A simple method for surface interpolation by means of spline functions

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Abstract

The paper describes a simple and computational-effective method for non-mathematical surfaces interpolation by means of spline functions. The method is intended for a CNC machine working environment in which the machines must create the surfaces using standard trajectory interpolators, capable of describing straight lines and circular arcs, thus allowing this kind of job to be performed on relative low cost machines, affordable to SMEs.

Keywords

CNC machining, surface interpolation, spline functions.

1.- INTRODUCTION

In the auto and aero industries, a standard shop procedure involves the machining of parts and pieces that have not a simple mathematic model, and whose surfaces are the result of empiric procedures, product of wind tunnel experiences or styling constraints. Those surfaces are generally known as point-to point models.

The purpose of this paper is to present a simple tool capable of making those surfaces machinable using CNC machines with simple (i.e. capable of straight lines and circular arcs) interpolators.

The interpolation method is outlined and the translation process (post-processing) is described.

2.- SURFACE DEFINITION.

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Several methods can be used to define a non-mathematical surface.

They differ from their approach, presenting piecewise interpolations or curve (i.e. sections) definition. Many of them are described in the works of Bèzier (1972), Giloy (1978) and Coons (1967).

2.1.- Curve definition of surfaces. It consists of a set of plane sections of a given surface, defined usually at constant intervals, measured on one of the coordinate axes of the figure, thus generating a set of planes parallell to the plane formed by the remaining coordinate axes as shown in Fig. 1. These sections are defined by means a set of points, usually measured on a model.

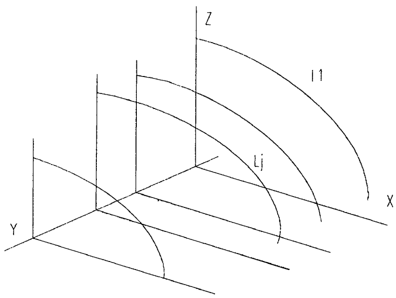

Figure 1
Surface definition by means of plane sections

In such a model, if the function defining a particular curve has coefficients of the form a_{ij} and the function representing section j is:

$$\sum_{i=0}^{n} a_{ij} x^i$$

then, for every y a function of the form

$$a_i (y)= \sum_{i=0}^{p} b_{ij} y^j$$

exists, and its value becomes a_{ij} when *y* has the value j.

Then, for each value of the coordinates x and y there is a value of z such as

$$z(x,y) = \sum_{i=0}^{n} [\sum_{i=0}^{p} b_{ij} y^j] x^i$$

The particular characteristics of the interpolation depend on which kind of curves i,j are chosen.

2.1. Surfaces defined by means of spline functions. It is not the purpose of this work to discuss spline functions, but some of the properties of these functions shall be noticed as discussed in the work of Ahlberg et al. (1967) and Niell (1978).

- They are a form of polinomial interpolation, usually of third order.
- The interpolation they produce mantains the continuity of the first and second order derivative on the knots.
- They are the smoothest interpolation possible for a certain set of data.

The last property means that the integral

$$B \int_l s^2 \, dl$$

is a minimun, being B the stiffness of the material drawing the curve, s its curvature and the integral is evaluated along all the lenght of the curve.

This set of properties make splines useful for the interpolation of surfaces of smooth curvature, typical of the auto and aero industries. The fact that the method provides first and second order derivative continuity makes it suitable for some fluid dynamic problems.

2.2.-Surface edge definition. To obtain an accurate interpolation of the surface and its derivatives with respect a certain axis, the first derivative with respect of the same axis of the line limiting the surface must be obtained. For convenience the coordinate axes are used for this purpose.

To obtain the values of the surface boundary, as shown in fig. 2, these values must be known:

* The value of the first derivative with respect to one given direction, in at least three points of the boundary curve.
* If there are corners, all the possible values of the second derivative with respect to both axes.

Thus, according with Fig. 2, for interpolating the interval [1]...[4], these values must be known:

$$\left.\frac{\delta z}{\delta x}\right|_1$$

$$\left.\frac{\delta z}{\delta x}\right|_4$$

$$\left.\frac{\delta z}{\delta x}\right|_b$$

$$\left.\frac{\delta^2 z}{\delta x \delta y}\right|_1$$

$$\left.\frac{\delta^2 z}{\delta x \delta y}\right|_4$$

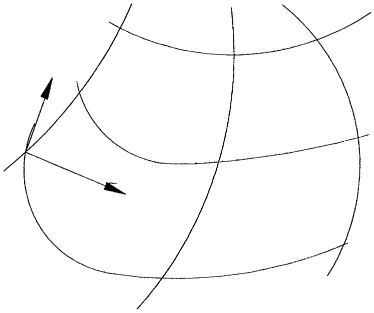

Figure 2.
Values of the boundary of the curve.

2.3.- Outlline of the method. The method used for surface interpolation is:

* A number of sections are established on the surface.
* A set of points are measured on each section.
* The derivatives on the surface boundary are obtained.
* The lines of the sections defining the surface are interpolated by means of splines.
* The surface is checked for regularity, i.e. that the sections are uniformly spaced. If it is not so, a new set of sections are added.

The first two steps are intuitive. The needed derivatives on the surface boundary are obtained applying Taylor difference expansion as:

$$f'i = \frac{f(i+\Delta) - f(i-\Delta)}{2}$$

where f is the function and Δ its increment.

It can be shown that the error of this estimation is of the order of Δ^2. Selecting a proper value of Δ, an acceptable error can be obtained.

Using the more compact expression for the numeric derivative:

$$f'i = \frac{f(i+\Delta) - f(i)}{2}$$

the order of the error rises to Δ, but calculations are more straightforward.

Making computations with this later expression, for a corner like the ones shown on Fig. 3, the values obtained are:

$$f'x_{|1} = f(h) - f(a)$$

$$f'y_{|1} = f(b) - f(a)$$

and:

$f'x_{|1+\Delta} = f(i) - f(b)$

$f'y_{|1+\Delta} = f(i) - f(h)$

and :

$f'' xy|_1 = f'x|_{1+\Delta} - f'x|_1$

$f'' yx|_1 = f'y|_{1+\Delta} - f'y|_1$

that can be rewriten as:

$f'' xy|_1 = f(i) - f(b) - f(h) + f(a)$
$f'' yx|_1 = f(i) - f(h) - f(b) + f(a)$

For increments corresponding to .1% of the function value, the derivative error is about 1%.

After obtaining the derivatives on the boundary of the surface, the line defining the section is interpolated. When all the sections are done, the resulting array of data is checked for regularity, and the missing sections are interpolated using the neighbor's data as input.

The resulting figure is a regular mesh of points, closely rendering the surface.

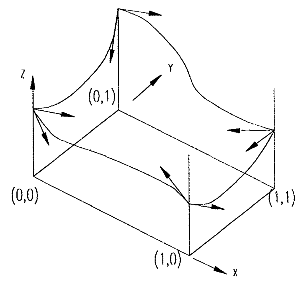

Figure 3.
Corners of the surface.

3. POSTPROCESSING

In the context of CNC machining, the term 'postprocessing' is the name of the procedure by mean of which, a set of data representing a geometric lucus is translated to a form suitable for a numeric control. This means to replace the original data for a series of straight lines and arcs of circle that 'fit' the desired curve within a certain error.

In the case of curves of non-mathematic description, a new problem arises: they have arbitrary spatial shapes and it poses a special problem for machining. Usually, a CNC machine has ways to control the velocity of its axes, thus being capable of generating a curve in space. In order to describe an arbitrary surface, aside of the contour of the piece, the machine must control the tool's attitude, that is, the angle it forms with the part being cut.

To attain such control, two aditional axis are added, each one being an angle that controls the spindle position. To govern these axis, the curvature of the surface must be translated into the laws of motion of these angles, usually lines and arcs being good aproximations.

The rules of motion of the auxiliar axes can be obtained as follows:

* The curvature of a surface can be described as the geometric composition of the curvatures of a set of two orthogonal axes.
* The direction of the curvature radius along one of the orthogonal axis is normal to the first derivative (in the geometric sense) of the curve with respect to that axis.
* The direction of the curvature radius of the surface on a given point, must be normal to the plane defined by the two first derivatives with respecto to the set of orthogonal axis in that point.

As result of he procedure described, it is possible to know the curvature of the surface in two directions. Of course the directions chosen are coincident with the movement of the servos controlling the angular displacements of the spindle of the CNC machine.

To obtain a suitable error, usually it is necessary to perform linear interpolations between points of known curvature, generating controlled rotations on the auxiliar axes of the machine. As linear interpolators are very simple to produce, it is possible to attain a very simple control procedure for machining spatial surfaces.

The programs implementing the algorithms were written in FORTRAN and they run with acceptable machine times on most current PCs.

4. CONCLUSSIONS

As a very simple way of interpolating surfaces, these programs make job preparations easier for some cases of metalworking processes.

The algorithms used are based on methods outlined in the references, and whose main characteristic is their simplicity and efficiency. The overall method is intended to work on small PC machines of the kind usually found in thechnical departments of most metalworking shops.

The heavier job is the postprocessing of the interpolated data. Depending of the error assumed as maximun, the process can last up to fifteen minutes for a 300 x 300 mm simple surface, with a mesh of one hundred points measured on it. The time corresponds to a 486 DX-based machine running at 66 Mhz. The same surface, with 200 points measured on it, is interpolated and the CNC programs generated by the post processor in six minutes of work on a 100 Mhz Pentium PC with only 8 MB of memory, using a LAHEY Fortran compiler running on DOS. This job preparation runs off line, thus using no CNC machine time.

Even with that penalty, it is possible to use a low cost machine for the process. The outcome of the whole system is a set of data that can be used to drive a CNC machine capable of performing only simple (straight lines and circular arcs) interpolations. All modern CNCs are capable of those, so with the addition of two controlled angles, driven each one as a CNC axis, the labor of machining "sculptured surfaces" can be faced at a reasonable cost. Applications for this process are found in the auto industry, specially in the construction of dies for body parts pressing.

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