On the Performance of the Unslotted CDMA-ALOHA Access Protocol for Finite Number of Users, With and Without Code Sharing

A. Kumar and D. Patil
Dept. of Electrical Communication Engineering
Indian Institute of Science
Bangalore - 560 012, INDIA
e-mail: anurag@ece.iisc.ernet.in

Abstract

We consider a system comprising a finite number of nodes, with infinite packet buffers, that use unslotted ALOHA with Code Division Multiple Access (CDMA) to share a channel for transmitting packetised data. We propose a simple model for packet transmission and retransmission at each node, and show that *saturation throughput* in this model yields a sufficient condition for the stability of the packet buffers; we interpret this as the *capacity* of the access method. We calculate and compare the capacities of CDMA-ALOHA (with and without code sharing) and TDMA-ALOHA; we also consider carrier sensing and collision detection versions of these protocols. In each case, saturation throughput can be obtained via analysis of a continuous time Markov chain. Finally, we also present some simulation results for mean packet delay.

Keywords

Optical CDMA, CDMA vs. TDMA, stability of multiple access protocols

1 INTRODUCTION

Direct Sequence Code Division Multiple Access (DS-CDMA, or just CDMA) is a technique for sharing the bandwidth of a channel among several sources of digital bit streams. In this paper we are concerned with the bandwidth efficiency, when the CDMA technique is used by several nodes to transmit packetised data. In particular, the question that we seek to answer is motivated by the following discussion.

S. V. Raghavan et al. (eds.), *Computer Networks, Architecture and Applications*

Suppose that the sources that need to share the channel require an access rate (or burst rate) of $\frac{1}{T}$ bits per second (say, 10 Mb per sec), i.e., each information bit emanating from a source occupies T secs (e.g., 0.1 μsec). The channel to be shared has a bandwidth of B Hz. The space of signals with bandwidth B Hz, and time duration approximately T secs, has dimension approximately $2BT$ [Lee *et al.*, 1988]. In this signal space we need to choose L orthogonal signals that can be practically used for CDMA signalling. It has been found [Salehi 1989] that to do this, a signal space dimension (i.e., $2BT$) much larger than L is required. For example, $2BT = 100$ may be required for getting L=10 useful orthogonal spreading sequences. Here $2BT$ becomes the spreading factor for CDMA; thus for L=10, we need a channel bandwidth $B = \frac{100}{2T}$, and a CDMA "chip" rate of $\frac{100}{T}$ chips per sec. Now this channel of bandwidth B can be used for direct digital signalling at about $2B$ bits per second (by the Nyquist criterion, [Lee *et al.*, 1988]). In the running example we have taken, $2B = \frac{100}{T} = 1000$ Mb per sec, i.e., a direct signalling rate of 1 Gb per sec.

The question therefore arises: why not use this channel directly to implement a TDMA system at a bit rate of about $2B$ bits per sec? This paper is motivated mainly by this question, i.e., we wish to compare the performance of random multiaccess of a channel by packetised traffic, when the channel bandwidth is shared using CDMA or TDMA. As one of the advantages ascribed to CDMA is the ease of asynchronous implementation [Green, 1993], for a fair comparison, in both cases random access is assumed, i.e., we compare CDMA-ALOHA and TDMA-ALOHA. We consider *unslotted ALOHA*, without and with carrier sensing (CS) and/or collision detection (CD).

We consider TDMA-ALOHA to be a special case of CDMA-ALOHA, i.e., one in which no more than one successful transmission can exist at one time on the channel. On bandwidth limited channels we can, following the argument above, assume that the TDMA system can be operated at a bit rate equal to the CDMA chip rate. All optical CDMA ([Salehi 1989], [Prucnal *et al.*, 1986]), however, can run at a chip rate much higher than the rate at which light pulses can be modulated and demodulated by electronics. Hence, when considering all optical CDMA-ALOHA and TDMA-ALOHA, TDMA bit rate may be less than CDMA chip rate. Consequently, in our analysis we normalise all times to packet transmission times, thus eliminating the factor of bit transmission rate. The transmission rate is factored back when making comparisons between CDMA and TDMA.

We consider the situation in which a finite number of nodes, with infinite packet buffers, share the channel. A simple model for attempts and retransmissions is used. The main contribution of this paper is the saturation throughput analysis with and without *code sharing* (i.e., more than one node transmits using the same spreading code), and with and without carrier sensing and collision detection. We show that saturation throughput yields a *sufficient* stability condition for the queues. Hence saturation throughput may be considered to be the *capacity* of the multiple access system, when there is symmetric loading of all the channels. We compare the capacities of various versions of CDMA-ALOHA and TDMA-ALOHA, including versions with collision detection and carrier sensing. Finally, we also show some results for mean packet delay, ob-

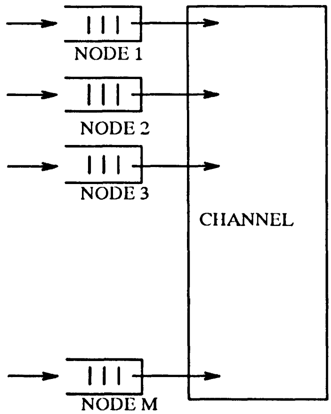

Figure 1: Model for a CDMA system with M users: The packets arriving at each user wait in a queue until they are transmitted on the channel.

tained using simulations.

There is a long history of analyses of random access systems. The work that is related to ours is the stability analysis of buffered finite user ALOHA (see [Sharma, 1990] for a list of references). The most general result for the stability of slotted ALOHA is presented in [Sharma, 1988]. See also [Szpankowski, 1990] for a survey of other related references. The result in [Sharma, 1988] basically asserts that saturation throughput yields a *sufficient* stability condition for finite user buffered slotted ALOHA. This result is easily extended to our model of unslotted CDMA-ALOHA.

In [Raychaudhari, 1981] the performance of slotted CDMA is considered. The number of nodes is finite but the queue length processes are not modelled. Throughput analysis is carried out for the case in which arrivals are blocked at backlogged nodes.

2 A MODEL FOR CDMA-ALOHA

There are M users and L CDMA codes, with each code being shared by $K = M/L$ users (it is assumed that K is an integer). The codes are assumed to be transmitter oriented, i.e., each user transmits on its own code and receives on all codes. Code-sharing (i.e., $K > 1$) reduces the complexity of the optical correlation receivers; we wish to study its impact on traffic performance. CDMA is interference limited [Lee *et al.*, 1988], and the bit error rate at the receiver deteriorates as the number of simultaneous transmissions on the channel increases. We assume that there is a desired bit error rate (e.g., the bit error rate obtainable by a TDMA system on the same channel with the same transmitter power). The desired bit error rate is achieved for a particular *interference limit* N; i.e., if more than N transmissions (involving one or more codes) take place simultaneously, they are all considered to be "bad".

The CDMA system can be very simply viewed as shown in Figure 1. Each node or user has a queue in which packets arrive and wait. The box representing "the channel" is a complex "server" that serves the head-of-the-line (HOL) packets in the queues. When a HOL packet from a user queue is transmitted, one of three things can happen to it:

1. It may be rendered "bad" owing to excessive interference if the number of active transmissions *at any time* during the transmission exceeds N.

2. It may collide with another packet transmitted by a user with the same CDMA code.

3. It may complete transmission successfully.

If case (i) or (ii) occurs, the packet is retransmitted after a random delay. The process repeats until the packet is successfully transmitted. The user then accesses the server (channel) for transmission of the next packet.

Thus transmissions from each user are either fresh transmissions or retransmissions in response to channel feedbacks. It is extremely complicated to work with exact models of the attempt, backoff and reattempt algorithms. To obtain an analysable model, we assume that when a node has a packet to send (and the packet is not already in transmission) the rate at which the HOL packet is attempted is α. There is thus a "refractory" period between successive packet attempts. This period models the channel feedback delay, and the retransmission back-off delay. Further, we assume that the refractory period is exponentially distributed with mean $1/\alpha$. All times are normalised to packet transmission time, which is assumed to be exponentially distributed with mean 1. Hence when a user has a packet to send its behaviour is modelled by the 2 state continuous time Markov chain (CTMC) shown in Figure 2; state 0 is the refractory state between attempts and state 1 is the transmitting state. Note that our model assumes that a packet arriving to an empty queue also experiences the refractory period delay (incidentally, the same phenomenon occurs in probabilistic attempt models for slotted ALOHA). While such a delay may actually occur occasionally, as the node may be waiting for a previous acknowledgement, in general it renders the model somewhat conservative at light loads. At heavier loads, however, the model is quite appropriate.

Since each receiver has correlation receivers for all the transmitting codes, it is possible to implement carrier sensing and collision detection in the following way. Each receiver monitors the output of all the correlation receivers. If the number of active outputs is N or more, then transmission from this node is deferred; this is carrier sensing. In spite of carrier sensing, owing to propagation delay, excessive interference owing to more than N active transmissions can still occur. If each node monitors the channel while transmitting, then it can detect a collision if more than N receiver correlator outputs become active. It can then abort transmission. We only consider carrier sensing (CS) and collision detection (CD) for the case $L = M$ (i.e., $K = 1$, no code-sharing).

Clearly, propagation delay must be modelled when analysing the carrier sensing and collision detection alternatives. We model the effect of propagation delay approximately as follows. When the number of active transmissions

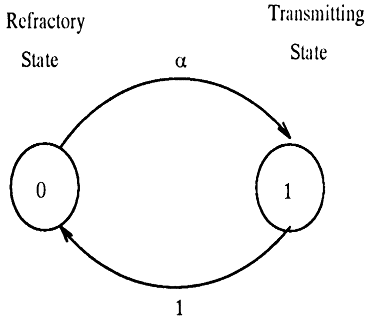

Figure 2: Channel attempt model for each user.

Table 1: Model parameters for the various access protocols; M is the number of nodes

	L	N
TDMA	1	1
CDMA, no code-sharing	M	> 1
CDMA, code-sharing	$< M$	> 1

becomes N then the system is said to be in a "vulnerable" state, as another transmission would corrupt all ongoing transmissions. We assume that after entering such a state, the system enters the safe state (in which all nodes have sensed the ongoing transmissions) after an exponentially distributed propagation delay with mean τ^{-1}; this models CS. Similarly when more than N transmissions are ongoing and hence are all corrupted, all transmissions get aborted after a propagation delay that is exponentially distributed with mean τ^{-1}; this models CD.

Note that, since we have normalised the rates to the packet transmission rate, the throughput will be obtained in packets per packet transmission time, and hence the actual transmission speeds need to be explicitly considered only when comparing TDMA and CDMA. Further, it is clear that from the point of view of our model, TDMA is just a special case of CDMA, obtained by setting $L = 1$ or $K = M$. Table 1 shows the values of L and N that yield the various access protocols that we wish to study.

3 SATURATION THROUGHPUT: THEORY AND ANALYSIS

For the queueing system of the type shown in Figure 1, saturation throughput is an important performance measure. Saturation throughput is defined as the channel throughput when the nodes always have a packet to send, i.e., the queues are *saturated.* In certain situations, saturation throughput has been shown to yield a sufficient stability condition for the queues.

The saturation condition yields the following model. Each user behaves as shown in Figure 2. When carrier sensing and collision detection is not being used then users *independently* alternate between the refractory state and the transmitting state. The saturation throughput of the system is accounted depending on the values of L and N. If during the transmission of a user, another user with the same code initiates transmission, then both users' packets are corrupted and they are counted as unsuccessful transmissions. If during the transmission of a user the total number of transmissions (of however many codes) exceeds N, then all participating transmissions are assumed to be unsuccessful. If neither of these events occur during the transmission of a packet, then the packet is counted towards channel (good) throughput. The overall throughput of good packets is then the saturation throughput. A two dimensional Markov chain can be defined for this model and its analysis yields the saturation throughput for CDMA without code-sharing, without and with CD and CS. We provide this analysis in Section 3.2. In Section 3.3 we provide a similar analysis for CDMA with code-sharing.

3.1 Significance of Saturation Throughput

We show, for CDMA-ALOHA, that, as may be expected, saturation throughput yields a sufficient stability condition for the packet buffers at the various nodes. We present detailed arguments for CDMA-ALOHA without code-sharing, or CD and CS. We use arguments similar to those used in [Sharma, 1988] and [Rao *et al.*, 1988].

We construct a "virtual attempt" process as follows. Consider M independent Poisson processes (indexed by $1 \leq j \leq M$) with rate $1+\alpha$ and epochs $\left\{T_0^{(j)} = 0,\, T_1^{(j)},\, T_2^{(j)}, \ldots, \right\}$. Associate with the interval $\left[T_{k-1}^{(j)},\, T_k^{(j)}\right)$ $(k \geq 1)$ of the j^{th} Poisson process the random variable $B_k^{(j)} \in \{0,1\}$. $B_0^{(j)} = 0$, and $\left\{B_k^{(j)},\, k \geq 2\right\}$ is a Bernoulli sequence, $B_k^{(j)} = 0$ with probability $(1+\alpha)^{-1}$ and $B_k^{(j)} = 1$ with probability $\alpha(1+\alpha)^{-1}$, and the sequences $\left\{B_k^{(j)},\, k \geq 1\right\}$, $1 \leq j \leq N$, are independent.

Define the random sequences:

$\left\{A_k^{(j)},\, k \geq 0\right\}$: $A_0^{(j)} = 0$, and, for $k \geq 1$, $A_k^{(j)}$ is the number of arrivals to the j^{th} node in the interval $[T_{k-1}, T_k)$.

$\left\{X_k^{(j)},\, k \geq 0\right\}$: $X_k^{(j)}$ is the queue length in the j^{th} queue at the epoch T_k.

$\left\{\overline{S}_k^{(j)},\, 1 \leq j \leq M,\, k \geq 1\right\}$: $\overline{S}_k^{(j)} = 1$ if $B_k^{(j)} = 1, B_{k+1}^{(j)} = 0$, and if during the run of ones, of which $B_k^{(j)} = 1$ is the last, at most $N-1$ of the $\left\{B_k^{(i)},\, i \neq j,\, k \geq 1\right\}$ processes were *ever* nonzero; otherwise $\overline{S}_k^{(j)} = 0$

$\left\{S_k^{(j)},\, 1 \leq j \leq M \quad k \geq 1\right\}$: $S_k^{(j)} = 1$ if a successful transmission attempt completes at $T_k^{(j)}$ in the actual queuing model; otherwise $S_k^{(j)} = 0$.

Now observe that owing to the assumptions that the time to attempt and the transmission time are exponentially distributed with rates α and 1, we have, for $k \geq 1$,

$$X_k^{(j)} = \left(X_{k-1}^{(j)} + A_{k-1}^{(j)} - S_k^{(j)}\right)^+$$

The only "tricky" point here is to verify that this equation is correct when an arrival joins an empty queue at a node. Suppose this arrival is to the j^{th} queue and arrives in the interval $\left[T_{k-1}^{(j)}, T_k^{(j)}\right)$. The earliest that it can get transmitted is the interval $\left[T_k^{(j)}, T_{k+1}^{(j)}\right)$. Thus it would already have waited for an amount of time distributed as exponential $(1+\alpha)$. It is attempted in the next interval with probability $\alpha(1+\alpha)^{-1}$, and not attempted with probability $(1+\alpha)^{-1}$. Hence the time until it is first attempted is distributed as exponential α.

Observe now, from the definitions above, that $S_k^{(j)} \geq \overline{S}_k^{(j)}$, $\forall j$, $1 \leq j \leq M$, $k \geq 1$, since, in the actual system, there is always a successful attempt that ends at an epoch at which $\overline{S}_k^{(j)} = 1$, but there can be successful attempts even when $\overline{S}_k^{(j)} = 0$. Using these bounds and $X_0^{(j)} = 0$, $1 \leq j \leq M$ we obtain

$$X_k^{(j)} \leq \left(\max_{1 \leq \ell \leq k} (\sum_{i=\ell}^{k} \left(A_{i-1}^{(j)} - \overline{S}_i^{(j)} \right) \right)^+$$

It then easily follows (see arguments in [Sharma, 1988], [Borovkov, 1976, pg.12]) that if the arrival processes $\{A_k^{(j)}\}$ are asymptotically stationary, and the "virtual service" processes $\{\overline{S}_k^{(j)}\}$ are asymptotically stationary, then $\{X_k^{(j)}\}$ converge in distribution to proper random variables provided that for each j, $1 \leq j \leq M$, the arrival rate is less than $(1+\alpha)\left\{\lim_{n\to\infty} \frac{1}{N} \sum_{k=1}^{N} \overline{S}_k^{(j)}\right\}$; but this latter limit is just the saturation throughput. The asymptotic stationarity of $\{\overline{S}_k^{(j)}\}$ is clear from the Markov chain analysis in Section 3.2.

3.2 Analysis for CDMA Without Code-Sharing

In the case of CDMA-ALOHA without code-sharing, there can be no collisions, but interference can arise when the total number of ongoing transmissions exceeds N, in which case all the transmissions are considered bad. We first do not consider carrier sensing and collision detection.

Let $x(t)$ denote the total number of ongoing transmissions, and $y(t)$ denote the number of these that are good. Consider the process $(x(t), y(t))$. Observe that, owing to exponential assumptions, this is a Markov chain on a finite state space.

$$S = \{(x,y) : 0 \leq x \leq N, 0 \leq y \leq x\} \ \cup \ \{(x,0) : N+1 \leq x \leq M\}$$

For $y(t) > 0$, a transition from $(x(t), y(t))$ to $(x(t)-1, y(t)-1)$ is obtained on the completion of a good transmission, i.e., at a rate $y(t)$ (recall that packet transmission times are exponentially distributed with mean 1). Similarly, a bad transmission completion gives rise to an $(x(t), y(t))$ to $(x(t)-1, y(t))$ transition, at a rate $x(t) - y(t)$, and, if $x(t) < N$, an arrival gives rise to an $(x(t), y(t))$ to $(x(t)+1, y(t)+1)$ transition at rate $(M - x(t))\alpha$. If $x(t) = N$, an arrival will lead to a transition to $(N+1, 0)$ since all transmissions become bad. Once $x(t)$ exceeds N, good transmissions can be initiated only after the total number of transmissions eventually falls below N. If there is any transmission initiation from a state $(x(t), 0)$, where $M > x(t) > N$, then it will result in a transition to state $(x(t)+1,0)$. This transition will occur at a rate $(M-x(t))\alpha$. Transitions from $(x(t),0)$ to $((x(t)-1),0)$ occur at the rate $x(t)$.

For $\alpha > 0$, this is an irreducible finite state CTMC. Let π denote its stationary distribution. The throughput of successful packets in this case is given by:

$$g_{CDMA\text{-}ALOHA}(L,M,N,\alpha) = \sum_{1\le i\le N, 1<j\le i} j\pi(i,j) \tag{1}$$

where j is the transition rate from state (i,j) to state $(i-1,j-1)$.

With collision detection, each transmitting user can detect that the number of transmissions has exceeded N, after a propagation delay distributed as $\exp(\tau)$ and it then aborts its own transmission. The packet is then reattempted after a delay distributed as $\exp(\alpha)$. The Markov chain is the same as in case 1 with the addition being that whenever $x(t)$ exceeds N, the system can go back to the state $(0,0)$ at the rate τ. The state space is the same as for case 1.

$$S = \{(x,y) : 0 \le x \le N, 0 \le y \le x\} \cup \{(x,0) : N+1 \le x \le M\}$$

The throughput of successful packets in this case is given by:

$$g_{CDMA\text{-}ALOHA\text{-}CD}(L,M,N,\alpha,\tau) = \sum_{1\le i\le N, 1<j\le i} j\pi(i,j) \tag{2}$$

where j is the transition rate from state (i,j) to state $(i-1,j-1)$

We can further include carrier sensing in this analysis. Now every node senses the channel before transmitting. If there are N users already transmitting, it refrains from transmitting, since otherwise, all transmissions will be rendered bad. The interference limit N may still be exceeded since a node may not perceive N ongoing transmissions, due to the finite propagation delay.

Now there are 2 sets of states corresponding to $x(t) = N$, denoted by (N,y) and (N',y). A state (N,y) is an "unsafe" or "vulnerable" state, and is entered from $x(t) = N-1$ upon a transmission initiation. After a propagation delay distributed as $\exp(\tau)$, it is assumed that all users are able to sense that the maximum limit has been reached and the remaining nontransmitting users refrain from transmitting; this is a transition from the unsafe state (N,y) to the corresponding "safe" state (N',y). This transition occurs at the rate τ. From the safe states there are only transmission completions. Any transmission initiation while in the unsafe state will, however, result in causing all the good ongoing transmissions to also become bad. The state space of the Markov chain $(x(t),y(t))$ is now given by

$$S = \{(x,y) : 0 \le x \le N, 0 \le y \le x\} \cup \{(N',y) : 0 \le y \le N\} \cup \{(x,0) : N+1 \le x \le M\}$$

The throughput of successful packets in this case is given by:

$$g_{CDMA\text{-}ALOHA\text{-}CD/CS}(L,M,N,\alpha,\tau) = \sum_{1\le i\le N, 1<j\le i} j\pi(i,j) + \sum_{1<j\le N} j\pi(N',j) \tag{3}$$

where j is the transition rate from state (i,j) to state $(i-1,j-1)$.

The saturation throughput results for the above three cases are given in the next section

Returning to the point of comparison with TDMA, an interesting observation can be made regarding the propagation delay. Let a be the propagation delay (between the furthest nodes in the network) normalised to the mean packet transmission time. Considering the running example in the Introduction, in which the CDMA bit rate is .01 times the TDMA bit rate, it is clear that for TDMA the factor a is 100 times the value of a for CDMA. With the numbers in that example in mind, while comparing throughput, we will compare the TDMA case with $a = 1$, with the CDMA case with $a = .01$. Recall that the propagation delay, though fixed in practical situations, is assumed to be distributed as exponential(τ), for simplification in the analysis. Note that a higher a will make the carrier sense and collision detection techniques less efficient.

3.3 A Reduced State Space Model for CDMA with Code-Sharing

In order to extend the CDMA analysis, to the case of CDMA with code-sharing, it appears necessary to keep track of the number of active users on each code. This will result in a Markov chain of higher dimensions and hence will complicate the analysis. For CDMA-ALOHA (without CS and CD) it is possible to work with a Markov chain that keeps track only of the transmissions of one code. This works since the problem is symmetric in the various codes.

Fix a particular code, and consider the Markov Chain $(x(t), y(t))$, where $x(t)$ is the total number of ongoing transmissions, while $y(t)$ is the number of ongoing transmissions of *that particular code.* A value $y(t) = 1$ indicates a good transmission, while a negative value $y(t) = -j$ indicates j bad ongoing transmissions using this code.

The state space is given by :

$$S = \{(x, 1) : 1 \leq x \leq N\} \cup \{(x, y) : 0 \geq y \geq -K, -y \leq x \leq M - K - y\}$$

Note that only one transmission on each code can be good; as soon as there is another transmission on the same code by any of the remaining $K-1$ users, both become bad, i.e., a transition from $(x(t), 1)$ to $(x(t)+1, -2)$ occurs at a rate of $(K-1)\alpha$. If the transmission on this code is completed successfully, a transition from state $(x(t), 1)$ to state $(x(t)-1, 0)$ is obtained at a rate of 1 (since rates have been normalised to packet transmission time). If $y(t) = 1$ and there is a transmission completion on any other code, then a transition occurs to state $(x(t)-1, 1)$ at the rate $x(t)-1$. If $x(t) < N, y(t) = 1$, then a transmission initiation on any other code leads to state $(x(t)+1, 1)$ at the rate $(M - K - x(t) - 1)\alpha$. If $x(t) = N, y(t) = 1$ then a transmission initiation on any other code leads to state $(N+1, -1)$ at the rate $(M-K-N-1)\alpha$. When the system is in a state $(x(t), y(t))$ such that $y(t) < 0$, four kinds of transitions can occur:

1. A transition to state $(x(t)+1, y(t))$ at a rate of $(M - K - x(t) - y(t))\alpha$ corresponding to a transmission initiation by a user on any other code.

2. A transition to state $(x(t)+1, y(t)-1)$ at a rate of $(K+y(t))\alpha$ corresponding to a transmission initiation by a user on this code.

3. A transition to state $(x(t)-1, y(t))$ at a rate of $x(t)+y(t)$ corresponding to the end of transmission of a user on any other code.

4. A transition to state $(x(t)-1, y(t)+1)$ at a rate of $-y(t)$ corresponding to the end of transmission of a user on this code.

When the system is in a state $(x(t), 0)$ with $x(t) < N$, a transmission initiation will cause a transition to state $(x(t)+1, 1)$ if it is on this particular code, else a transition to state $(x(t)+1, 0)$ is obtained. However, if $x(t)$ were more than N, then a transmission initiation on this code would give rise to a transition to state $(x(t)+1, -1)$.

The above chain also correctly captures the interdependence among the codes, while separating the model for each code. The channel throughput is given by :

$$g_{CDMA-ALOHA-Code-Sharing}(L, M, N, \alpha) = L \sum_{i=1}^{N} \pi(i, 1)$$

where $\pi(i, j)$ is the stationary probability vector, and the term involving the summation is the throughput per code.

Note that for $L = 1$, the above model will correspond to TDMA-ALOHA. Also, with $L = M$, it corresponds to CDMA ALOHA without code-sharing. It is expected that the results in this case should match exactly with the results obtained in Section 3.2. This is indeed true, as shown by the results in the next section.

4 SATURATION THROUGHPUT: NUMERICAL RESULTS

The results of the saturation throughput analysis are presented in Figures 3 through 8.

Each Markov chain is seen to be irreducible and is defined over a finite state space. Hence it is positive recurrent and there is a stationary probability distribution $\pi(i, j)$. The saturation throughputs have been obtained using the equations given in Sections 3.2 and 3.3. The computations were carried out with various values of $\alpha, M, N,$ and L.

4.1 CDMA Without Code-Sharing

Figure 3 shows the saturation throughput results for CDMA without code-sharing, for $N = 1$, and various values of M, the number of users. Recall that $N = 1$ corresponds to pure TDMA-ALOHA. Note that α is the attempt rate per node, and the curves are plotted against $M\alpha$ the aggregate attempt rate when all the nodes are in the refractory state, to enable comparison with the classical infinite user ALOHA model. Observe the peak of about 0.2 near $M\alpha = 0.5$. Note, however, that in this model $M\alpha$ is not the attempt rate on the channel.

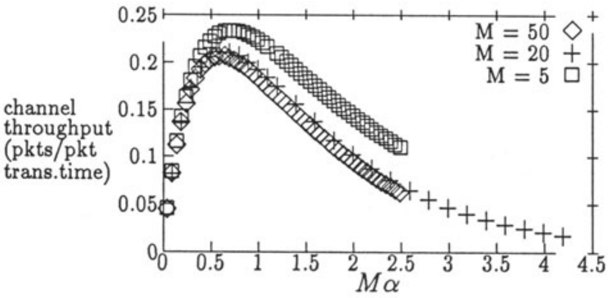

Figure 3: Saturation throughput analysis of unslotted finite user TDMA-ALOHA. Graph of total channel throughput (in packets per packet time) vs. $M\alpha$, for various values of M, the number of users.

Figure 4 shows saturation throughput results for CDMA without code-sharing, but with its collision detection and carrier sensing enhancements, for $M = 20, N = 5$, with $\tau = 200$ (normalised to packet transmission time). It is observed that the throughput increases progressively as collision detection and carrier sensing features are added. Although not shown in the curves, as α increases further, the CD and CS curves decrease owing to excessive collisions.

In Section 3.1 we have seen that saturation throughput yields a sufficient stability condition for the packet buffers. We interpret this saturation throughput as channel capacity. But then it follows that channel capacity depends on α, a parameter that models the backoff algorithm used by the nodes. In a real system, if adaptive backoff is used, it may be expected that α will be adapted on-line till throughput is maximised. The throughput will then correspond to the peak of the corresponding saturation throughput curve. It is therefore meaningful to compare the peaks of the saturation throughput curves.

From Figures 3 and 4 observe that for $M = 20$ (and $K = 1$) without CD or CS, TDMA-ALOHA has a peak normalised throughput of about 0.2, whereas CDMA-ALOHA has a peak normalised throughput of about 2 for $N = 5$, and 5 for $N = 10$ (see Figure 5). If CDMA chip rate is the same as the TDMA bit rate then for a spreading factor of 100 (say), TDMA-ALOHA has a channel capacity of 20 packets per CDMA packet transmission time, compared with, for CDMA, 2 for $N = 5$ and 5 for $N = 10$. On the other hand if CDMA chip rate can be higher than the maximum achievable TDMA bit rate (as may be the case with optical CDMA), then the channel capacity with CDMA-ALOHA can exceed that with TDMA-ALOHA.

Table 2 shows how the peak saturation throughput varies with M for the TDMA case with $\tau = 1, a = 1$ and $N = 1$, and for the CDMA case with $\tau = 100, a = .01$ and $N = 10$, both with CD and CS. For $M = 20$, the maximum throughput for CDMA ALOHA is about 8 packets per packet transmission time, while for TDMA ALOHA the maximum throughput is about 0.35 packets per packet transmission time. For TDMA bit rate equal to CDMA chip rate, and a CDMA spreading factor of 100, it follows that the ratio of CDMA throughput

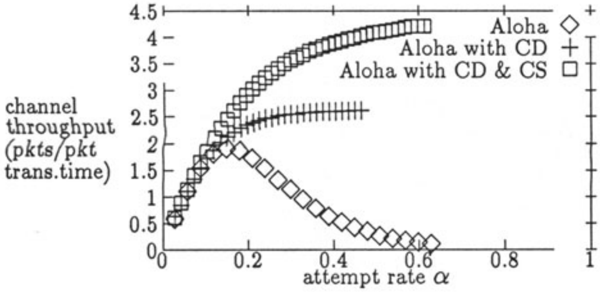

Figure 4: Saturation throughput analysis of unslotted finite user CDMA-ALOHA, CDMA-ALOHA with collision detection, and CDMA-ALOHA with collision detection and carrier sensing, for CDMA system without code-sharing. Graph of total channel throughput (in packets per packet time) vs. α, for $M = 20$ users, with interference limit $N = 5$, and $\tau = 200$ (normalised to packet transmission time).

to TDMA throughput is $\frac{8}{100 \times 0.35} \approx \frac{1}{4.5}$.

It may have seemed at the outset that since there are 10 codes, after taking into consideration the bandwidth expansion factor, CDMA would yield an aggregate throughput that is $(1/10)^{th}$ of the TDMA throughput; owing to the different impacts of ALOHA overheads, however, it turns out that the CDMA throughput is (1/4.5) of the TDMA throughput.

4.2 CDMA with code-sharing

Figure 5 shows the saturation throughput results for CDMA-ALOHA without code-sharing with $M = 20, N = 10$, obtained from the analysis in Section 3.2, and the total throughput for all codes for CDMA with code-sharing for $M = 20, N = 10, K = 1$, obtained from the analysis in Section 3.3. The throughput values match exactly thus showing the consistency of the two analyses.

Figure 6 shows the throughput per code versus the attempt rate α, for various values of K, the number of users sharing each code. Figure 7 shows the total throughput of the system versus the attempt rate α, for various values of K. The total throughput has been obtained simply by multiplying the throughput per code (as plotted in Figure 6), by the number of codes (L), since the system is symmetric in the various codes. The nature of the curve is observed to be similar to the usual ALOHA throughput curve. At low values of α the throughput increases with α; it peaks and then reduces with increasing attempt rate since the interference and collisions increase. For $K = 1$, the reduction in the throughput per code with α is solely due to the interference limit set by N. If there was no such limit, then the throughput would increase to 1 and saturate. At higher K, increasing α reduces throughput due to an increase in collisions, in addition to an increase in the probability of exceeding the interference limit N.

Table 2: Variation of maximum throughput with M, for TDMA with $\tau = 1$ & $N = 1$, and for CDMA with $\tau = 100$ & $N = 10$, considering the case of ALOHA with CS & CD

M	Maximum throughput	
	TDMA	CDMA
10	0.3581	10
11	0.3563	8.99
12	0.3555	8.72
13	0.3541	8.54
14	0.3537	8.40
15	0.3524	8.28
16	0.3519	8.19
17	0.3516	8.125
18	0.3510	8.06
19	0.3497	8.01
20	0.349	7.97

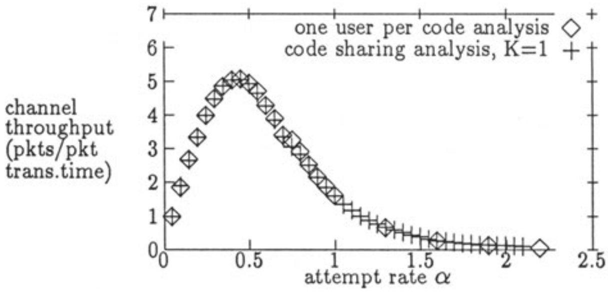

Figure 5: Saturation throughput analysis of unslotted CDMA-ALOHA without code-sharing. Comparison of results obtained from the analysis of the two Markov chain models, for $M = 20 and N = 10$.

Figure 7 shows that the value of α at which the curves peak reduces as K increases, i.e., the maximum permissible attempt rate per user reduces as the level of code-sharing increases. It is also observed that the system throughput decreases for any α as the level of code-sharing increases.

Finally Figure 8 again shows more explicitly the decrease in the total throughput when the level of code-sharing is increased.

5 PACKET DELAY: SIMULATION STUDY

The system was modelled as M stations connected to each other via L links, made to represent codes. A station can transmit on only one link (its own code) but can receive on all the links (codes) simultaneously. Each code is shared by $K = M/L$ users.

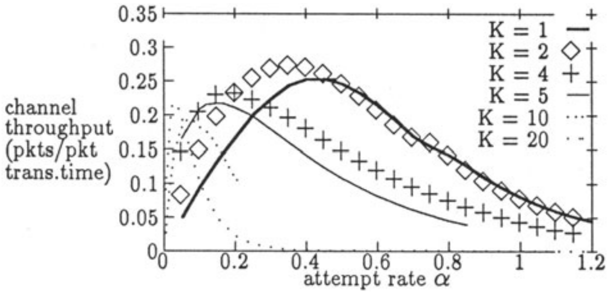

Figure 6: Saturation throughput analysis of unslotted CDMA-ALOHA with code-sharing(i.e. $K > 1$). Graph of channel saturation throughput per code(in packets per packet time) vs. α, for $M = 20$ users and the interference limit $N = 10$, for various values of K, the number of users per code.

Each node operates in accordance with the model presented in Section 2 and shown in Figure 1 and Figure 2. We assume Poisson arrival processes in our simulation. The value of α for each user was set to the value that gave the peak saturation throughput for the particular access protocol with the corresponding values of $L, M,$ and N.

The results are shown in Figure 9. The results assume that TDMA bit rate is the same as CDMA chip rate. The arrival rate has been normalised to the number of packets arriving per packet transmission time of CDMA. Hence the curves take into consideration the fact that packet transmission time for CDMA is 100 times the packet transmission time for TDMA.

Figure 9 contains the plots for the delay versus the normalised arrival rate. The results obtained are as expected. The delay increases to a large value as the arrival rate is increased to the saturation throughput. This stable region is upto 22 packets per CDMA packet transmission time for TDMA-ALOHA (see Figure 3), 5 for CDMA-ALOHA without code-sharing, and 2.5 for CDMA-ALOHA with code-sharing and $K = 2$ (see Figure 7). The mean delay increases when code-sharing is done.

6 CONCLUSION

This study was motivated by the question of the comparative efficiency of CDMA-ALOHA and TDMA-ALOHA when they are used as bandwidth sharing mechanisms by a finite number of nodes for transmitting packetised data. We have proposed a simple stochastic model for channel attempts by the nodes, we have shown that saturation throughput in this model yields a sufficient stability condition for the packet buffers, and we have used Markov chain models to obtain saturation throughputs for several versions of these medium access protocols; i.e., CDMA-ALOHA with or without code-charing, and with or without collision detection or carrier sensing.

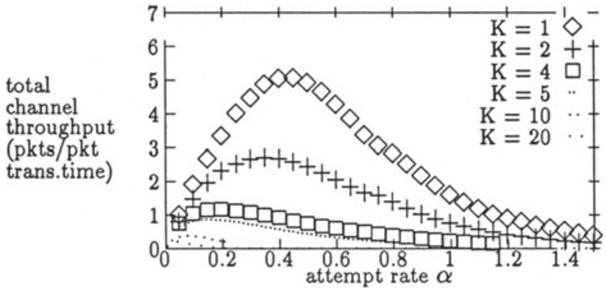

Figure 7: Saturation throughput analysis of unslotted CDMA-ALOHA with code-sharing(i.e. $K > 1$). Graph of total channel saturation throughput (in packets per packet time) vs. α, for $M = 20$ users and the interference limit $N = 10$, for various values of K, the number of users per code.

If the best that can be done for CDMA-ALOHA is that its chip rate be the same as the maximum achievable TDMA-ALOHA bit rate, then the channel utilisation efficiency is quite poor with CDMA-ALOHA, though not as poor as it may seem from a simple minded calculation based on the spreading factor and the interference limit N. In CDMA-ALOHA, code sharing further reduces the bandwidth efficiency but makes for a cheaper implementation of the correlation receivers. If CDMA-ALOHA chip rate can substantially exceed the achievable TDMA-ALOHA bit rate, as may be possible in optical CDMA, then CDMA-ALOHA will have the combined advantages of high efficiency, less sensitivity to propagation delays (owing to the lower transmission rate from the nodes), and ease of implementation.

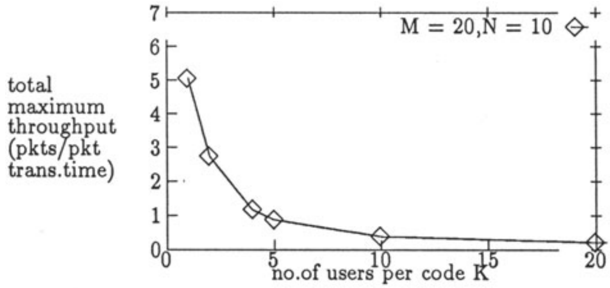

Figure 8: Saturation throughput analysis of unslotted CDMA-ALOHA with code-sharing. Graph of maximum total channel saturation throughput (in packets per packet time) vs. number of users sharing a code (K). The number of users $M = 20$ and interference limit $N = 10$.

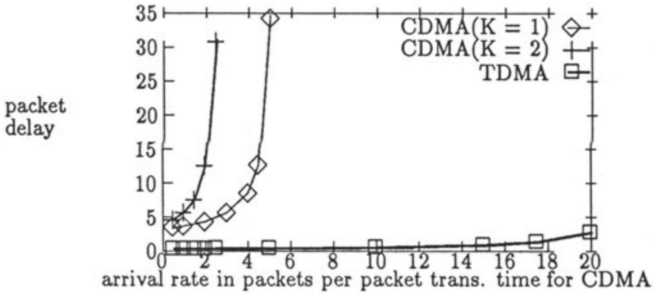

Figure 9: Simulation analysis of packet delay for unslotted finite user TDMA-ALOHA and CDMA-ALOHA, with and without code-sharing. Graph showing the mean packet delay, in number of packet transmission times for CDMA, vs. normalised arrival rate. The number of users $M = 20$ and the interference limit $N = 10$.

ACKNOWLEDGEMENTS

This problem was motivated during a discussion with Prof. A. Selvarajan, who is himself developing an optical CDMA network. We are thankful to Drs. Utpal Mukherji, Vinod Sharma, and Vivek Srinivas for discussions and exchange of ideas that have benefited this work.

REFERENCES

[Borovkov, 1976] A.A. Borovkov. *Stochastic Processes in Queuing Theory* . Springer-Verlag, New York, 1976.

[Rao *et al.*, 1988] Ramesh R. Rao and A.Ephremides. "On the Stability of Interacting Queues in a Multiple Access Environment". *IEEE Tran. on Inf. Thy.*, Vol.34, No.5, pp 918-930, Sept.1988.

[Lee *et al.*, 1988] E.Lee and D.Messerschmitt. "*Digital Communication*". Kluwer Academic Publishers, Boston, 1988.

[Salehi 1989] J.A. Salehi. "CDMA Techniques in Optical Fibre Networks-Part 1 & 2". *IEEE Transactions on Communications*, Vol.37, No.8, pp. 824-842, rAug. 1989.

[Prucnal *et al.*, 1986] P.R.Prucnal and M.A.Santoro. "Spread Spectrum Fibre-Optic Local Area Network using Optical Processing". *IEEE Journal of Lightwave technology*, Vol.LT-4, No.5, May 1986.

[Raychaudhari, 1981] D.Raychaudhary. "Performance Analysis of Random Access Packet Switched CDMA systems". *IEEE Transactions on Communications*, vol.29, No.6, pp. 895-901, June 1981.

[Green, 1993] P.E.Green. "*Fibre Optic Networks*". Prentice Hall, New Jersey, 1993.

[Sharma, 1988] V. Sharma. "Stability and Continuity for Slotted ALOHA with Stationary Non-independent Input Traffic". *Journal of Appl. Prob.*, Vol.25, pp 797-807, 1988.

[Sharma, 1990] V.Sharma. "Stability of Multiple Access Packet-Switched CDMA Systems". *Sadhana*, Proceedings of the Indian Academy of Sciences, V.15, Parts 4 and 5, pp 365-380, Dec 1990.

[Szpankowski, 1990] W. Szpankowski. "Towards Computable Stability Criteria for Some Multidimensional Stochastic Processes". *Stochastic Analysis of Computer and Communication Systems*, Hideaki Takagi (ed.), Elsevier, North-Holland, pp.131-172, 1990.

BIOGRAPHIES

Anurag Kumar has a B.Tech. in E.E. from the Indian Institute of Technology at Kanpur, and a PhD from Cornell University. He was a Member of Technical Staff at AT&T Bell Labs, Holmdel for over 6 years. Since 1988 he has been with the Indian Institute of Science (IISc), Bangalore, in the Dept. of Electrical Communication Engineering, where he is now Associate Professor. He is also the Coordinator at IISc of the Education and Research Network Project, which has set up a country-wide computer network for academic and research institutions, and conducts R&D in the area of communication networks. His own research interests are in the area of modelling, analysis, control and optimisation problems arising in communication networks and distributed systems.

Deepak Patil has a B.Tech. in Electronics and Telecommunication Engg. from V.J.T.I., Bombay, and an M.E. in Electrical Communication Engg. from the Indian Institute of Science. He is currently a Senior Engineer software R&D in Wipro Infotech, Bangalore.