

# Five –Axis Control Sculptured Surface Machining Using Conicoid End Mill

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Abstract: The study deals with five-axis control tool path generation for the sculptured surface machining with a conicoid end mill. In general, ball end mills are utilized for the sculptured surface machining. However, it takes much time to finish such complicated surfaces with high accuracy due to the necessity of small amount of pick feed, which causes the reduction in machining efficiency. Thus, special cutting tools called conicoid end mill were devised to meet the requirement of relatively large amount of pickfeed. The adequate selection of tool attitude allows a large amount of pick feed, and yields good surface quality in surface finishing.

## 1. INTRODUCTION

In the recent years, industrial products are changing into the complicated ones consisting of numerous sculptured surfaces so as to meet the demands for improvements of design and mechanical performance. In regard to the above background, it is necessary to establish a new technique for the efficient machining of sculptured surfaces.

In case of machining sculptured surfaces, ball end mills are inevitably utilized, since they allows us to machine complicated surfaces with a wide variety of curvature by selecting a suitable radius one. However, in cutting the complicated curved surface accurately, the ball end mill with the smallest radius should be selected so as to prevent the gouging between tool and surface. Furthermore, in order to keep the cusp height below the specified tolerance, a very small amount of pickfeed must be set according to the surface curvature and the tool radius. The above problems indicate that the complicated surface machining with ball end mill is not necessarily effective and is liable to lead to the increase in machining time and the reduction in machining efficiency.

In previous publications, five-axis control heel cutting with a flat end mill or a face-milling cutter has been reported. The methods allow us to accommodate the *Cutter Location* (CL) data to the surface curvature change by modifying the tool attitude and the imaginary radius of tool at each cutting point<sup>1,2,3</sup>. In our previous paper, we proposed the introduction of a *conicoid end mill* and the application to the five-axis control machining of sculptured surfaces<sup>4</sup>. Since the curvature of the cutting edge changes continuously, the tool attitude can be adjusted for the tool radius to be equal to the curvature at each cutting point on a workpiece by five-axis control. Thus, we produced a *paraboloid end mill* as a kind of the conicoid end mills, whose outline is formed by a revolutional body of parabola, and a certain availability was experimentally confirmed. However, it is difficult to say that the method is widely applicable to all surfaces, since all tool paths are generated along parameter lines on the sculptured surface. The paper deals with the adaptability of our tool path generation method mentioned in the previous paper.

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## 2. SCULPTURED SURFACE FINISHING WITH CONICOID END MILL

In cutting by ball end mills, wave-like remains take place between two neighbouring cutter paths, which are called *culp*. The height of *culp* is determined by a few factors such as the surface curvature, the tool radius and the amount of pickfeed, thus influencing the surface accuracy. In case that the change in surface curvature is quite small compared with the amount of pickfeed, the *height of culp*  $h$  is approximately expressed by the following equation;

$$h = r - \sqrt{r^2 - \frac{f^2}{4}} \approx \frac{f^2}{8r} \quad (1)$$

, where  $r$  and  $f$  are a *tool radius* and an *amount of pickfeed* respectively. When machining the complicated surface, whose curvature varies largely, we must select a cutter with as small radius as possible so that it can cut the maximum curvature part of surface without gouging. Since the height of *culp*  $h$  increases with decreasing the tool radius  $r$ , as seen in Equation (1), a small amount of pickfeed must be required to make the *culp* height below the tolerance, which causes the drastic increases in data length and machining time. Furthermore, the actual height of *culp* varies according to the surface curvature change, which results in the difficulty to obtain the uniform surface appearance.

In regard to the above problems, it is convenient to change both the tool radius and the amount of pickfeed according to the surface curvature change. However, it causes such problems as the tool setting error resulting from the tool change, which deteriorates the surface quality. In addition, the working time increases with increasing the number of tool setup. Judging from the above consideration, the method dependent on the tool change is not always practical.

The solution is to use the special end mill whose radius changes continuously and to adjust the cutter location and the amount of pickfeed at each cutting point so as to keep the *culp* height constant. It allows us to perform the effective surface finishing with a uniform *culp* height. The utilization of five-axis control enables to adjust the tool location to the surface configuration. As the special end mill, we devised conicoid end mills that consist of a revolutionary body of conic section, such as ellipsoid, hyperboloid and paraboloid. Figure 1 shows the comparison of a new machining method by use of paraboloid end mill with a conventional one by use of ball end mill. Conicoid end mills enable to cut the complicated surface with tool radius and pickfeed as large as possible, thus performing the finishing process efficiently.

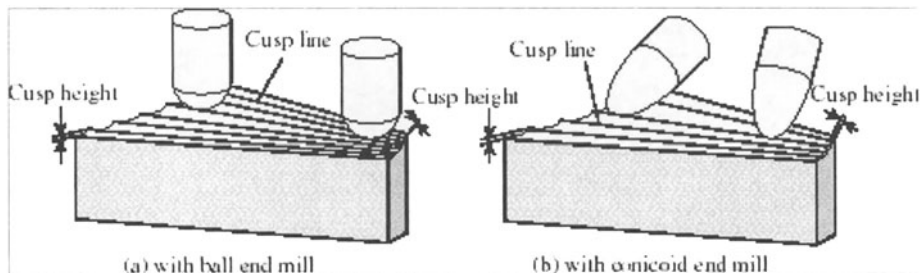


Figure 1. Two kinds of machining method for sculptured surface

In our latest research, the shape of conicoid end mills is limited to paraboloid end mills. Figure 2 shows two paraboloid end mills actually made, based on different parabola functions,  $f(x) = x^2/2$  and  $x^2/5$ . We have proposed the basic method of tool path generation, and obtained a measure of success. However, several problems take place for cause that the tool path was

generated along parameter lines on the surface. Thus, it is improper to apply the method to curved surfaces that change their width drastically in the tool feed direction. If the problem is solved, the method offers the potential of effective machining of complicated surface.

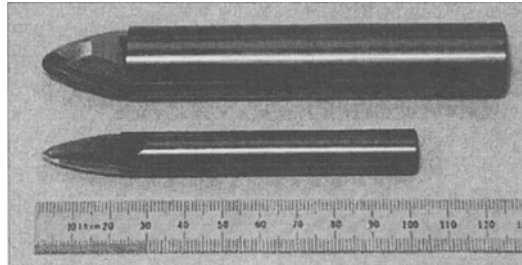


Figure 2. Paraboloid end mill

The paper deals with the improvement of the method for paraboloid end mill, and the expansion of the algorithm so as to generate tool path without influences of parameter lines on the surface.

### 3. TOOL PATH GENERATION FOR MACHINING WITH PARABOLOID END MILL

#### 3.1 Determination of tool attitude

At first, we set up the *tool coordinate system*  $Ot$ , which has its *origin*  $O$  at the *tool center point*, and the  $z$  coordinate axis corresponds with the *tool axis*  $t$ , as illustrated in Figure 3

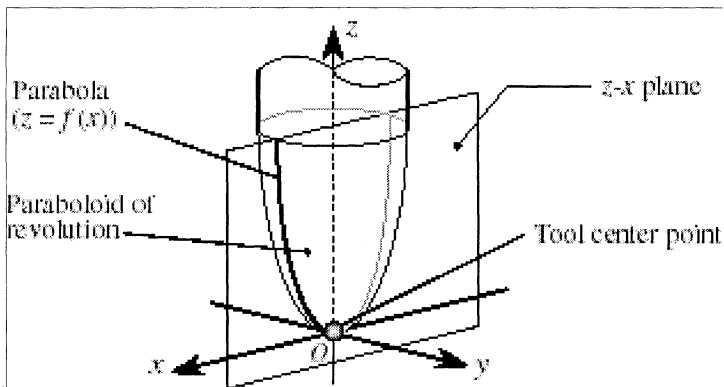


Figure 3. Tool coordinate system  $Ot$

Next, at each *cutting point*  $C_c$ , let us assume a local orthogonal coordinate system  $O_c$ . This coordinate system is called *cutting point coordinate system* and has its origin at the cutting point  $C_c$ , with the *direction of surface normal*  $nc$ , the  $z$  coordinate axis, and the *tool feed direction*  $fc$ , the  $x$  coordinate, as illustrated in Figure 4. The tool is located on the cutting surface so that the cutting point of tool can contact with the surface at  $C_c$ .

At the time, the *tool center point*  $P_o$  is determined in the *coordinate system*  $O_t$ , as illustrated in Figure 5. The point  $P_o$  can be calculated by the following equation.

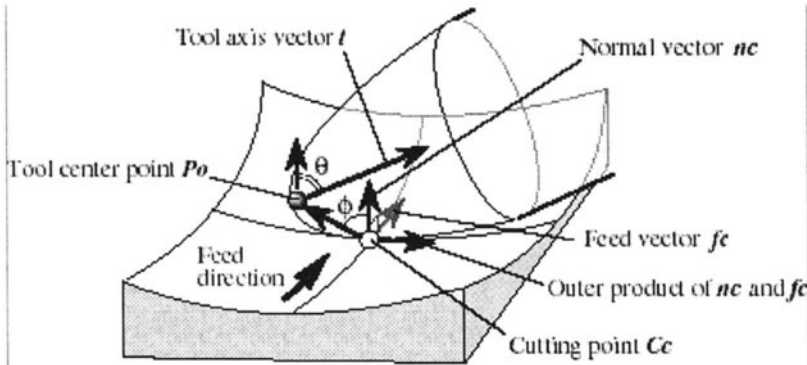


Figure 4. Cutting point coordinate system

$$P_o = C_c + |d| R^{fc(-\phi)} \bullet n_c \quad (2)$$

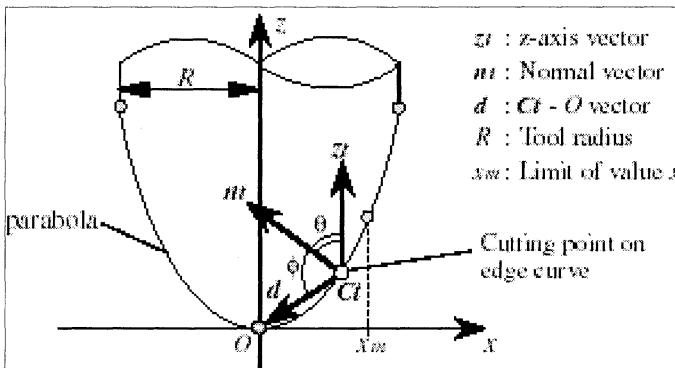


Figure 5. Positional relationship in tool coordinate system

Where  $nt$ : the normal vector of parabola at the cutting point  $Ct$

$zt$ : a vector parallel to z-axis that starts from point  $Ct$

$q$ : an angle between the vector  $nt$  and  $zt$

$f$ : an angle between the vector  $d$  directing from point  $Ct$  to  $O$  and  $nt$

$Rv(a)$  represents a *rotational transformation matrix* by a *rotational angle*  $a$  around the vector  $v$ , and  $|d|$  is the *scalar of the vector*  $d$ . In the similar manner, the *tool axis vector*  $t$  in the coordinate system  $O_c$  is represented by the following equation,

$$t = R^{fc(\theta)} \bullet n_c \quad (3)$$

### 3.2 Calculation of tool path

In general, sculptured surfaces are defined by two parameters,  $u$  and  $v$ , which change from  $0$  to  $1$ . Thus, each cutting point on the surface is calculated from  $u$  and  $v$ . The tool

attitude is determined on the basis of the *cuspl line* that is like ridge on the surface. At the time, to perform the surface cutting in good condition, the tool attitudes must be generated so as to change smoothly. In the following paragraphs, the tool path generation process is described in detail.

### 3.2.1 Cutter locations in the first tool path

Cutting points for the first tool path are generated on the *iso-parametric curve*  $u$  ( $v=0$ ; constant) along the surface edge. Cutting parts on cutting edge and tool attitudes are calculated on the basis of curvatures in the *direction of parameter*  $v$ . At the time, the interference between tool and surface is checked on each cutting point. If the interference is detected, the tool attitude is modified by shifting the cutting part on tool edge. Repeating the above check and modification for all cutting points generates the first interference-free first tool path.

### 3.2.2 Cusp line generation

Except for the above first path, almost of the all paths are generated on the basis of the former path. First, the parabolas, which correspond to the tool shape, are arranged on the cutting surface according to the cutter locations in the former tool path.

Next, at each cutting point, the *cuspl point*  $Pcs$ , which is distant from the cutting surface to the specified cusp height, is searched by using a convergent algorithm, as illustrated in Figure 6. The cusp lines are generated by interpolating cusp points obtained above, as shown in Figure 7.

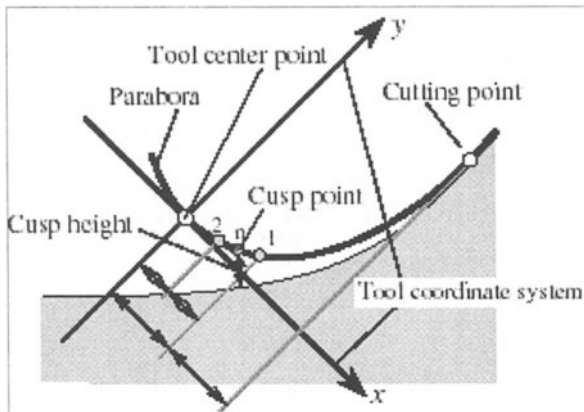


Figure 6. Calculation of Cusp Point

### 3.2.3 Determination of tool attitude based on cusp line

Cutter locations for each tool path are determined so that the cutting edge may contact with the cusp line.

First, we assume a coordinate system  $Oc$ , as illustrated in Figure 8. At this time, let  $vc$  be the vector that directs to the *intersection point*  $Sc$  between the cusp line and the  $x$ - $z$  plane in the coordinate  $Oc$ . In addition, the value  $p$  is the  $x$  component of the cutting point  $Ct$  in the tool coordinate system  $Ot$ . Then, the following equation is satisfied.

$$l^2 = (1 + \beta)(4p^2 - 4p \frac{\beta}{a} + \frac{\beta^2}{a^2}) \quad \left( \beta \equiv \frac{2ap \bullet \sin + \cos \zeta}{-2ap \bullet \cos \zeta + \sin \zeta} \right) \quad (4)$$

, where  $\zeta$  is the angle between the vector  $\mathbf{vt}$  and the vector  $\mathbf{nt}$  in the coordinate  $Ot$ ,  $l$  and  $a$  are the scalar of the vector  $\mathbf{vt}$  and the constant that is used in a function of parabola,  $z = f(x) = ax^2$ , respectively. Thus, the tool attitude is obtained by the suitable value  $p$  of Equation (4) on the basis of values,  $\zeta$  and  $l$ .

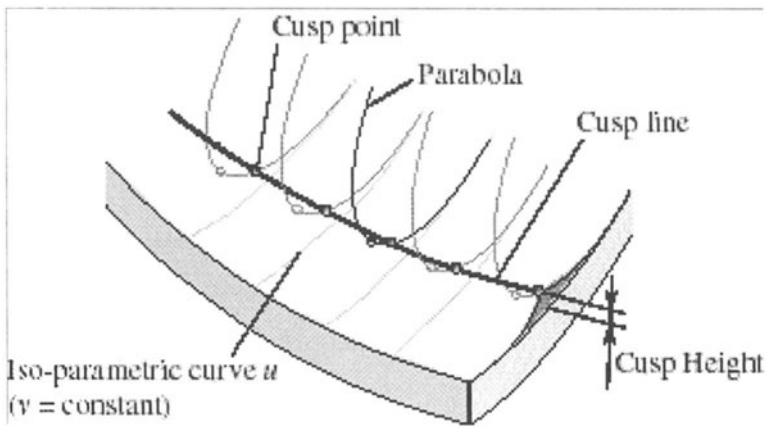


Figure 7. Generation of Cusp Line

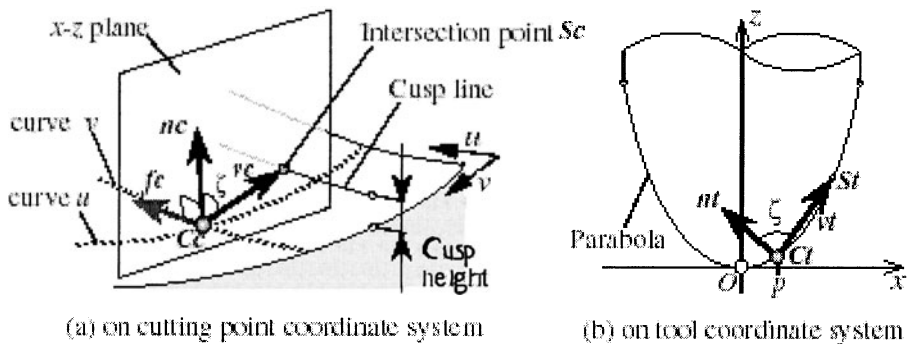


Figure 8. Decision of tool attitude based on the cusp line

### 3.3 CL data with smooth change in tool attitude

By searching sets of possible cutting point and tool axis on an *iso-parametric curve*  $u$  ( $0 \leq u \leq l, u = \text{constant}$ ) in consideration of the tool interference, the sets of parameters  $u, v$ , and  $p$  can be obtained, which determine the tool attitude in Equation (4).

First, the possible cutting range is searched on a curve  $u$ , where the tool contacts with the cusp line, as illustrated in Figure 9. At the time, we collect sets of parameters  $v$  corresponding to possible tool attitudes along each curve  $u$  and the parameters  $p$  that are  $x$  values of cutting points on the tool shape parabola. By interpolating obtained parameter sets in two-dimensional plane, the  $v$ - $p$  curve is generated, as shown as Figure 10. The above processes are repeated on several curves  $u$ , the obtained  $v$ - $p$  curves are arranged on the *imaginary three-dimensional orthogonal space*  $F(u, v, p)$  that formed by three coordinate axes,  $u, v$  and  $p$ , as illustrated in Figure 11.

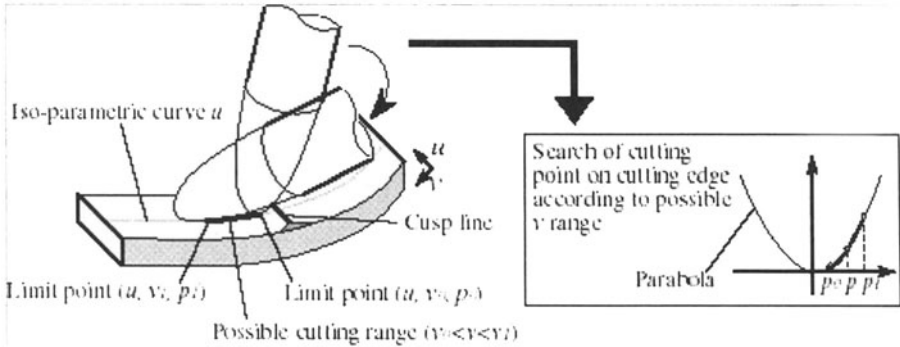


Figure 9. Search of possible cutting point

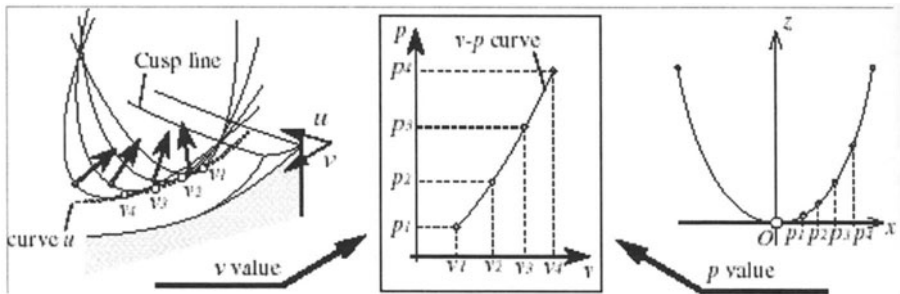


Figure 10. Generation of v-p curve

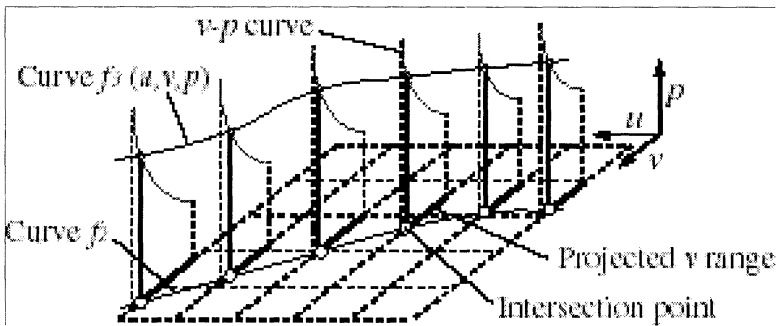


Figure 11. Three-dimensional imaginary space  $F(u, v, p)$  and curve  $f(u, v, p)$

### 3.3.1 Tool attitude in the last tool path

In this machining method, parameter values are unequal in the direction of pickfeed, therefore it leads to the disconnection of tool path. Then, imaginary cutting points are added to the parameter  $u$ , where the path is disconnected.

At the initial stage, the tangent vector  $\mathbf{v}_t$  is calculated at the end point of the iso-parametric curve  $u$ , and the curve  $u$  is extended in the direction of  $\mathbf{v}_t$ , as illustrated in Figure 12. Next, the possible range of parameter  $v$  is calculated so as to form the assumed cusp line. After determining the cutting line on the basis of the possible  $v$  range including extended one

as the mentioned above, we can obtain the suitable last path without the undesirable retraction of tool.

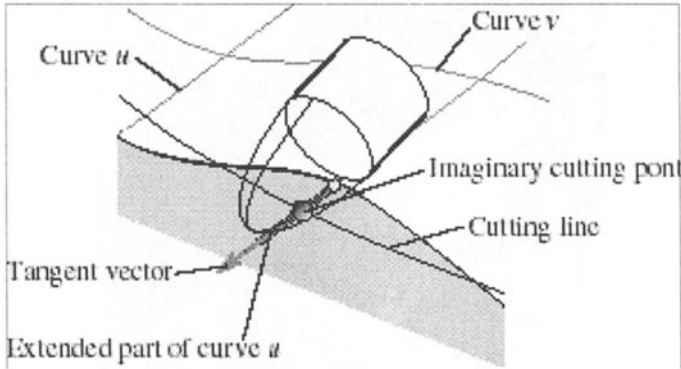


Figure 12. Supposition of imaginary cutting point

#### 4. APPLICATION TO SURFACE FINISHING

In order to confirm whether the extension of the method is effective or not, a simple experiment was conducted with a sculptured surface defined by a three-dimensional geometric model, *Ricoh DESIGNBASE*<sup>5</sup>. The shape of paraboloid end mill used for the experiment was designed by appointing a coefficient  $a = 0.5$  and the tool radius,  $R = 6mm$ , as already shown in Figure 2. Utilising the main-processor extended in this study, the suitable CL data can be generated automatically on the basis of the specified strategy, i.e. the regular cusp height. The calculated CL data was transformed to NC data by a *post-processor* that was previously developed<sup>6</sup>, and then the experiment was carried out on a five-axis control machining center.

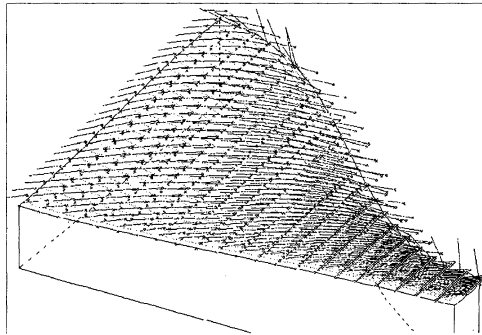


Figure 13. Generated CL data for milling with conicoid end mill

The surface was machined with the paraboloid end mill, and compared with that machined with a conventional  $R5$  ball end mill. The CL data for machining with paraboloid end mill is calculated so as to keep each cusp height  $10\text{ mm}$  constant, as shown in Figure 13. On the other hand, the CL data for ball end mill is generated so as to make the maximum cusp height below  $10\text{ mm}$ . The workpiece material is a duralumin, *A7075*. Figure 15 shows the actual five-axis control milling on the machining center.



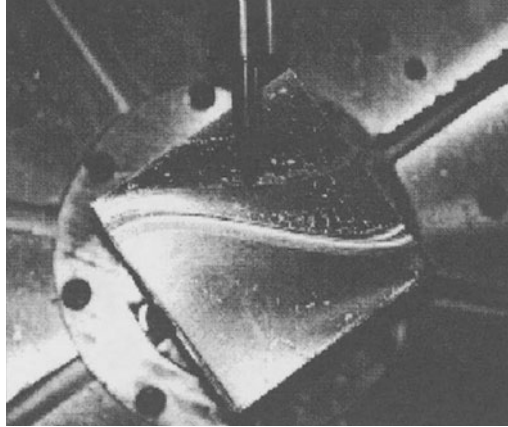


Figure 14. Five-axis machining with conicoid end mill

Table 1 lists the comparison of the cutting length. It is found that the use of paraboloid end mill drastically reduces the length. Figure 15 (a) and (b) show the surface appearance for each machining method. On the surface of Figure 15 (a) that is machined with the paraboloid end mill, the cusp lines are found as we assume in the calculation process. Also, the smooth and continuous change in the each pickfeed amount was confirmed. On the contrary, the ball end mill requires the very small amounts of pickfeed to make each cusp height below the specified height, which causes the reduction in the machining efficiency.

Table 1. Comparison of tool path length

End mill	Path length
Paraboloid	5565 mm
Ball	8856 mm

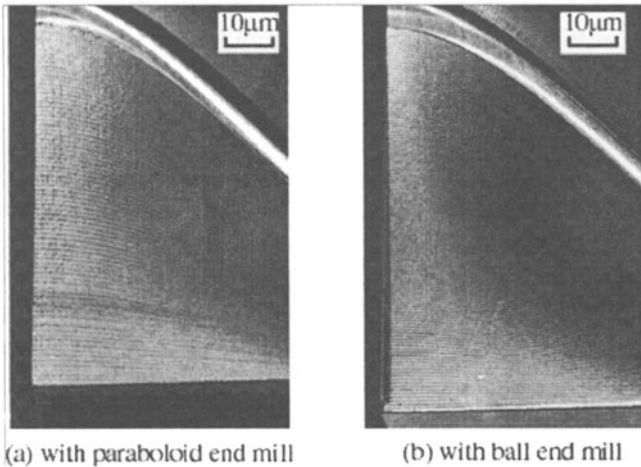


Figure 15. Machined Surface

## 5. CONCLUSIONS

In the study, to achieve the efficient surface finishing without the deterioration in the accuracy, we proposed the five-axis control machining by using a conicoid end mill. The main-processor developed on the basis of devised method allows us to automatically generate CL data for the surface finishing with paraboloid end mill. As a result of several cutting experiments, the reduction in cutting length was obviously found, compared with the machining with ball end mill, which indicates the improvement of machining efficiency.

## REFERENCES

- 1 Vickers, G. W., and Quan, K.W., (1989) Ball-mills versus end-mills for curved surface machining, *Journal of Engineering for Industry, Transactions of the ASME*, 111, 22-26.
- 2 Choi, B. K., Park, J. W., and Jun, C. S., (1993) Cutter-location data optimization in 5-axis surface machining, *Computer-Aided Design*, 25, 377-386.
- 3 Kruth, J.-P., and Klewais, P., (1994) Optimization and Dynamic Adaptation of the Cutter Inclination during Five-Axis Milling of Sculptured Surface, *Annals of the CIRP*, 43/1, 443-448.
- 4 Takeuchi, Y., Morishige, K. and Nakakarumai, N., (1997) Effective 5-Axis Control Machining Using Conicoid End Mill, *Rapid Product Development*, Chapman & Hall. 313-322.
- 5 Toriya, H. and Chiyokura, H., (1993), *3D CAD - Principle and Applications*, Computer Science Workbench, Springer-Verlag, Tokyo.
- 6 Takeuchi, Y and Watanabe, T., (1992) Generation of five-axis Control Collision-Free Tool Path and Postprocessing for NC Data, *Annals of the CIRP*, 41/1, 539-542.