Generalized Morphological Operators Applied to Map-Analysis*

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Abstract. The concept of strictness of morphological operators is discussed, and it is shown that the ordinary morphological operators have extreme strictness which leads to sensitivity to noise and digital artifacts. Based on this observation new morphological operators that generalize the ordinary morphological operators, are defined. The generalized operators have controllable strictness and so excessive erosion and dilation may be prevented. Some properties of the generalized morphological operators are discussed, and it is shown that they may have a linear filtering interpretation. The paper concludes with some preliminary examples demonstrating the advantages of the generalized morphological operators.

1 Introduction

Computerized representation of maps and line-drawings enables computer aided design and facilitates efficient updating for urban development, architecture, land use management, and similar disciplines. The conversion of printed maps into computerized data bases is an enormous task since the process requires scanning and analysis of extremely large volumes of data. Therefore, automation of the conversion process is essential. Even though substantial research and development efforts have been devoted to map and line-drawings conversion [5, 3], the problem is still unresolved. The problem has become more important with the development of advanced CAD and GIS tools, which lead to a situation in which the processing possibilities are much more progressed than the data input facilities.

Since directional information has a very strong meaning in line-drawing images, the processing of line-drawing images may be performed by decomposing the input image into directional edge-planes, and then using directional morphological operators for the processing of these planes [7, 6]. Directional morphological operators could be classified as a subset of the general morphological operators, in which the morphological kernel is non-isotropic. When a fine selectivity of the directional morphological operators is required, a large morphological kernel (or successive application of small kernels) may be required [1, 2]. In

* This work was partially supported by The Paul Ivanier Center for Robotics and Production Automation, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel.

such cases effects of digital artifacts and noise may damage the expected results due to extreme strictness of the ordinary morphological operators.

In this work it is suggested to define new morphological operators that generalize the ordinary morphological operators, and so to achieve control over their strictness. By controlling the strictness of the morphological operators it is possible to improve their resistivity to noise and digital artifacts, and so to prevent excessive erosion or dilation even when large morphological kernels are used.

2 Generalized Morphological Operators

2.1 The Strictness of Morphological Operators

Given two sets $A, B \subset \mathcal{Z}^N$, the morphological dilation and morphological erosion of A by B are defined [4] respectively by:

$$A \oplus B \equiv \{x \mid \exists a \in A \, , \, b \in B \; : \; x = a + b\} = \bigcup_{a \in A} (B)_a \tag{1}$$

$$A \ominus B \equiv \{x \mid \forall b \in B \;\; \exists a \in A \; : \; x = a - b\} = \bigcap_{b \in B} (A)_{-b} \tag{2}$$

where in these definitions $(B)_a \equiv \{x \mid \exists b \in B \; : \; x = a + b\}$.

When comparing these definitions it is possible to observe that they differ by the sign in the condition, and by the strictness of the condition. While the sign difference in the condition only causes a reflection of the kernel set, the strictness difference is what determines their nature. The strictness in the dilation definition is very loose ($\exists$) and so elements are added to the dilated result. The strictness in the erosion definition is very strict ($\forall$) and so elements are removed from the eroded result. Based on this observation it is possible to generalize the morphological operators by controlling the strictness of the operators with higher flexibility.

2.2 Generalized Dilation

In order to develop the generalized dilation definition, a different interpretation for dilation is considered.

Proposition 1. *The dilation of A by B may be obtained as the union of all the possible shifts for which the reflected and shifted B intersects A :*

$$A \oplus B = \{x \mid (A \cap (\check{B})_x) \neq \emptyset\} \tag{3}$$

where $\check{B}$ is the reflection of B given by: $\check{B} \equiv \{x \mid \exists b \in B \; : \; x = -b\}$.

Proof. By developing the right side of the proposition we get:

$$\begin{aligned} \{x \mid (A \cap (\check{B})_x) \neq \emptyset\} &= \{x \mid \exists a \in A \, , \, b \in \check{B} \; : \; a = b + x\} \\ &= \{x \mid \exists a \in A \, , \, b \in B \; : \; a = -b + x\} \equiv A \oplus B \end{aligned}$$

By using (3), the morphological dilation of A by B can be generalized by combining the size of the intersection into the dilation process. In that sense, a given shift is included in the dilation of A only if the intersection between A and the reflected and shifted B is big enough. The obtained advantage of the generalized dilation is the elimination of excessive dilation caused by small intersections. That is, the mass of an intersection should be big enough in order to cause a change.

Definition 2. The generalized dilation of A by B with strictness s is defined by:

$$A \overset{s}{\oplus} B \equiv \{x \mid \#(A \cap (\breve{B})_x) \geq s\} \; ; \; s \in [1, \min(\#A, \#B)] \tag{4}$$

where the symbol # denotes the cardinality of a set.

It should be noted that since $\forall x : \#(A \cap (\breve{B})_x) \leq \min(\#A, \#B)$, the strictness s is bounded by $\min(\#A, \#B)$.

Proposition 3. *The ordinary dilation is obtained as a special case of the generalized dilation when* $s = 1$ *:*

$$A \oplus B = A \overset{1}{\oplus} B \tag{5}$$

Proof. Results directly from the definition of the generalized dilation, since the cardinality of $(A \cap (\breve{B})_x)$ is greater or equal to one if and only if the intersection between A and $(\breve{B})_x$ is not empty.

Proposition 4. *The generalized dilation is decreasing with respect to the strictness* s *:*

$$A \overset{s1}{\oplus} B \subseteq A \overset{s2}{\oplus} B \iff s1 \geq s2 \tag{6}$$

Proof. According the definition of the generalized dilation, when $s1 \geq s2$ it follows that:

$$\forall x \in A \overset{s1}{\oplus} B : \#(A \cap (\breve{B})_x) \geq s1 \geq s2$$

and so $x \in A \overset{s2}{\oplus} B$. However, there may exist $x \in A \overset{s2}{\oplus} B$ such that $s1 > \#(A \cap (\breve{B})_x) \geq s2$ and so $x \notin A \overset{s1}{\oplus} B$.

Corollary 5. *The generalized dilation results in a subset of the ordinary dilation:*

$$A \overset{s}{\oplus} B \subseteq A \oplus B \tag{7}$$

Lemma 6. *The cardinality of the intersection between a set and a reflected and shifted set remains the same when reflecting and shifting the first set instead of the second set:*

$$\#(A \cap (\breve{B})_x) = \#(B \cap (\breve{A})_x) \tag{8}$$

Proof. By developing the left side of the lemma we get:

$$\begin{aligned}\#(A \cap (\breve{B})_x) &= \#\{a \in A \mid \exists b \in B : a = -b + x\} \\ &= \#\{b \in B \mid \exists a \in A : b = -a + x\} = \#(B \cap (\breve{A})_x)\end{aligned}$$

Proposition 7. *The generalized dilation is commutative:*

$$A \overset{s}{\oplus} B = B \overset{s}{\oplus} A \tag{9}$$

Proof. Results directly from the definition of the generalized dilation, by using (8).

2.3 Generalized Erosion

In order to develop the generalized erosion definition, a different interpretation of erosion is considered.

Proposition 8. *The erosion of A by B may be obtained as the union of all the possible shifts for which the shifted B does not intersect A^c :*

$$A \ominus B = \{x \mid (A^c \cap (B)_x) = \emptyset\} \tag{10}$$

Proof. By developing the left side of the proposition based on (3) and the duality between erosion and dilation we get:

$$\begin{aligned}A \ominus B = (A^c \oplus \breve{B})^c &= \{x \mid (A^c \cap (B)_x) \neq \emptyset\}^c \\ &= \{x \mid (A^c \cap (B)_x) = \emptyset\}\end{aligned}$$

By using (10) the morphological erosion of A by B can be generalized by including in the erosion of A only such shifts for which the intersection between A^c and the shifted B is small enough. The obtained advantage of the generalized erosion is the elimination of excessive erosion caused by small intrusions. That is, the mass of an intersection should be big enough in order to cause a change.

Definition 9. The generalized erosion of A by B with strictness s is defined by:

$$A \overset{s}{\ominus} B \equiv \{x \mid \#(A^c \cap (B)_x) < s\} \ ; \ s \in [1, \#B] \tag{11}$$

where it is assumed that: $\#A < \infty$.

It should be noted that since it is assumed that $\#A < \infty$ then $\forall x : \#(A^c \cap (B)_x) \leq \#B$, and so the strictness s is bounded by $\#B$.

Proposition 10. *The ordinary erosion is obtained as a special case of the generalized erosion when $s = 1$:*

$$A \ominus B = A \overset{1}{\ominus} B \tag{12}$$

Proof. Results directly from the definition of the generalized erosion, since the cardinality of $(A^c \cap (B)_x)$ is less than one if and only if the intersection between A^c and $(B)_x$ is empty.

Proposition 11. *The generalized erosion is increasing with respect to the strictness s :*

$$A \overset{s1}{\ominus} B \subseteq A \overset{s2}{\ominus} B \iff s1 \leq s2 \tag{13}$$

Proof. According the definition of the generalized erosion, when $s1 \leq s2$ it follows that:

$$\forall x \in A \overset{s1}{\ominus} B \ : \ \#(A^c \cap (B)_x) < s1 \leq s2$$

and so $x \in A \overset{s2}{\ominus} B$. However, there may exist $x \in A \overset{s2}{\ominus} B$ such that $s1 \leq \#(A^c \cap (B)_x) < s2$ and so $x \notin A \overset{s1}{\ominus} B$.

Corollary 12. *The generalized erosion results in a superset of the ordinary erosion:*

$$A \overset{s}{\ominus} B \supseteq A \ominus B \tag{14}$$

Ordinary erosion and dilation are dual in a sense that the morphological erosion of A by B can be obtained by dilating the complement of A with the reflected B, and then taking the complement of the result. This property remains valid for the generalized operators.

Proposition 13. *The generalized dilation and erosion are dual in the same sense that exists for the ordinary dilation and erosion:*

$$A \overset{s}{\ominus} B = (A^c \overset{s}{\oplus} \breve{B})^c \tag{15}$$

Proof. By developing the right side of the proposition according to the generalized dilation definition we get:

$$\begin{aligned} (A^c \overset{s}{\oplus} \breve{B})^c &= \{x \mid \#(A^c \cap (B)_x) \geq s\}^c \\ &= \{x \mid \#(A^c \cap (B)_x) < s\} = A \overset{s}{\ominus} B \end{aligned}$$

2.4 Linear Filtering Interpretation

Given two images $\underline{A} \equiv \{\underline{A}(k,l)\}_{k,l=-M}^{M}$ and $\underline{B} \equiv \{\underline{B}(k,l)\}_{k,l=-P}^{P}$ where $P < M$, the linear filtering of $\underline{A}$ by $\underline{B}$ is given by the linear convolution between them: $\{\underline{A}(k,l) * \underline{B}(k,l)\}_{k,l=-M}^{M}$. The linear convolution between $\underline{A}(k,l)$ and $\underline{B}(k,l)$ is defined by:

$$\underline{A}(k,l) * \underline{B}(k,l) \equiv \sum_{m=-P}^{P} \sum_{n=-P}^{P} \underline{B}(m,n)\underline{A}(k-m,l-n) \tag{16}$$

where it is assumed that $\underline{A}(k,l)$ is zero-padded so that $\underline{A}(k,l) = 0$ for each k,l that is not in the range $[-M,M]$.

Definition 14. The respective set A of the binary image $\underline{A} \equiv \{\underline{A}(k,l)\}_{k,l=-M}^{M}$ is defined by:

$$A \equiv \{(k,l) \ \mid \ k,l \in [-M,M] \ , \ \underline{A}(k,l) = 1\} \tag{17}$$

The binary image $\underline{A}$ is called the respective image of the set A .

Lemma 15. *Given two sets $A, B \subset \mathcal{Z}^2$, the cardinality of the intersection between A and the reflected B shifted by (k,l) may be obtained as the value at location (k,l) of the linear convolution between the respective images $\underline{A}$ and $\underline{B}$:*

$$\#(A \cap (\breve{B})_{(k,l)}) = \underline{A}(k,l) * \underline{B}(k,l) \tag{18}$$

Proof. By developing the right side of the lemma according to the linear convolution definition we get:

$$\begin{aligned}\underline{A}(k,l) * \underline{B}(k,l) &= \textstyle\sum_{m=-P}^{P} \sum_{n=-P}^{P} \underline{B}(m,n)\underline{A}(k-m,l-n) \\ &= \textstyle\sum_{m=-P+k}^{P+k} \sum_{n=-P+l}^{P+l} \underline{B}(-m+k,-n+l)\underline{A}(m,n) \\ &= \textstyle\sum_{(-m+k,-n+l)\in B} \underline{A}(m,n) = \sum_{(m,n)\in(\breve{B})_{(k,l)}} \underline{A}(m,n) \\ &= \#(A \cap (\breve{B})_{(k,l)})\end{aligned}$$

Proposition 16. *Given two sets $A, B \subset \mathcal{Z}^2$, the generalized morphological dilation (erosion) of A by B may be obtained by thresholding the linear convolution between the respective binary images $\underline{A}$ and $\underline{B}$:*

$$A \overset{s}{\oplus} B = \{(k,l) \ \mid \ \underline{A}(k,l) * \underline{B}(k,l) \geq s\} \tag{19}$$

$$A \overset{s}{\ominus} B = \{(k,l) \ \mid \ \underline{A^c}(k,l) * \underline{\breve{B}}(k,l) < s\} \tag{20}$$

Proof. Results directly from the definition of the generalized dilation (erosion), by using (18).

Following the last proposition, it is possible to observe that the non-linear nature of morphological operators is derived by a threshold operation, where for the ordinary morphological operators the threshold is one, and for the generalized morphological operators the threshold is higher. It should be noted that the properties in this subsection are discussed for sets in $\mathcal{Z}^2$ in order to simplify the transcription of indexes. These properties can be easily extended to sets in $\mathcal{Z}^N$.

2.5 Extended Duality Proposition

The generalized dilation of A by B may be interpreted as the union of all the possible shifts for which the intersection between A and the reflected and shifted B is big enough. The following proposition states that the same interpretation may be applied to the generalized erosion when using the reflection of the set B and the complement of the strictness s.

Definition 17. The complement of the strictness s relative to the set B is defined by:

$$\overline{s_B} \equiv \#B - s + 1 \tag{21}$$

Lemma 18. *The generalized erosion of A by B with strictness s may be obtained by:*

$$A \overset{s}{\ominus} B = \{x \mid \#(A \cap (B)_x) \geq \overline{s_B}\} \quad ; \quad s \in [1, \#B] \tag{22}$$

Proof. By developing the left side of the lemma according to the generalized erosion definition, we get:

$$\begin{aligned} A \overset{s}{\ominus} B = \{x \mid \#(A^c \cap (B)_x) < s\} &= \{x \mid \#(A \cap (B)_x) \geq \#B - s + 1\} \\ &= \{x \mid \#(A \cap (B)_x) \geq \overline{s_B}\} \end{aligned}$$

Proposition 19. *The extended dilation and erosion may be obtained from each other by reflecting the kernel set and complementing the strictness relative to the kernel set:*

$$A \overset{s}{\oplus} B = A \overset{\overline{s_B}}{\ominus} \breve{B} \tag{23}$$

$$A \overset{s}{\ominus} B = A \overset{\overline{s_B}}{\oplus} \breve{B} \tag{24}$$

where $s \in [1, \#B]$.

Proof. Results directly from (22) when using the fact that the complement of the strictness $\overline{s_B}$ relative to B is s.

Corollary 20. *When B is symmetric to reflection (that is* $B = \breve{B}$*), the dilation and erosion of A by B give identical results when using the strictness:* $s = (\#B + 1)/2$.

Following the last proposition, and the fact that the generalized dilation (erosion) is decreasing (increasing) with respect to the strictness s, it is possible to observe that the generalized dilation (erosion) is turned into erosion (dilation) when increasing the strictness s (assuming that the kernel is symmetric to reflection). That is, a generalized dilation (erosion) which is too strict is turned into erosion (dilation).

An example to generalized dilation is presented in Figure 1, where the original image is presented at the top left, and following it towards the bottom right are the resulting images when performing generalized dilation with strictness of 1–9 respectively. The light gray background in the resulting images represents the original image. The morphological kernel that is used in this example is a 3×3 square with the origin at its center. Since the kernel in that example is symmetric to reflection, the presented results are also the results obtained for generalized erosion with strictness of 9–1 respectively. As could be observed, while in ordinary morphology it is possible to get only the edges of the sequence in Figure 1, in generalized morphology it is possible to get any element of the sequence by selecting the required strictness. It is also possible to observe that the generalized dilation and erosion are essentially the same, and differ only by the degree of strictness and by a reflection of the kernel.

Based on the extended duality proposition, and the fact that the generalized dilation is commutative, it is possible to construct a proposition concerning the exchange between the arguments of a generalized erosion operation.

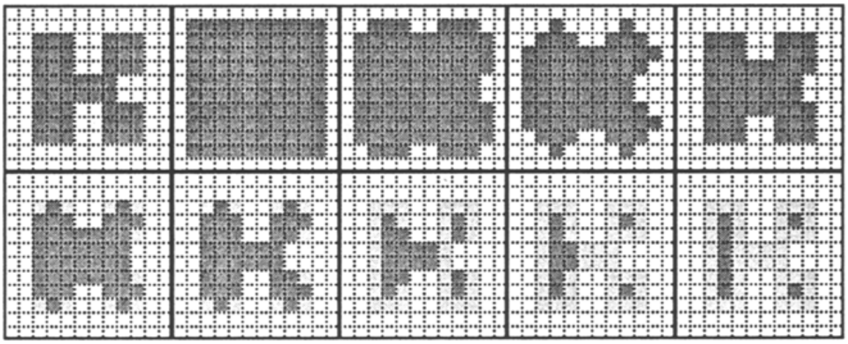

Fig. 1. Demonstration of generalized morphological operations.

Proposition 21. *It is possible to exchange the arguments of a generalized erosion operation provided that the arguments are reflected, and that the strictness is updated respectively:*

$$A \overset{s}{\ominus} B = \breve{B} \overset{\#A-\#B+s}{\ominus} \breve{A} \tag{25}$$

where $s \in [1, \#B]$.

Proof. By developing the left side of the proposition according to the extended duality proposition, and using the fact that the generalized dilation is commutative, we get:

$$\begin{aligned} A \overset{s}{\ominus} B &= A \overset{\overline{s_B}}{\oplus} \breve{B} = \breve{B} \overset{\#B-s+1}{\oplus} A \\ &= \breve{B} \overset{\#A-(\#B-s+1)+1}{\ominus} \breve{A} = \breve{B} \overset{\#A-\#B+s}{\ominus} \breve{A} \end{aligned}$$

3 Conclusion

This section presents some preliminary results obtained by using the generalized morphological operators. Using the generalized operations in existing algorithms with strictness greater than one, may increase the the resistivity of the algorithms to noise and small intrusions. The example in Figure 2 demonstrates a simple skeletonization algorithm [9] where Figure 2-a presents the original image, Figure 2-b presents results obtained by the algorithm when using ordinary morphological operators, and Figure 2-c presents the results obtained when using the same algorithm with the generalized morphological operators and strictness greater than one. As can be observed, the results obtained when using the generalized operators are less influenced by the noise on the shape.

Figure 3 presents the results of an ordinary close and a generalized close [8] that is based on the generalized morphological operators, where a directional kernel (in direction of 45° is used). The original image is presented in Figure 3-a, the result of an ordinary close is presented in Figure 3-b, and the result of a generalized close with strictness greater than one is presented in Figure 3-c.

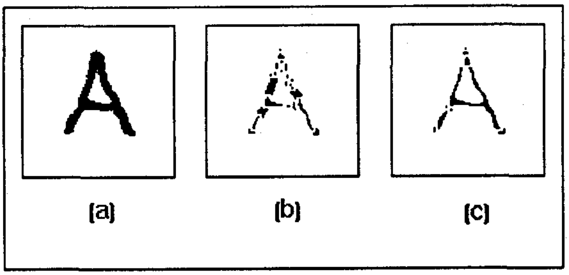

Fig. 2. Demonstration of skeletonization based on generalized operators.

As could be observed the generalized close operation managed to connect the dashed lines in direction of 45° without influencing the horizontal dashed lines.

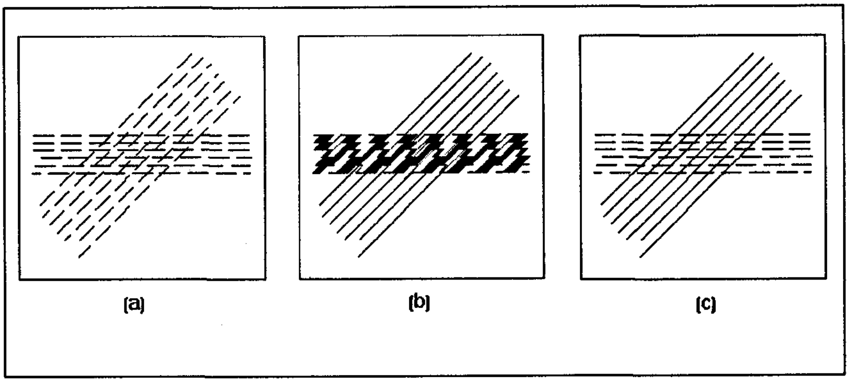

Fig. 3. Demonstration of generalized close with extreme and relaxed strictness.

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