16 The computer in mathematics teaching: scenes from the classroom

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Abstract

In this paper we analyse, through a case-study approach, the role assigned by mathematics teachers to the use of educational software. We considered cases in which teachers autonomously chose and used the software. Our analysis was carried out by the direct observation of classroom activities. Two areas of particular note were those of how the computer was used, e.g., in the development of mathematical knowledge and/or for reinforcing ideas already taught, and the nature of the teacher-pupil classroom discourse. Results indicate that even experienced users make limited use of the exploratory potential of the technology, and the dynamics of communication (teachers, pupils, computer) is still very much that of a 'teacher directed and led' classroom.

Keywords

Classroom practice, software, integration, teaching methods, attitudes.

INTRODUCTION

This paper is based on research we have conducted over a number of years which considers the impact of computers on mathematics teachers' behaviours and beliefs. The focus in this work has been at the upper secondary school level (students aged from 14 to 19). Initially we studied the conceptions that mathematics teachers have developed about the introduction of basic elements of

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informatics in their schemes of work. These conceptions have been compared with the conceptions teachers have of mathematics and its teaching (see Bottino and Furinghetti, 1996a).

At present we are studying how mathematics teachers use educational software in their classroom work. Here the term educational software is used to denote both packages explicitly designed with didactic aims and other more generic software, such as, for example, spreadsheets, when used for educational purposes. While studies which consider teachers' attitudes towards the use of software in mathematics education have been reported in the literature, in general their focus has been on teachers who have been trained in the use of specific software packages and then guided in the development of classroom experimentation. Our work differs in that the context is one in which software is autonomously chosen and used by teachers in their classroom work, that is to say, their choice and use is independent of guided and controlled experimentation.

The main phases in our programme of work can be outlined as follows:

1. A general investigation (through questionnaires) aimed at collecting data on the use of educational software in mathematics teaching at upper secondary school level (diffusion, type of software used, topics afforded, etc.).

2. An analysis of interviews conducted with a significant sample of teachers aimed at providing initial insights on teachers' expressed/perceived beliefs and choices in the use of educational software.

3. Case studies involving classroom observations aimed at providing rich descriptions of teachers' behaviour when using software in school practice.

4. A synthesis aimed at illuminating emerging beliefs and conceptions.

Results in Stage 2 indicate that the range of software used is quite limited (see Bottino and Furinghetti, 1996b). The most widely used tool is the spreadsheet which would appear to be appreciated because of its potential use in a number of topic areas. Symbolic manipulation systems (especially Derive) have a certain degree of diffusion among the teachers most experienced in the use of the computer, with the topic areas mainly in the field of calculus and analytical geometry.

The present paper concentrates mainly on Stage 3, the classroom observations. Here we take into consideration issues related to both educational computing and mathematics education:

- from an educational computing perspective, the work provides elements for the analysis of the role that computers can assume in the teaching/learning processes; and
- from a mathematics education perspective the research provides a framework for analysing teachers' beliefs and conceptions on mathematics and its teaching (see also Hoyles, 1992); moreover it offers elements for informing teacher education and training as well as the work of curriculum developers.

METHODOLOGY

The research involved classroom observation of computing activities with the focus on teacher behaviour. Two experienced mathematics teachers, competent in the classroom use of computers, one from each of two classes with the same curricula (where the mathematics program is rather rich) were observed on a number of occasions. The teachers both used the computer quite regularly during the school year (approximately one hour per week). They agreed to host us in their classrooms when the computer would be used. The teachers were also interviewed prior to the lesson observations. With each of them we chose a topic that would be developed when we would be present. In each situation, the lessons carried out with the computer took place in a laboratory equipped with about 10 personal computers (connected with a local network) and a printer. Both researchers were present in the lessons and took notes separately and independently without intervening in the activities.

Here we present the observations made during different lessons grouping them in two 'scenes', one for each teacher. The observations were collected and analysed according to a number of issues grouped into the four main areas listed below. We identified these areas as crucial in studying how the use of technology affects the way in which mathematics is taught, the way in which teachers perceive their role in classroom and the types of interactions which take place.

Setting

- school, classroom level, students' age, number of computers available in the laboratory
- presence of a computer-laboratory assistant (who takes care of the technical aspects of the work)
- topics developed in the computer laboratory
- software used

Role of the computer in the development of mathematical knowledge

- types of problems and the ways in which they are proposed
- types of mathematical activities: exploring, conjecturing, verifying, ...
- autonomy left to students (in mathematics and in the use of the computer)
- types of answers required from the students
- integration with classroom activities
- support for laboratory work—in addition to software tools, e.g., manuals, written worksheets, ...

Social interaction

- team-work: spontaneous aggregation or suggested by the teacher, responsibility, ...
- guidance offered by the teacher to the development of the activity: to give a general outline, to guide step-by-step, to offer a spectrum of possibilities, ...

- time left to students
- teacher's behaviour: stays at the blackboard, passes among the computer desks, asks the students individually or poses questions to all the class, discusses the answers or considers only the right ones, ...
- attention paid by the teacher to the students' behaviour during the work
- role of the laboratory assistant (if present)

Assessment and evaluation

- evaluation of the laboratory work: focus on 'computer abilities', on mathematical abilities, on both, ...
- quantitative and/or qualitative assessment
- attention paid by the teacher to students' response during the work: if they follow the activity, if it is necessary to help some of them individually, ...
- how the students' work is collected and stored

SCENES FROM THE CLASSROOM

Scene A

The setting is a fourth-year class of 20 students (aged 17) of a technical institute. A computer-laboratory assistant is present during the work. The teacher leaves the students free to organise themselves in groups: the majority form groups of two, some form groups of three, two students work alone. The topic studied is 'function'; the notions of function and its graph have been already developed without the use of the computer. The educational software used are Derive and Lotus; students have already used both.

The teacher writes on the blackboard the function

$$f(x) = x^3 + x^2 + x$$

Students are asked to use Derive to find its first and second derived functions and to graph them. Then they are asked to make conjectures about the graph of f starting from the derived functions.

Standing at the blackboard, the teacher asks the students about some mathematical notions useful to solve the exercise and some technical issues about the use of Derive. Since students do not remember some points about Derive (derivatives, variables, notations, ...), she gives a general account of them.

Students begin to work with the computer. The teacher walks around the desks. Since some students try to directly graph the function with Derive, the teacher, a bit disappointed, asks them to withdraw and to follow the instructions given. The teacher gives some indications about how to plan out the organisation of the screen (axis, unit of measures, etc.) in order to obtain 'good' graphs. She tries to fix students' attention on some elements that can be useful for making conjectures (students at the computer can see both the graph and the algebraic expression of the functions); the conjecturing process is directed by the teacher with such questions as: "Has the second derivative zeros? Well, then, what is the

concavity of the function like? ...". The teacher every now and then sums up orally for all the students. She does not pay particular attention if students do or do not use strategies that take better advantage of software opportunities. The laboratory assistant intervenes only when students ask technical questions.

The teacher gives a new problem:

> "Find the zeros of the function $f(x) = x^3 + x - 1$ in the interval [0, 1], with an error less than 0.01".

She suggests that they use Lotus. Some students ask if they may solve the problem using Pascal. The teacher agrees.

Students do not remember Lotus and Pascal very well and look frequently at their written notes. The teacher, going around the desks, intervenes on the mathematical aspects (recalling what has been done before without the use of the computer); the computer laboratory assistant gives suggestions about Pascal programming and about the use of Lotus.

Each group of students prints the work developed on the computer; printouts are collected in a folder. The teacher tells us that during oral examinations on these topics she does not usually look at these folders and that she does not ask questions about the work done with the computer.

Scene B

The setting is a third-year class of 19 students (aged 16) of a technical institute. There is no computer-laboratory assistant. The teacher asks the students to form themselves in groups of two. The topic developed is geometrical plane transformations; this topic has already been studied without the computer (the teacher states that it is necessary to first develop topics without the computer to ensure they understand what they are doing in the laboratory sessions). The software used is Derive; students have used this previously.

Standing at the blackboard, the teacher reviews some ideas about reflections in the plane (reflections in the origin and in any point of the plane, corresponding equations, ...); she poses some questions to the class, only some students answer, she goes on quickly; students take notes on their exercise-books while the teacher is at the blackboard. She asks them to use Derive to verify the mathematical concepts she has outlined. For example, she asks them to use Derive to investigate the reflection in a point of the plane. After students have worked out with an assigned straight-line and with assigned points, the teacher asks them to infer a general rule using the algebraic functions of Derive. The teacher goes around the desks only when the students have difficulties with the computer, otherwise she remains at the blackboard.

The teacher gives another problem: "Find the reflection of a parabola in a point of the plane". The teacher does not ask the students to make conjectures about the result; students, following her suggestions, go on graphing a given parabola and its reflection in a given point. When they have obtained the graphs

she asks them to conjecture if it is possible to generalise. She guides them in finding the generalisation.

Few students follow the work tempo of the teacher; she does not look at the work of all the groups and does not make adjustments if some students fall behind.

COMMENTS

Scene A

The computer was not integrated in the mathematics teaching in the sense of creating new 'scenarios', but was utilised for enriching topics previously taught. The teacher proposed a traditional type of problem. The use of the computer prompted activities such as working with images to explore properties of functions. The contemporary use of two registers (graphical and symbolic) is interesting. The teacher seemed aware of the problem played by symbols seen as procepts (concepts + processes) and used the computer to pass from the processes (e.g. calculation of derivatives) to the concepts (e.g. meaning of the sign of the derivative and its relation with the graph of its primitive). We can interpret this behaviour as an attempt to promote the 'flexible meaning of symbols', as described in Gray and Tall (1993).

The computer would seem to have been used to 'scaffold' students' learning of mathematical topics through the provision of concrete support for their progression, step-by-step, through the activities. See Kutzler, 1994, for further discussion on the use of Derive in relation to the theoretical aspects of scaffolding.

The teacher directed students' interaction and reflection on their actions according to the feedback provided by the computer (she provided some indications as to how to plan graphics on the screen, fixed their attention on elements useful for making conjectures, summed up orally, etc.). The phase of free exploration was limited (as a matter of fact, the first exercise was not very suitable for development through free exploration). It would seem that the computer was perceived as a learning aid that helps in structuring knowledge.

Research, such as that reported in Artigue (1995) and Balacheff (1991), has stressed that the computer, acting as a third pole in the dynamics of the classroom, can change the traditional relationship between the student and the teacher. In Scene A, even if the teacher maintained a prominent role in classroom interaction, students were given a certain degree of freedom in the activity with the computer. This was evidenced, for example, by the fact that they were allowed to use software which was different from that chosen by the teacher (Pascal instead of Lotus). This was an indication that the teacher was flexible in her interactions with the students and was confident in her ability to establish a fruitful communication which was independent of the software used. We observe here that the presence of the laboratory assistant freed the teacher from any anxiety

which might have arisen in regard to the use of the technology (hardware and software).

Explicit acknowledgement of the existence of two types of skills were exhibited in the teacher's behaviour; those related to mathematics and those related to the use of the computer. She ascribed importance only to the first of these. The way in which assessment was performed also reflected this attitude.

Scene B

Two key elements orientated our analysis of this scene: the way in which the teacher carried out the communication with the students and the time allowed for students to answer questions. She stood at the blackboard and asked questions of the students collectively for the majority of the time. Individual dialogue only occurred when students made specific requests for help. The teacher explained the topic at hand and asked the students to use the computer whenever there was a need to use examples to verify a conjecture or to find a counterexample. The time allowed for answering questions was very short.

The communication with the students was limited and did not seem affected by the use of the computer. The teacher did not consider all the students' answers, only taking into account the first correct response and then continuing with her discourse. She often drew sketches on the blackboard of what she presumed students had obtained with the computer, with the students taking notes, etc.

The teacher does not seem convinced of the fact that the use of the computer can be really effective for students' mathematics learning (see, for example, her claim that it is necessary to first develop the topics without the computer since otherwise they may not have a real understanding of what has taken place). The computer is perceived as a teaching aid but its impact on classroom reality is very limited.

CONCLUSIONS

The two scenes described above provide insights on the teacher's 'every day life' in school. They demonstrate that somewhere between the categories of teachers who refuse to use the computer and those who carry out advanced experiences using it, there is an intermediate level of teachers who, step-by-step, are building what probably will be the standard use of educational software in mathematics secondary school teaching for the majority of teachers. While they will have integrated the computer in their teaching, exploiting some of the opportunities brought about by this innovation, a large part of the potential is likely to be neglected.

Both of the teachers in this research took into consideration the use of the computer to promote students' visualisation capabilities. In one case (Scene B) the focus was more on an exemplification of the mathematics concepts (e.g. reflections in the plane) while in the other (Scene A) it was more on the students'

personal observations (e.g., inferring properties of a function from the properties of its derivative functions). In this second case (Scene A) students were given a certain degree of responsibility in that they were able to choose the software to work with. In Scene B the teacher kept complete control of the situation as it appeared that she was not confident about allowing students to work on their own and hence deemed it necessary to assume a role which might be described as 'prescriptive'.

In both the scenes the use of the computer was strictly adapted to pre-fixed objectives in mathematics, the alternative of starting from the opportunities offered by the computer to pursue mathematics objectives, was not followed. This explains, for example, why the activities of conjecturing and exploring were only partially contemplated or even not exploited.

The two teachers conceived the computer as a tool and assigned it a secondary role as a mediator in the teacher-student relationship. This attitude was particularly evident in Scene B, but also occurred in Scene A, in that the reciprocal role of the teacher and of the students was not changed. For example, in no case did the teacher assume a role of co-explorer, but rather remained in the position of a 'knowledge keeper'.

In the interviews the teachers pointed out the heavy constraints that the school system had on their behaviour. They particularly stressed the problem of student assessment and the unfavourable ratio among the number of topics present in the mathematics curriculum and the time allowed to develop them. This form of curriculum-anxiety could be one of the elements that hinders the exploitation of the new forms of communication that are now possible with the computer.

The results of our work provide some indications as to possible strategies for use in training and re-training. In Bottino and Furinghetti (1995) we stress the importance of starting from the mathematical difficulties encountered by the students in planning for the use of educational software in the classroom. Here we add the need to make teachers aware of the opportunities offered to classroom work by the new forms of communication.

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