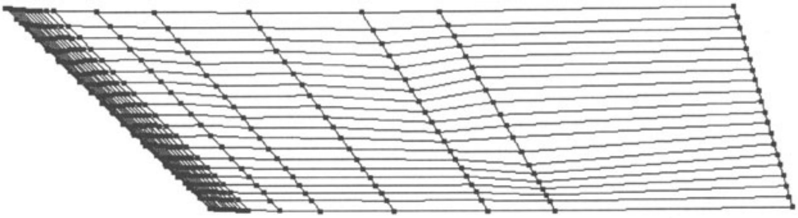

Figure 7. Foreplane after modification by sanding

5. ACKNOWLEDGEMENTS

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