One-Pass, Optimal Tree Parsing —

With Or Without Trees

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Abstract

This paper describes the theory behind and implementation of wburg, a code-generator generator that accepts tree grammars as input and produces a code generator that emits an optimal parse of an IR tree in just a single bottom-up pass. Furthermore, wburg eliminates the need for an explicit IR tree altogether. The grammars that wburg-generated parsers can parse are a proper subset of those that two-pass systems can handle. However, analysis indicates that wburg can optimally handle grammars for most instruction sets (e.g., SPARC, MIPS R3000, and x86).

1 Introduction

Compilers often use intermediate representation (IR) trees to represent expressions. A compiler's front end generates an IR tree, and the back end walks the tree, emitting appropriate assembly language instructions and operands.

Several automatically generated code generators perform pattern matching on IR trees to emit optimal code. The code-generator generator consumes a cost-augmented tree grammar with associated semantic actions. The resulting code generator requires two passes over the IR tree to determine a least-cost parse and to execute the associated semantic actions. Code-generator generators based on this model include BEG [ESL89], twig [AGT89], burg [FHP92b, Pro92], iburg [FHP92a], and 1burg [FH95].

This paper describes the theory behind and implementation of wburg, a code-generator generator that accepts tree grammars as input and produces a code generator that emits an optimal parse of an IR tree in just a single bottom-up pass. Furthermore, wburg eliminates the need for an explicit IR tree altogether. The generated parser emits optimal code, and can do so without retaining an entire IR tree during its single pass. The grammars that wburg-generated parsers can parse are a proper subset of those that the two-pass systems can handle. However, analysis indicates that wburg can optimally handle grammars for most major instruction sets, including the SPARC, the MIPS R3000, and the x86.
Our system has the advantage of not requiring a dynamically allocated IR tree, but sometimes suffers from its small amount of additional bookkeeping. Preliminary experiments indicate that \texttt{wburg}'s one-pass parsers run over 30\% faster than two-pass \texttt{burg}-generated parsers that use \texttt{malloc} and \texttt{free}. Even when the overhead of dynamic allocation is completely factored out, the one-pass parsers ranged from over 50\% faster to 3\% slower than \texttt{burg}'s parsers.

2 Related Work

Many code-generator generators use cost-augmented instruction patterns to produce code generators that optimally parse IR trees. The resulting code generator uses tree pattern matching and dynamic programming to find least-cost instruction sequences for realizing the given IR tree's computations. The various code-generator generators differ in the pattern-matching technology they use, and in when they perform dynamic programming. Pattern-matching techniques vary widely in theoretical efficiency [HO82]. Previous code generators have employed many of these pattern-matching technologies, ranging from the slowest, naive [ESL89, FHP92a, FH95], to top-down [AGT89], to the fastest, bottom-up [FHP92b, Pro92, BDB90, PLG88]. Most systems perform dynamic programming at compile-time [ESL89, AGT89, FHP92a, FH95]; those based on bottom-up rewrite system (BURS) technology do all dynamic programming at compile-compile time [FHP92b, Pro92, BDB90, PLG88].

Previous tree-pattern matching systems required two passes over the IR tree: one for labeling the tree with dynamic programming information, and another for selecting the least-cost parse based on that information. To enable two tree walks, this design requires allocating and building an explicit IR tree. In contrast, \texttt{wburg}'s parsers can find an optimal parse in a single pass. Surprisingly, an explicit IR tree is not needed at all. The procedure invocations necessary to build a tree in a bottom-up fashion form a trace of the tree's structure — much like a recursive-descent parser traces out a parse tree — which is all that is necessary for our system to produce optimal code.

The Oberon compiler's code generator works in a single pass without the benefit of an explicit IR tree [WG92]. In order to generate complex addressing modes, the code generator retains a constant amount of information to guide subsequent instruction selection decisions. \texttt{wburg}'s parsers must also retain a constant amount of information for subsequent decisions. Oberon's code generator, however, does not generate optimal code for all expressions, and is not automatically generated.

3 \texttt{burg} Automata

\texttt{burg} is a code-generator generator that accepts a cost-augmented tree grammar as input and automatically produces a tree-parser that gives an optimal parse of an IR tree using a two-pass algorithm [FHP92b]. \texttt{burg} does all dynamic programming at compile-compile time and creates a state machine for guiding
stmt: \text{ASGN}(addr, \text{reg}) = 1 (1);
addr: \text{ADD}(\text{reg}, \text{con}) = 2 (0);
addr: \text{reg} = 3 (0);
reg: \text{ADD}(\text{reg}, \text{con}) = 4 (1);
reg: \text{con} = 5 (1);
con: \text{CONST} = 6 (0);

Figure 1: A burg-Style Grammar, \(G\)

all subsequent pattern matching and instruction selection decisions. Rules in a burg grammar have the form

\textit{nonterminal} : \textit{rule} = \textit{rule\_number} (\textit{cost});

Consider the burg grammar, \(G\), in Figure 1. The fourth rule specifies that the result of adding an immediate value to a value held in a register can be placed in a register at the cost of one. The second rule in Figure 1 corresponds to the "register-offset" addressing mode supported by many architectures, which is free. A burg grammar describes the operations and addressing modes of a particular architecture. Rules in a burg grammar have one of two forms. A chain rule has only a single nonterminal on its right-hand side, while a base rule always has a terminal (operator) on its right-hand side. In Figure 1, rules 3 and 5 are the only chain rules.

We restrict our analysis to grammars in normal form [BDB90]. A grammar is in normal form if all patterns are either chain rules, or base rules of the form \(n_0 \rightarrow \theta(n_1, \ldots, n_k)\) where \(n_i\) are all nonterminals, \(\theta\) is an operator, and \(k\) is the arity of \(\theta\). Normal-form grammars are no less expressive than other tree grammars, and this restriction greatly simplifies our discussion. We also restrict our discussion to binary, unary, and leaf operators.

Parsers produced by burg use a two-pass algorithm to give an optimal (least-cost) parse of a tree. The first, bottom-up pass over the tree labels each node with a state. A state encodes, for each nonterminal in the grammar, the rule to apply at the current node to derive that nonterminal at least cost. (The burg states and transition function implicitly encode the cost of each rule, and therefore explicit cost comparisons are avoided completely at compile-time — this is one of burg's major advantages.) The second, top-down pass finds the least-cost parse based on those states. This second pass reduces the tree by applying the appropriate rules at nodes in the tree.

Figure 2 shows the set of burg-generated states for grammar \(G\) (Figure 1). State 1 encodes the fact that an ADD node labeled with state 1 is reduced to an addr nonterminal using rule 2 and to a reg using rule 4. State 3 represents the label for all CONST nodes. CONST nodes are reduced directly to a con nonterminal via rule 6, but require the application of chain rules for addr and reg nonterminal reductions. Note that reducing a CONST node to an addr would require two
Figure 2: The \texttt{burg} States for $G$

chain rule applications (rules 3 and 5) followed by the base rule 6. Also note that there is not necessarily a 1:1 correspondence between between states and operators; more than one state corresponding to a particular operator may exist. All reductions at a state follow this pattern of zero or more chain rules followed by exactly one base rule. (Bottom-up reductions would reverse that order.) The application of a base rule at a node with children will cause those children to be reduced to the nonterminals on the right-hand side of the base rule.

4 One-Pass Tree Parsers

A two-pass system defers all reduction decisions until labeling is complete. Because every node in the tree contains a label, an unbounded amount of information must be retained prior to starting the reduction process. If reductions could be done during the bottom-up labeling pass, any subtree that is reduced could be discarded since reduction is the last phase of pattern matching. The labeler would only retain unreduced subtrees until they could be reduced.

(Note: Reducers normally apply rules in a top-down order, but the semantic actions associated with those rules typically are applied in bottom-up order — the order in which code is emitted. From this point forward, we will assume that reducers reduce trees bottom-up.)

For some states, optimal reductions can be made during the bottom-up labeling pass. Because rule 1 is the only rule in state 2, it must be used to reduce any node labeled with state 2. Therefore, the labeling pass can \textit{immediately} apply rule 1 to any node it labels with state 2. \texttt{wburg} creates an array, \texttt{reduce\_now[ ]}, that is indexed by state numbers. Whenever a state contains only one base rule,
**Figure 3: Array reduce_now for Grammar G**

wburg places the appropriate nonterminal (the nonterminal appearing on the left-hand side of that base rule) into the table.

Reducing a node via a base rule will cause all descendents of that node to be reduced, and thus no longer needed. Although not true for state 2, applying a base rule does not always completely reduce the given node. Subsequent reductions of ancestor nodes may require the application of chain rules at this node in order to derive a nonterminal other than the left-hand side of the base rule. Therefore, the node itself must be retained for the possible chain-rule applications later.

Now consider state 1, which contains two base rules, 2 and 4. The labeler cannot immediately determine the appropriate reduction when it reaches such a node. Therefore, wburg must mark state 2's entry in reduce_now[] as unknown (⊥). Figure 3 gives reduce_now[ ] for grammar G. Formally,

$$reduce\_now[A] \equiv N \quad \text{iff state } A \text{ contains only one base rule, } N \to \phi(...)$$

Although state 1 contains two base rules, and a decision about which to apply depends on reductions at ancestor nodes, the state does contain enough information to guide the reductions of its children. Both base rules have the same vector of nonterminals — (reg, con) — on the right-hand side. Therefore, any node labeled with state 1 can immediately reduce its left child to reg and its right child to con. This fully reduces its children. The labeler must defer reductions of the state 1 node, however.

wburg could generate vectors like reduce_now[ ] to map states to the nonterminals to which their children should be reduced. Instead, we generalize this notion and create tables that guide child reductions based on the states of both the child and the parent. Two two-dimensional arrays, left_nt[ ][ ] and right_nt[ ][ ], hold the nonterminals to which a node's children should be reduced. left_nt[B][A] holds the nonterminal to which a node labeled with B should be reduced if it is the left child of a node labeled with A.

An entry is put in left_nt[B][A] if every left-most nonterminal on the right-hand side of every base rule in A derives from the same base rule in B via zero or more chain rules. Under these conditions, the nonterminal on the left-hand side of B's base rule, N, is put in the table. This condition ensures that wburg knows which base rule in B ultimately will be applied when B is the left child of A. right_nt is similar. Formally,

$$left\_nt[B][A] \equiv N \quad \text{iff } X \to \theta(Y, -) \in A \text{ implies } N \to \phi(...) \in B \text{ and } Y \Rightarrow N \text{ in } B.$$ $right\_nt[B][A] \equiv N \quad \text{iff } X \to \theta(-, Y) \in A \text{ implies } N \to \phi(...) \in B \text{ and } Y \Rightarrow N \text{ in } B.$

Figure 4 gives the tables for G. A "⊥" indicates that no immediate reduction is possible.
4.1 Characteristic burg Graph

For each cost-augmented tree grammar, burg generates at compile-compile time a function state that, when invoked at compile-time, computes the state of a particular node given its operator and the states of its children.

\[
\text{state} : \text{operator} \times \text{state}_{\text{left}} \times \text{state}_{\text{right}} \rightarrow \text{state}
\]

Using state, we can generate a characteristic directed graph to represent states and state transitions. Nodes represent states, and edges represent possible state transitions. Edges are annotated with either left or right, depending on whether the target node represents the left or right child of the source node. Formally,

\[
A \xrightarrow{\text{left}} B \iff \exists \theta, X \text{ such that state}(\theta, B, X) \equiv A
\]

\[
A \xrightarrow{\text{right}} B \iff \exists \theta, X \text{ such that state}(\theta, X, B) \equiv A
\]

Figure 5 shows the characteristic graph for G. A characteristic graph depicts which states can be derived from combinations of operators and certain child states. For instance, the given graph indicates that a node in state 1 must have a state 3 right child, but can have a state 1 or 3 left child. We can use the characteristic graph to generate “trees” of states by starting out at a root node (state 2 in this example), and doing a traversal of the graph, following one left- and one right-labeled edge each time we encounter a node. (Different edges may be chosen each time a given node is encountered in a traversal.) The nodes visited and the edges followed represent a tree of states. In general, the characteristic graph can generate a superset of the trees that a grammar describes because not all combinations of left and right-labeled arcs leaving a particular state can actually be combined to generate that state. Only cyclic graphs generate trees of unbounded size.

A bound on the sizes of the trees generated by an acyclic graph is simply the size of the largest tree in the graph. Therefore, this bound is the maximum amount of information that must be retained between passes of a two-pass parser.

4.2 Arc Pruning

Analysis of the characteristic graph leads to a method of doing one-pass parsing that retains only a constant amount of information about the reduction of any
Figure 5: Transition Graph for 

Figure 5: Transition Graph for burg States of Grammar G

labeled node and the subtree it roots. burg will prune an arc from the graph if
the arc corresponds to a state transition for which a correct base-rule reduction
of the child node is known. The remaining arcs represent the reductions that
the analysis cannot immediately determine and therefore must postpone until
the parser can safely reduce an ancestor. If the pruned graph is acyclic, then
only a bounded number of nodes must ever be retained for subsequent reduction.
Because the original graph (Figure 5) is cyclic, one-pass parsing with grammar
G is not possible without arc pruning.

Arcs leaving state 2 may be pruned because the parser can immediately
reduce the left and right children of a state 2 node. After the parser reduces
the children of a state 2 node, the parser would retain those nodes, but not
their children (the node's grandchildren). The children may require subsequent
chain-rule reductions, but the grandchildren will be completely reduced at this
point. (The condition for pruning arcs only guarantees that one of the child’s
base rules can be applied immediately — application of the child’s chain rules
may be deferred, and so it is necessary to record the last nonterminal to which
the child was reduced.)

The entries in left_nt and right_nt correspond directly to the pruned arcs.
An arc \( A \xrightarrow{\text{left}} B \) may be pruned if \( \text{left}_\text{nt}[B][A] \neq \perp \) because the labeler can
immediately reduce \( B \) to the given nonterminal, which forces the reduction by
its base rule.

If, after applying the arc pruning rule, the resulting graph contains no cycles,
then the grammar can be parsed optimally during the single, bottom-up labeling
pass while retaining only a constant number of unreduced nodes at any given
point. A conservative upper bound on the number of unreduced nodes is the
size of the largest tree generated by the pruned graph plus information about
defered chain rule reductions of the nodes just off the leaves of that tree.

If the pruning rule removes all directed edges from the graph (as it will for
grammar \( G \)), then the largest subtree whose reduction may have to be deferred
consists of a tree node and its children. Its children will lack, at most, chain-rule applications.

5 A Code-Generator Generator

5.1 One-Pass, Optimal Parsing of IR Trees

If the application of the pruning rule creates an arc-free graph, then labeling and reducing the tree can be done in a single-pass while deferring at most three nodes worth of reductions at the most recently labeled node: the reductions at that node, and possible chain rule reductions at its children. All descendents further down the tree must be completely reduced.

\textit{wburg} is a system for generating optimal, single-pass tree-parsers for grammars for which the characteristic graph can be completely pruned of arcs. Note that this is the same class of grammars for which \textit{wburg} can fill all entries in \textit{left\_nt} and \textit{right\_nt} that correspond to valid state transitions.

Because the reductions induced by \textit{wburg}'s \textit{reduce\_now}, \textit{left\_nt}, and \textit{right\_nt} tables are identical to the reductions that \textit{burg}'s parser would apply, both systems find identical least-cost parses.

Consider the procedure \textit{Compose} in Figure 6, which creates a new node from an operator and two child nodes. \textit{Compose} labels a node and attaches that node to its children. Then, it initializes the node's nonterminal field to zero, indicating that the current node has not yet been reduced. The procedure checks the array \textit{reduce\_now} to see whether the node itself can be reduced immediately. If so, \textit{Compose} will immediately reduce the entire subtree rooted at this node (except, of course, previously reduced nodes).

If \textit{reduce\_now} cannot help, the parser consults \textit{left\_nt} and \textit{right\_nt} to obtain the appropriate nonterminals to which the children should be reduced and performs those reductions. In this case, \textit{Compose} does no reductions at node itself, but it does invoke reductions of its children. The node's reductions and the children's chain rule reductions must wait until this node's parent is labeled and causes subsequent reductions.

Figure 7 gives the code for \textit{reduce}, which performs reductions. Nodes previously reduced to the desired nonterminal will terminate the reduction. \textit{arity(rule\_number)} gives the number of nonterminals on the right-hand side of \textit{rule\_number}, which represents the number of subsequent reductions this rule must invoke. For chain rules, \textit{kids[]} simply holds \textit{node}; for base rules, it is a vector of the node's children. \textit{nts[]} is a table of the nonterminals to which the \textit{kids[]} should be reduced based on the rule number.

5.2 One-Pass, Optimal Parsing Without Trees

Because \textit{burg} and other tree-parsing algorithms require two-passes over the IR tree, the IR tree must be retained until the second pass is complete. Because a tree may be of arbitrary size, the tree nodes must be dynamically allocated and deallocated. \textit{wburg}'s parsers parse an IR tree in a single bottom-up pass, but
procedure Compose(out node_ptr node, in int op, in node_ptr left_child, in node_ptr right_child)

node.left = left_child // Assemble tree.
node.right = right_child
node.state = state(op, left_child.state, right_child.state) // Label tree.
node.nt = 0 // Node is not reduced.
if (reduce_now[node.state] ≠ ⊥) then
  reduce(node, reduce_now[node.state]) // Reduce immediately.
else
  left_nt = left_nt[left_child.state][node.state]
  right_nt = right_nt[right_child.state][node.state]
  reduce(left_child, left_nt) // Reduce children’s base rules.
  reduce(right_child, right_nt)
end if
end procedure

Figure 6: Procedure Compose

procedure reduce(in/out node_ptr node, in int nt)
if nt = node.nt then return // Previously reduced.
rule_number = rule(node.state, nt) // Find rule to apply.
for i = 1 to arity(rule_number) do
  reduce(node.kids[i], nts[rule_number][i]) // Invoke subsequent reductions.
end for
// Invoke semantic action for rule here.
node.nt = nt // Record reduction.
end procedure

Figure 7: Procedure reduce

reductions at a given node may require reducing child nodes further. This seems to indicate that the subtrees descending from each node must be retained until the entire tree is reduced, but that is not the case.

Retaining entire subtrees is overly conservative. The analysis described in Section 4 guarantees that when Compose processes a node, the only deferred reductions possible for the subtree rooted at that node are chain rule reductions at its children, and reductions at itself. No deferred reductions can exist below its children. Therefore, all deferred reductions at a given node can be summarized by the state and nt fields of that node and its children. Of course, this information can be kept (via copying) in the node itself, thus eliminating any need to retain child nodes at all after creating a new node. A node exists only until its parent is created. Thus, it is possible for the parser to eliminate the creation of the explicit IR tree. Furthermore, the parser can use the run-time stack to store nodes, eliminating the need for any dynamic heap allocation or deallocation.

Consider the modified Compose procedure, Compose', in Figure 8. Compose'
procedure Compose'(out node_ptr node,  
in int op, in node_ptr left_child, in node_ptr right_child)  
node.self.state = state(op, left_child.self.state, right_child.self.state)  
node.self.nt = 0  
If (reduce_now[node.self.state] ≠ ⊥) then  
  node.self.left = left_child.self  // Create (tiny) tree for  
  node.self.right = right_child.self  // immediate reduction of  
  reduce(node.self, reduce_now[node.self.state])  // node (and its children).  
else  
  left_nt = left_nt[left_child.self.state][node.self.state]  
  right_nt = right_nt[right_child.self.state][node.self.state]  
  reduce(left_child.self, left_nt)  // Reduce children.  
  reduce(right_child.self, right_nt)  
  node.retain_left = left_child.self  // Retain children for later  
  node.retain_right = right_child.self  // chain rule applications.  
  node.self.left = addressof node.retain_left  // node represents 3-node tree.  
  node.self.right = addressof node.retain_right  
end if  
end procedure

Figure 8: Procedure Compose'

invokes reduce whenever possible. Nodes retain all necessary information for the deferred reductions of their children. The node retains the summary of its state (node.self) and the retained state of its children (node.retain_left and node.retain_right) — the pruning analysis guarantees that this is sufficient for one-pass parsing. Each node maintains pointers (left and right) to its children to give the reducer the illusion of an actual tree. Compose' must maintain these pointers, invoke the reducer when possible, and copy the states of the children into the root node when necessary. (Note that the left and right children of node.retain_left/right are never accessed because the only rules they may defer are chain rules.)

If the array reduce_now indicates that an immediate reduction of the current node can be made, then pointers to the two children are copied into the node, and the entire subtree rooted at the current node is reduced. If the reduction of the current node must be deferred, the left and right children are reduced, and their root nodes are copied into node.retain_left and node.retain_right for later chain-rule reductions. The node’s left and right pointers are set to point to those fields.

Compose' constructs nodes and invokes parsing reductions. Each node retains all the information necessary for any deferred reductions at, or below, itself. Therefore, the nodes from which it was built are no longer needed. Bottom-up tree building may require maintaining multiple nodes that each represent the unreduced portion of an entire subtree. Fortunately, such nodes can often be kept on the run-time stack.
Appendix A contains a complete one-pass parsing example.

6 Implementation

wbureg is an extension of bureg. wbureg and bureg, therefore, have identical specification languages. wbureg uses the output of bureg to build and prune the characteristic graph. For suitable grammars, it creates reduce_now, left_nt, and right_nt tables.

6.1 Experimental Results

While the set of grammars that can be parsed optimally using wbureg's parsers is a proper subset of the grammars that can be parsed optimally using bureg's, analysis indicates that most useful grammars, including those describing the SPARC, the MIPS R3000, and the x86 architectures, fall within this subset.

We compared the speed of wbureg's one-pass parsers with bureg's two-pass parsers. To the best of our knowledge, bureg produces faster two-pass parsers than any other parser-generator system. We present speeds for bureg's parsers both with and without dynamic memory allocation. For dynamic allocation, we use malloc and free. We also present wbureg's parsers both with explicit trees (using Compose) and without trees (using Compose'). wbureg's parsers with trees do not do dynamic memory allocation. Tests utilized modified versions of lcc grammars for the SPARC and the x86 [FH95].

The tests consisted of labeling and reducing the 1,234 unique trees lcc generates when it compiles itself. Each test parsed a set of 1,234 trees (a total of 11,587 nodes) 500 times. The tests were performed on lightly loaded DEC Alpha and Sun SPARCstation workstations. The times in Table 1 were measured using the system clock.

Clearly, wbureg's parsers without trees are faster than bureg's when bureg's trees are allocated via malloc and free.

When both bureg and wbureg parsers operate on trees and do no dynamic memory allocation, wbureg's parsers must do bookkeeping to avoid redundant reductions that bureg's do not need to do. wbureg's parsers avoid a second tree walk, but occasionally must visit a node a second time to do deferred chain-rule
reductions. When measured with pixie on the Alpha, the number of cycles executed for wburg's parsers was only 1-2% greater than those for burg's. The difference in actual running time ranged between +3% and -7%, and we believe this is due to cache-effects, which are not measured by pixie.

On the SPARC, wburg's parsers were over 50% faster than burg's even though they were saddled with additional bookkeeping to avoid redundant reductions. While very pleased by these results, we cannot currently explain them other than to speculate. Possible explanations include reduced register-window spilling from only one recursive tree walk and beneficial cache-effects.

The relative speeds of burg's parsers and wburg's parsers appear to be independent of grammar, which is what we expected. Because of the overhead of copying child states into parent nodes, wburg's parsers without trees are slower than those with trees.

6.2 lcc Grammars

The lcc grammars are written for the lburg code-generator generator, and we modified them for burg. lcc grammars allow rule costs to be determined at runtime, which burg cannot handle, so we had to alter the grammar to use constant costs. We also put lcc grammars into normal form. After the changes, all three grammars met the sufficient restrictions for one-pass optimal code generation without further modification. lcc puts all trees in a canonical form (e.g., all constant operands appear as the right child of commutative operators), and this greatly simplifies the code generation grammars. Tests demonstrate, however, that wburg can handle non-canonical grammars for all possible permutations of complex addressing modes.

7 Conclusion

We have developed the theoretical basis for optimal, one-pass tree pattern matching. Using this theory, we have developed wburg, a code-generation system based on optimal tree-pattern matching that has two important advantages over previous systems: the code generator does labeling and reducing in a single parsing pass, and the code generator does not need to build an explicit IR tree. Both advances result in time and space advantages.

References


A Example One-Pass Parsing

We will outline the one-pass construction and reduction of the seven-node tree given in Figure 9. This example utilizes the grammar and tables described throughout this paper. burg states appear underneath the operators in the tree. burg's left-to-right, bottom-up reduction of the example tree appears in Figure 10. Each rule in the reduction is annotated with the node at which the rule was applied.

Figure 11 gives the necessary calls for building and reducing the example tree using a wburg-derived one-pass parser. A–G are the tree nodes being composed, which correspond to the labels in Figure 9. The final call to reduce guarantees that the root node is fully reduced to the goal nonterminal, stmt.

Figure 12 gives the (nearly identical) calls for reducing the same tree, only without actually constructing the tree. Note that the nodes X–Z are reused after