Image Segmentation by Means of Fuzzy Entropy Measure

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Abstract
The paper describes an algorithm for image segmentation using fuzzy entropy measure. The relation between the fuzzy entropy of an image domain and the fuzzy entropy of its subdomains is explored as a uniformity predicate. With the aim of implementing the model, we have introduced a well known technique of Problem Solving. The most important roles of our model are played by the Evaluation Function (*EF*) and the Control Strategy. So the *EF* is related to the ratio between the fuzzy entropy of one region or zone of the picture and the fuzzy entropy of the entire picture. The Control Strategy determines the optimal path in the search tree (quadtree) so that the nodes of the optimal path have minimal fuzzy entropy. The paper shows some comparisons between the proposed algorithm and classical edge detection techniques.

1. Introduction

Segmentation is a significant issue in the field of image processing and image understanding. The segmentation is the process, both human and automatic, that isolates in a pictorial scene zones or regions, edges or contours and angles with respect to a certain uniformity predicate.
In the last years several approaches have been proposed in the literature and may be classified as follows:

- Local (or atomistic) approach:

Such a method takes advantages of grey level discontinuities that are considered relevant features of image. In order to extract these features several local operators have been introduced and the most important are Sobel [1, 2], Marr-Hildreth [3]. In this case the discontinuities assume the shape of monodimensional step function. In [1] Gonzales and Wintz propose an effective algorithm that looks at regions in image with non-unimodal histogram, using local thresholds.
To the aim of resolving the strong simplification that associates edges with a monodimensional step function, a lot of corrections have been introduced. In this manner local operators can process image with smoothing and shading effects [2, 4, 5]. In [6, 7] any angle of a region in a digital image is approximated by means of a series of segments or half edges, where a half edge can be characterised by its position and orientation angle. So the angle is a point at which two (or more) half edges cross [7-9].

Recently some techniques based on image iterated smoothing have been proposed. Such a method eliminates higher frequencies of image by iterated sampling, preserving -as much as possible- the shape and the edge positions [10, 11]. This approach presents some obvious limits such as an high computing time, and the strictly depending on threshold and on smoothing parameters. For this reason usually local operators are not utilised to process medical images.
In [12] Higgins presents and compares three different methods to detect grey level discontinuities by utilising structural local informations.

- Global (or structural) approach:

For its simplicity the global threshold is the oldest technique for image segmentation. In this approach it is customary to utilise one threshold for the whole image (global information) or one threshold for each image region (contextual method) [13-16]. The threshold is based on the following rules: each region in image may be associated with a peak histogram [17-20]. Of course such a rule is very restrictive.
In order to extract the objects that make up an image, entropy -high order entropy or conditional entropy- is often adopted as uniformity predicate [21,22].
The Gestalt theory and Neurophysiologic theory affirm that in the human perceptive process the eye aims to minimise objects and background variations -homogenise-enhancing transitions regions, i.e. edges.
The goal of this paper is to propose a new algorithm for image segmentation using the fuzzy entropy, namely FISE, exploiting the ratio between each image region and the whole image [23, 24, 32]. In other words such a ratio is minimum on the regions and maximum on the edges.

2. Fuzzy Entropy Measure for Edge Detection

The task of the segmentation is to enhance the different regions of the image. In order to segment the image into regions each of them has to satisfy the uniformity predicate. Now it is necessary to analyse some aspects of the uniformity predicate before we introduce it.
In our opinion the uniformity predicate couldn't be determined without considering the features of the whole image. In other words the measures calculated to determine the uniformity predicate have to be related to the same measures calculated on the whole image. In this paper the fuzzy entropy has been chosen as the uniformity predicate. In particular the fuzzy entropy of a region (or a background) -that is a region too- is always lower than the entropy of whole image or, in other words, the fuzzy entropy of a region is always greater or equal than the entropy of its subdomains.
The FISE adopts a strategy to segment the image similar to that introduced in [25]: the merge and the split are applied in cascade on the image using a region growing algorithm.
A search tree of any hierarchical structure, as for example t-ari trees, can be utilised. In our work we have chosen the quadtree.
The strategy we'll propose segments into regions whose fuzzy entropy is nearly zero. More precisely, the zones extracted as regions are those whose evaluation function is gradually goes to zero.
We have defined the evaluation function as a measure of the fuzzy entropy of a region according to the theorem of fuzzy entropy [33].

The theorem of fuzzy entropy defines the entropy of a fuzzy set A as a measure of the subset A *and notA* (A **and** A^c)in relation to the subset A *or notA* (A **or** A^c) as showed in Figure 1.

Let a_i be an internal node of quadtree whose grey tones are $g_1, g_2, \ldots, g_n$ ($g_i < g_j$, $i < j$, $i, j = 1, \ldots, n$) and frequencies $f_1, f_2, \ldots, f_n$, respectively.

Let a_{4i+t}, $t \in [1,4]$, a child node of a_i, whose greytones are $g'_1, g'_2, \ldots, g'_m$ ($g'_i < g'_j$, $i < j$, $i, j = 1, \ldots, m$) and frequencies $f'_1, f'_2, \ldots, f'_m$, respectively.

For each grey tone g_i, $i = 1, \ldots, n$, of the node a_i we define

$$\Delta f_i = \left| f_i - f'_j \right|$$

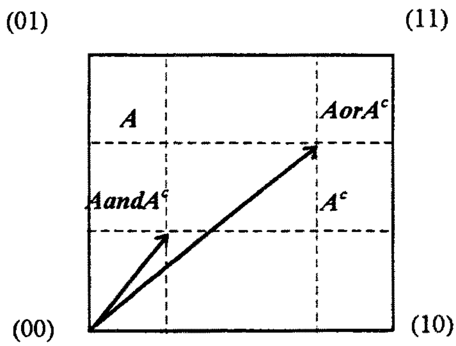

Figure 1 - The entropy fuzzy theorem.

where f_i is the frequency of the grey tone g_i while f'_j is the frequency of the grey tone of the node a_{4i+t}, with $g_i = g'_j$.

Then we evaluate

$$\Delta^{100} f_i = \sqrt{(100 - \Delta f_i)^2 + 100^2}$$

and finally we define the fuzzy entropy of the greytone g_i as:

$$e_i = \frac{\Delta f_i}{\Delta^{100} f_i}.$$

So, the fuzzy similarity of the node a_i respect to its child node a_{4i+t} is:

$$E(a_i, a_{4i+t}) = \frac{1}{K} \sum_{i=1}^{K} e_i \quad (1)$$

where $K = \max(n, m)$.

Now we can define the fuzzy homogeneity of the node a_{4i+t}, $t \in [1,4$ as the average fuzzy similarity of a_{4i+t} and its four children nodes:

$$EM(a_{4i+t}) = \frac{1}{4}\left(E(a_{4i+t}, a_{(4i+t)+1}) + E(a_{4i+t}, a_{(4i+t)-2}) + E(a_{4i+t}, a_{(4i+t)+3}) + E(a_{4i+t}, a_{(4i+t)+4}) \right)$$

We can observe that if $EM(a_{4i+t})$ is nearly zero then the node a_{4i+t} is "totally homogeneous".

So, the evaluation function of the node a_{4i+t} is given by:

$$F(a_{4i+t}) = \frac{EM(a_{4i+t})}{E(a_i, a_{4i+t})}.$$

$F(a_{4i+t})$ evaluates the fuzzy homogeneity of the node a_{4i+t} respect to the fuzzy similarity to its father node a_i, assuming minimal value inside a region and maximal value along the contour of a region.

So, if a_i is an internal node of a quadtree and the evaluation functions of its four children nodes are $F(a_{4i+1})$, $F(a_{4i+2})$, $F(a_{4i+3})$, $F(a_{4i+4})$, the control strategy expands the child node a_s, with $s = 4i+1,\ldots,4i+4$, whose $F(a_s)$ is minimal:

$$F(a_s) = \min\{F(a_{4i+1}), F(a_{4i+2}), F(a_{4i+3}), F(a_{4i+4})\}.$$

According this rule, along the search path of the quadtree, $F(a_s) = \frac{EM(a_s)}{E(a_i, a_s)}$ is different from zero, in particular $EM(a_s) \neq 0$ and $E(a_i, a_s) \neq 0$.

Applying this rule at each level of the search tree, we'll individuate a node a_j whose $EM(a_j) \cong 0$ and so $F(a_j) \cong 0$. Then we can state that the subdomain relative to the node a_j belongs to a region (or an object) R.

If, inside the region R the evaluation function $F(a_{i'})$ of an expanded node $a_{i'}$ is nearly zero and the evaluation function $F(a_{j'})$ of its child node $a_{j'}$ to expand is different from zero then we can state that the subdomain relative to the node $a_{i'}$ is the smallest subdomain including the partition element of the region. In other words, if the dimension of the subdomain relative to the node $a_{i'}$ is $k_{i'} \times k_{i'}$ and its initial coordinates in the entire image domain are $(x_{i'}, y_{i'})$, then the dimension of the partition element P of the region R will be $dx \times dy$, with $k_{i'/2} \le dx \le k_{i'}$, $k_{i'/2} \le dy \le k_{i'}$ and $dx = \min\{x\}$, $k_{i'/2} \le x \le k_{i'}$ and $dy = \min\{y\}$, $k_{i'/2} \le y \le k_{i'}$ such that the evaluation function computed on the subdomain with dimension $x \times y$, included in the domain of the node $a_{i'}$, and initial coordinates $(x_{i'}, y_{i'})$ is nearly zero.

So, the region R of the image will be the union of the elements (or subdomains) $P_1, P_2, \ldots, P_N$ fuzzy similar to the partition element P:

$$R = \bigcup_{i=1}^{N} P_i$$

where P_i has coordinates (px_i, py_i) and dimension $dx_i \times dy_i$, with $dx_i = dx$ and $dy_i = dy$.

3. Experimental Results and Discussions

In order to evaluate the performance of the proposed edge detection algorithm we have tested it on several theoretical and real 512x512x8 bits images. For the sake of brevity in this Section objective results obtained on theoretical images are presented. Moreover, FISE has been widely compared with the segmentation algorithm based on entropy (namely ISE) proposed by Vitulano et alt. in [32] and some of the most useful local

operators, i.e. Sobel, DOG zero-crossing, Haralick [1, 2], Anisotropic diffusion and three methods proposed by Higgins [12].
We describe briefly ISE segmentation method.
Let a_s be a node at level z of quadtree and a_{4s+t}, $t \in [1,4]$ a child node of a_s. Let $f_k(a_s)$ with $k \in [1,n]$ be the maxima frequencies of the histogram of the image area relative to the node a_s and $f_l(a_{4s+t})$ with $l \in [1,m]$ be the maxima frequencies of the histogram of the image area relative to the node a_{4s+t}.
For simplicity we denote the set of these frequencies with $\{f_k\}$ and $\{f_l\}$. With each maximum frequency f_i in $\{f_k\}$ (or in $\{f_l\}$) we associate a gaussian function whose standard deviation is defined as:

$$\sigma_i = \frac{1}{\sqrt{2\pi f_i}}. \qquad (2)$$

So, with each frequency f_i in $\{f_k\}$ (or in $\{f_l\}$) we can associate an interval of graytones of the histogram of a_s (or of a_{4s+t}):

$$[g_i - \sigma_i, g_i + \sigma_i]$$

where g_i is the greytone with frequency f_i and standard deviation σ_i defined as in (2).
If g_i is the graytone with frequency $f_i \in \{f_k\}$ and g_j is the graytone with frequency $f_j \in \{f_l\}$ then the following relations could be satisfied:

$$|g_i - g_j| \le |a - b| \le 2\sigma^* \qquad (3)$$

where a and b are the extrema of the intersection interval $[g_i - \sigma_i, g_i + \sigma_i] \cap [g_j - \sigma_j, g_j + \sigma_j]$ and $\sigma^* = \min(\sigma_i, \sigma_j)$,
otherwise

$$[g_i - \sigma_i, g_i + \sigma_i] \cap [g_j - \sigma_j, g_j + \sigma_j] = \varnothing \qquad (3')$$

We consider now the sets $\{f_k\}$ and $\{f_l\}$ sorted according increasing graytones and we state that if $n = m$ and $|g_i - g_j| \le 2\sigma^*$, where g_i is the greytone with frequency $f_i \in \{f_k\}$ and standard deviation σ_i and g_j is the greytone with frequency $f_j \in \{f_l\}$ and standard deviation σ_j, and $\sigma^* = \min(\sigma_i, \sigma_j)$, then the histograms of a_s and a_{4s+t}, $t \in [1,4]$, have the maxima frequencies in correspondence to the same graytones. In terms of multivariate analysis the histograms have the same variables.
Moreover, if

$$\forall i = j \in [1,n], \; f_i = f_j \text{ and } |g_i - g_j| = 0 \qquad (4)$$

then the histograms of a_s and a_{4s+t} have the same graytones distribution.

If (3) and (4) are satisfied we define the domain relative to a_s "totally homogeneous" to the domain relative to a_{4s+t}.

Obviously, the relations (3) and (4) are valid for theoric images and for a limited number of nodes of the quadtree. Instead for real images it's necessary the introduction of an evaluation function to determine the homogeneity degree of two different domains.

The control strategy prefers the nodes with minimum *E.F.* along the search tree as in FISE method.

So, for each $f_i \in \{f_k\}$ and $f_j \in \{f_l\}$ satisfying the relation (3), we define the entropy e_i relative to the frequencies f_i and f_j as:

$$e_i = \frac{|a-b|}{|a'-b'|}$$

where a and b are the extrema of the intersection interval $[g_i-\sigma_i, g_i+\sigma_i]\cap[g_j-\sigma_j, g_j+\sigma_j]$, a' and b' are the extrema of the union interval $[g_i-\sigma_i, g_i+\sigma_i]\cup[g_j-\sigma_j, g_j+\sigma_j]$ and g_i, σ_i, g_j and σ_j are defined as in (3). So, we define the *E.F.* of a_s in relation to a_{4s+t} as:

$$E(a_s, a_{4s+t}) = \frac{1}{p}\left(\sum_{i=1}^{p}(1-e_i)+\alpha\right) \qquad (5)$$

where a is the number of frequencies of $\{f_k\}$ and $\{f_l\}$ not satisfying the relation (3) and $p = \frac{n+m-\alpha}{2}$.

For the test cases we assume the image f is made up of disjoint constant intensity region corrupted by additive Gaussian noise (AWGN); i.e. if point $(x,y)\in$ region R_k, than $f(x,y)=\mu_k+\eta_k(x,y)$, where μ_k is the constant intensity value for points in R_k and $\eta_k(x,y)$ is a sample of AWGN having statistics $N(0,\sigma_k^2)$.

In order to determine an objective parameter to test the quality of processed images, as proposed by Haralick [31], we use two performance metrics to compare the various methods: *P(AETE)* and *P(TEAE)*. The first metric is the conditionally probability of a point being assigned as an edge point, given that the point is a true edge point, while the second one the conditional probability of a point being a true edge point, given that the point is assigned as an edge point. Assigned edge points are those points that a particular edge detection method assigns. True edge points are defined to be the points within the two-point wide region in which each point is adjacent to some point having a value different from it on the uncorrupted checkerboard. For each method, except for our algorithm, a threshold is adjusted until *P(AETE)*≈*P(TEAE)*. This equalisation in practice represents an even better trade-off between detecting true edge points and rejecting non-edge points.

The numerical comparison results proposed by Higgins [12], ISE and FISE are illustrated in Table 1.

Table 2 presents the numerical results obtained by applying ISE and FISE on a checkerboard image corrupted by AWGN, mean=0 and standard deviation=60.

	P(TEAE)	*P(AETE)*	Average
Sobel gradient	0.660	0.656	0.658
DOG	0.865	0.833	0.849
Haralick operator	0.760	0.759	0.759
Anisotropic Diffusion	0.889	0.898	0.894
Higgins-Method 1	0.948	0.920	0.934
Higgins-Method 2	0.828	0.823	0.825
Higgins-Method 3	0.866	0.847	0.857
FISE Method	**0.457**	**0.598**	**0.528**
ISE Method	**0.974**	**1.000**	**0.987**

Table 1 - Results for noisy checkerboard using local structure operators, ISE and FISE.

	P(TEAE)	*P(AETE)*	Average
FISE Method	**0.579**	**0.518**	**0.549**
ISE Method	**0.669**	**0.956**	**0.812**

Table 2 - Results for noisy checkerboard using ISE and FISE methods.

4. CONCLUDING REMARKS

The FISE method seems that it should be helpful both for edge detection and segmentation of regions.

The most important features of such method could be summarised as follows:

- high noise tolerance both for real and theoretic image;
- threshold independence;
- low computing time (*n*log*n*) where *n* is the size of the image;
- the edges detected don't present steps usually introduced by local operators.

We underline that the method doesn't need the choice of thresholds or operators dimensions and requires short consuming time. The control strategy makes totally automatic the entire process.

The checkerboard image represents the most significant test image. Infact, since it is a theoric image we know all its edge points and the regions contained in it. The theoretic image has been corrupted by additive Gaussian noise (AWGN) in order to test the goodness of different algorithms. Local operators require high computing time ([12]) and the choice both of thresholds and of operators dimensions (i.e. DOG 21x21, DOG 69x69) doesn't correspond to deterministic criteria. Through a visual analysis we have observed that the contours are irregular and present many breakings. Many points that are not true edge points are assigned as edge points. In general the edge image is smaller than the input image: this decreasing depends on the dimensions of the local operator. The numerical results confirm the limits of the local operators and the goodness of the global FISE and ISE methods. We have observed that FISE method works better as the complexity of the image increases. So, in the next future we are going to improve the experimentation of the method on complex real test images.

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