Reconstruction
in Different Classes of 2D Discrete Sets

Attila Kuba

Department of Applied Informatics, József Attila University,
Árpád tér 2, H-6720 Szeged, Hungary
`kuba@inf.u-szeged.hu`

Abstract. The problem of reconstruction of two-dimensional discrete
sets from their two projections is considered in different classes. The re-
construction algorithms and complexity results are summarized in the
case of hv-convex sets, hv-convex polyominoes, hv-convex 8-connected
sets, and directed h-convex sets. We show that the reconstruction al-
gorithms used in the class of hv-convex 4-connected sets (polyominoes)
can be used, with small modifications, for reconstructing hv-convex 8-
connected sets. Finally, it is shown that the directed h-convex sets are
uniquely reconstructible with respect to the row and column sum vectors.

1   Introduction

One of the most frequently studied reconstruction problems is the construction of
2-dimensional discrete sets from their row and column sum vectors. This kind of
questions can arise, for example, in electron microscopy [9], image processing [18]
and radiology [16]. The general problem of reconstructing binary matrices (or,
equivalently, discrete sets) has been extensively studied. Existence, consistence
and uniqueness results are known (as a summary see [3]) and reconstruction
algorithms are published (e.g., [6,17]). Since, in general, the number of solutions
can be very high, there is an interest to reconstruct binary matrices/discrete
sets having special properties. Kuba published an algorithm to reconstruct so-
called two-directionally connected discrete sets [15]. Del Lungo *et al.* studied
the reconstruction of different kinds of polyominoes [11,1]. Recently, Chrobak
and Dürr found reconstruction algorithms for special polyominoes. Since the
reconstruction in certain classes can be too complex (see [19]), the classes, where
the reconstruction can be performed in polynomial time, are the most important
for the applications.

This paper deals with the reconstruction problem in different classes of dis-
crete sets. The most frequently used property is some kind of discrete version of
the convexity. For example, we can suppose that the elements of the discrete sets
in each row and column are consecutive. Since this property itself is not enough
to have a reconstruction problem which is solvable in polynomial time (see [19]),
further properties can be included (e.g., connectedness). From the literature five
properties are collected (h-convex, v-convex, 4-connected, 8-connected, and di-
rected discrete sets) and the classes constructed from these combinations are

studied from the viewpoint of reconstruction. As a result of this survey study, we have found classes where known algorithm, after small modifications, can be used for reconstructing discrete sets of other classes. Also, a new reconstruction algorithm is given.

First, the necessary definitions and notations are introduced. Then in the next sections the reconstruction problem in the following classes are discussed: hv-convex sets, hv-convex polyominoes, hv-convex 8-connected sets, and directed h-convex sets.

2   Definitions and notations

Let $\mathbb{Z}^2$ denote the 2-dimensional integer lattice. In this paper the elements of $\mathbb{Z}^2$ will be called points or positions. The finite subsets of $\mathbb{Z}^2$ will be called *discrete sets*. The class of discrete sets will be denoted by $\mathcal{F}$.

Let F be a discrete set. Then there is a *discrete rectangle* T of size $m \times n$ (m and n are positive integers),

$$T = \{1, \ldots, m\} \times \{1, \ldots, n\}, \tag{1}$$

such that T is the smallest discrete rectangle containing F. The discrete set F can also be represented as a binary matrix $(f_{ij})_{m \times n}$, $f_{ij} \in \{0, 1\}$, such that

$$f_{ij} = \begin{cases} 1, & \text{if } (i,j) \in F, \\ 0, & \text{otherwise.} \end{cases}$$

For any subset of T we define the *i-th row* of the subset, $1 \le i \le m$, as its intersection with $i \times \{1, \ldots, n\}$. Similarly, the *j-th column*, $1 \le j \le n$, of the subset is its intersection with $\{1, \ldots, m\} \times j$.

Let $\mathbb{N}$ denote the set of positive integers. For any discrete set F we define its *projections* by the operations $\mathcal{H}$ and $\mathcal{V}$ as follows. $\mathcal{H} : \mathcal{F} \longrightarrow \mathbb{N}^m$, $\mathcal{H}(F) = H = (h_1, \ldots, h_m)$ where

$$h_i = \sum_{j=1}^{n} f_{ij}, \qquad i = 1, \ldots, m, \tag{2}$$

and $\mathcal{V} : \mathcal{F} \longrightarrow \mathbb{N}^n$, $\mathcal{V}(F) = V = (v_1, \ldots, v_n)$ where

$$v_j = \sum_{i=1}^{m} f_{ij}, \qquad j = 1, \ldots, n. \tag{3}$$

The vectors H and V will be called the *row* and *column sum vectors* of F, respectively.

Let F and F' be discrete sets. We say that F and F' are *tomographically equivalent* (w.r.t. the row and column sum vectors) if

$$\mathcal{H}(F) = \mathcal{H}(F') \quad \text{and} \quad \mathcal{V}(F) = \mathcal{V}(F'). \tag{4}$$

The discrete set F is *unique* (w.r.t. the row and column sum vectors) if there is no discrete set F' ($\neq F$) being tomographically equivalent to F.

Let $\mathcal{G}$ be a class of discrete sets. The discrete set $F \in \mathcal{G}$ is determined (by their projections) if there is no tomographically equivalent set in the class $\mathcal{G}$.

We are going to study the following problem in different classes of discrete sets.

RECONSTRUCTION($\mathcal{G}$).

Instance: *Two vectors $H \in \mathbb{N}^m$ and $V \in \mathbb{N}^n$.*

Task: *Construct a discrete set $F \in \mathcal{G}$ such that $\mathcal{H}(F) = H$ and $\mathcal{V}(F) = V$.*

Different classes of discrete sets will be introduced by the following definitions.

Let P and Q be two points in $\mathbb{Z}^2$. Let us consider the Euclidean distance between P and Q,

$$\|P - Q\| = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2}. \tag{5}$$

The points P and Q are said to be *4-adjacent* if $\|P - Q\| \leq 1$ and *8-adjacent* if $\|P - Q\| \leq \sqrt{2}$. (Explicitly, the 4-adjacent points of (i, j) are $(i - 1, j)$, $(i, j - 1)$, $(i, j + 1)$ and $(i + 1, j)$, i.e. the next points in directions north, west, east and south, respectively, and the point (i, j) itself.) The sequence of distinct points $(i_0, j_0), \ldots, (i_k, j_k)$ is a *4-path/8-path* from point (i_0, j_0) to point (i_k, j_k) in a discrete set F if each point of the sequence is in F and (i_l, j_l) is 4-adjacent/8-adjacent, respectively, to (i_{l-1}, j_{l-1}) for each $l = 1, \ldots, k$. Two points are 4-connected/8-connected in the discrete set F if there is a 4-path/8-path, respectively, in F between them. A discrete set F is *4-connected/8-connected* if any two points in F are *4-connected/8-connected*, respectively, in F. The 4-connected set is also called *polyomino* (see, for example, Golomb [12]). The classes of 4-connected and 8-connected sets will be denoted by (c_4) and (c_8), respectively. From the definitions it follows that $(c_4) \subset (c_8)$.

The discrete set F is *horizontally convex* (or, shortly, *h-convex*) if its rows are 4-connected. The class of h-convex sets will be denoted by (h). Similarly, a discrete set F is *vertically convex* (or, shortly, *v-convex*) if its columns are 4-connected. The class of v-convex sets will be denoted by (v). The h- and v-convex sets will be called *hv-convex* and their class will be denoted by (h, v). Clearly, $(h, v) = (h) \cap (v)$. Using this notation, the hv-convex polyominoes are the class of (c_4, h, v).

The sequence of distinct points $(i_0, j_0), \ldots, (i_k, j_k)$ is an *EN-path* from point (i_0, j_0) to point (i_k, j_k) in a discrete set F if each point of the sequence is in F and (i_l, j_l) is in east or north to (i_{l-1}, j_{l-1}) for each $l = 1, \ldots, k$. The discrete set F is *directed* if there is a particular point of F, called *source*, such that there is an EN-path in F from the source to any other point of F. It follows from the definition that the source point of a directed set is necessarily the point $(m, 1)$. The class of directed sets will be denoted by (d). Clearly, $(d) \subset (c_4)$.
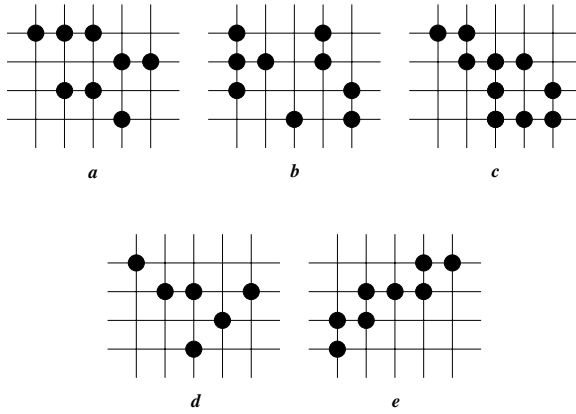
Fig. 1. Examples of discrete sets in different classes. a) h-convex discret set; b) v-convex discrete set; c) 4-connected discrete set; d) 8-connected discrete set; f) directed discrete set.

3   Reconstruction of hv-convex sets

The reconstruction of hv-convex sets was studied by Kuba [15]. He gave a greedy type heuristic algorithm, by which the set F to be reconstructed could be approximated by two sequences of discrete sets, $\{K^{(l)}\}$ and $\{S^{(l)}\}$. The first sequence is nondecreasing and it consists of hv-convex sets supposed to be the subsets of F, called *core* sets. The other sequence is nonincreasing and its consists of discrete sets supposed to contain F, called *envelope* sets. As initial core and envelope sets we can use

$$K^{(0)} = \emptyset \quad \text{and} \quad S^{(0)} = T. \tag{6}$$

The core set in each iteration will be constructed from the (previous) envelope set S and from the projections H and V as follows. First, we create the discrete set $K_h(S, H)$, the intersection of all h-convex sets contained by S and having horizontal row sum H. (It can be computed easily row by row. For example, if the i-th row of S consists of $(i, s_i'), (i, s_i' + 1), \dots, (i, s_i'')$ $(1 \leq s_i' \leq s_i'' \leq n)$ and $h_i \leq s_i'' - s_i' + 1 \leq 2 * h_i - 1$ then the i-th row of $K_h(S, H)$ is the set of $\{(i, s_i'' - h_i + 1), (i, s_i'' - h_i + 2), \dots, (i, s_i' + h_i - 1)\}$.) Similarly, using the envelope S and the column sums V the set $K_v(S, V)$ can be defined. Clearly, if $F \subseteq S$ then $K_h(S, H) \subseteq F$ and $K_v(S, V) \subseteq F$. Therefore, we can select the intersection of all hv-convex sets containing $K_h(S, H) \cup K_v(S, V)$ as the new core. (See Fig. 2.)

Analogously, the envelope set in each iteration will be constructed from the (previous) core set K and from the projections H and V as follows. First, we create the discrete set $S_h(K, H)$, the union of all h-convex sets containing K and having horizontal row sum H. (It can be computed easily row by row. For example, if the i-th row of K consists of $(i, k_i'), (i.k_i' + 1), \dots, (i, k_i'')$ $(1 \leq$
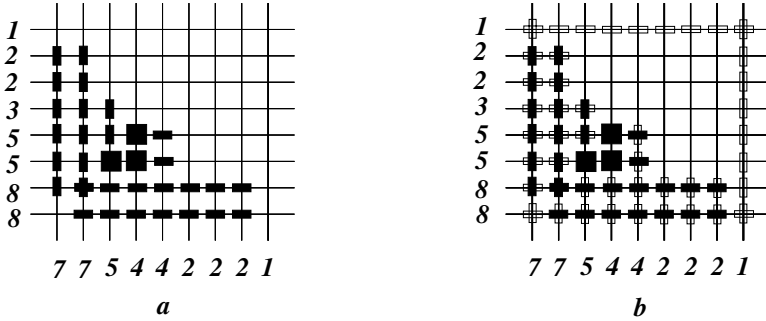
Fig. 2. Construction of core and envelope sets. a) The construction of the core set from the envelope set $S = T$ and from the projections. The elements of $K_h(S, H)$ and $K_v(S, V)$ are denoted by horizontal and vertical thick line segments, respectively. The squares denote the new elements of the intersection of all hv-convex sets containing $K_h(S, H) \cup K_v(S, V)$. b) The construction of the envelope set from the core set constructed in a). The elements of $S_h(K, H)$ and $S_v(K, V)$ are denoted by horizontal and vertical line segments, respectively.

$k'_i \leq k''_i \leq n$) and $k''_i - k'_i + 1 \leq h_i$ then the i-th row of $S_h(K, H)$ is the set of $\{(i, k''_i - h_i + 1), (i, k''_i - h_i + 2), \ldots, (i, k'_i + h_i + 1)\}$.) Similarly, using the core K and the column sums V the set $S_v(K, V)$ can be defined. Clearly, if $K \subseteq F$ then $F \subseteq S_h(K, H)$ and $F \subseteq S_v(K, V)$. Therefore, we can select the set $S_h(K, H) \cap S_v(K, V)$ as the new envelope.

During the iterations we have

$$K^{(l)} \subseteq F \subseteq S^{(l)}. \tag{7}$$

If we have $K^{(l)} = S^{(l)}$ for some l then $F = K^{(l)} = S^{(l)}$ is a solution. It may also happen that during the iterations we reach a situation when core or envelope do not change or $K^{(l)} \not\subseteq S^{(l)}$. In the first case, we can take an arbitrary element of the set $S^{(l)} \setminus K^{(l)}$, add it to the core and continue the algorithm with this new core set. In the second case we have a contradiction indicating that there is no solution between the current core and envelope. Then we can return to the arbitrary selected element (if we made such a formation trial earlier), take it away from the envelope set and continue the algorithm with this reduced envelope.

4   Reconstruction of hv-convex polyominoes

This class of discrete sets was studied first by Del Lungo *et al.* [11,1]. The method was improved [2,4] and the latest version is able to reconstruct hv-convex polyominoes in time $O(nm \cdot \log(nm) \cdot \min\{m^2, n^2\})$.

This algorithm uses also the iterative procedure applied for hv-convex sets, but it is basically different in two points: selection of the initial core and handling

the situation when the core and envelope can not be changed in the way described in Section 3.

For the selection of the initial core the concepts of feet of polyominoes are introduced. The columns of F that have element in the first row of T are called the *north foot* of F, denoted by P_N. Analogous definitions can be given for the *south*, *east*, and *west feet*, P_S, P_E. and P_W, respectively. The algorithm starts with choosing the positions of the four feet. (There are at most $O(\min\{m^2, n^2\})$ possible cases. Some of them can be sorted out soon at the beginning of the reconstruction.) Then the *hv*-convex polyomino containing the selected feet (as an initial core) is constructed. Using the iterative procedure described in Section 3 the core set is increased and the envelope set is decreased.

If the core and envelope sets can not be changed then the reconstruction is reformulated as a 2-Satisfiability problem (also referred to as 2SAT). Such situations can be described by a 2SAT expression, i.e., a Boolean expression in conjunctive normal form with at most two literals in each clause. The 2SAT expression is satisfiable if and only if there is an *hv*-convex polyomino solution of the reconstruction problem.

Recently, Chrobak and Dürr have found a new algorithm [8] reconstructing *hv*-convex polyominoes in $O(mn \cdot \min\{m^2, n^2\})$ time. The basic idea of their algorithm is to rewrite the whole reconstruction problem as a 2SAT problem (see Section 5).

5   Reconstruction of *hv*-convex 8-connected sets

This class of discrete sets was studied first by Brunetti *et al.* [4]. They showed that a modification of the algorithm suggested for reconstructing *hv*-convex polyominoes can be used even in this class. The complexity of the algorithm is $O(nm \cdot \log(nm) \cdot \min\{m^2, n^2\})$ in the worst case.

In order to show the strong connections between the classes of *hv*-convex 8-connected sets and *hv*-convex 4-connected sets (i.e., polyominoes), we present the corresponding lemmas for both classes.

Lemma 1. *Let F be a hv-convex polyomino with row and column sums H and V. If*

$$\sum_{l=1}^{j} v_l \geq \sum_{k=1}^{i-1} h_k, \quad \sum_{k=1}^{i} h_k \geq \sum_{l=1}^{j-1} v_l, \quad \sum_{k=1}^{i} h_k \geq \sum_{l=j+1}^{n} v_l, \quad \sum_{l=j}^{n} v_l \geq \sum_{k=1}^{i-1} h_k \quad (8)$$

then $(i, j) \in F$.

Proof. See [2].

Lemma 2. *Let F be a hv-convex 8-connected set with row and column sums H and V. If*

$$\sum_{l=1}^{j} v_l > \sum_{k=1}^{i-1} h_k, \quad \sum_{k=1}^{i} h_k > \sum_{l=1}^{j-1} v_l, \quad \sum_{k=1}^{i} h_k > \sum_{l=j+1}^{n} v_l, \quad \sum_{l=j}^{n} v_l > \sum_{k=1}^{i-1} h_k \quad (9)$$

then $(i,j) \in F$.

Proof. See [4].

In this section we show that the modification of the algorithm suggested by Chrobak and Dürr [8] for reconstructing *hv*-convex polyominoes can be used in the more general class of (c_8, h, v). They showed that the reconstruction of a *hv*-convex polyomino was equivalent to the evaluation of a suitable constructed 2SAT expression. Now, we give the description of the modified algorithm following the same idea as Chrobak and Dürr. Since the statements can be proved in the same way, we omit the proofs.

We say that the discrete set A is an *upper-left corner region* in the discrete rectangle T containing F if $(i+1, j) \in A$ or $(i, j+1) \in A$ implies $(i, j) \in A$. The *upper-right*, *lower-left* and *lower-right regions* can be defined analogously. Let $\overline{F}$ denote the complement of F (in T).

Lemma 3. $F \in (c_8, h, v)$ *if and only if*

$$\overline{F} = A \cup B \cup C \cup D, \quad (10)$$

where A, B, C, and D are disjoint corner regions (upper-left, upper-right, lower-left and lower-right, respectively).

Proof. See [8].

Remark 1. Lemma 3 is not valid in the class (c_4, h, v). In the class (c_4, h, v) two further conditions should be satisfied:

(i) $(i-1, j-1) \in A$ implies $(i, j) \notin D$,
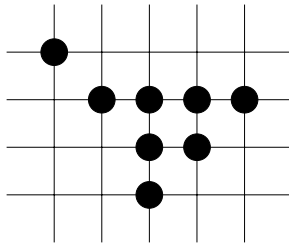(ii) $(i-1, j+1) \in B$ implies $(i, j) \notin C$.

See Fig. 3.

Fig. 3. An 8-connected *hv*-convex set which is not 4-connected.

We say that F is *anchored* at (k,l) $(1 \le k,l \le m)$ if $(k,1),(l,n) \in F$. The basic idea of the reconstruction method is to construct a 2SAT expression $F_{k,l}(H,V)$ with the property that $F_{k,l}(H,V)$ is satisfiable if and only if there is an $F \in (c_8,h,v)$ that is anchored at (k,l). The suitable $F_{k,l}(H,V)$ in the class (c_8,h,c) can be constructed as

$$F_{k,l}(H,V) = Cor \wedge Dis \wedge Anc \wedge LBC \wedge UBR, \tag{11}$$

where Cor, Dis, Anc, LBC, and UBR are sets of clauses describing the properties of "Corners", "Disjointness", "Anchors", "Lower bound on column sums", and "Upper bound on row sums", respectively, in the following way.

$$Cor = \bigwedge_{i,j}(a_{ij} \Rightarrow a_{i-1,j} \ \wedge \ a_{ij} \Rightarrow a_{i,j-1}) \wedge \bigwedge_{i,j}(b_{ij} \Rightarrow b_{i-1,j} \ \wedge \ b_{ij} \Rightarrow b_{i,j+1}) \wedge$$

$$\bigwedge_{i,j}(c_{ij} \Rightarrow c_{i+1,j} \ \wedge \ c_{ij} \Rightarrow c_{i,j-1}) \wedge \bigwedge_{i,j}(d_{ij} \Rightarrow d_{i+1,j} \ \wedge \ d_{ij} \Rightarrow d_{i,j+1}),$$

$$Dis = \bigwedge_{i,j}\{x_{i,j} \Rightarrow \overline{y}_{i,j} \mid \text{for symbols } X,Y \in \{A,B,C,D\}, \ X \ne Y\},$$

$$Anc = \overline{a}_{k,1} \wedge \overline{b}_{k,1} \wedge \overline{c}_{k,1} \wedge \overline{d}_{k,1} \wedge \overline{a}_{l,n} \wedge \overline{b}_{l,n} \wedge \overline{c}_{l,n} \wedge \overline{d}_{l,n},$$

$$LBC = \bigwedge_{i,j}(a_{ij} \Rightarrow \overline{c}_{i+v_j,j} \ \wedge \ a_{ij} \Rightarrow \overline{d}_{i+v_j,j}) \wedge$$

$$\bigwedge_{i,j}(b_{ij} \Rightarrow \overline{c}_{i+v_j,j} \ \wedge \ b_{ij} \Rightarrow \overline{d}_{i+v_j,j}) \ \wedge \ \bigwedge_{j}(\overline{c}_{v_j,j} \ \wedge \ \overline{d}_{v_j,j}),$$

$$UBR = \bigwedge_{j}\left(\bigwedge_{i \le \min\{k,l\}}(\overline{a}_{i,j} \Rightarrow b_{i,j+h_i}) \ \wedge \bigwedge_{l \le i \le k}(\overline{a}_{i,j} \Rightarrow d_{i,j+h_i})\right) \wedge$$

$$\bigwedge_{j}\left(\bigwedge_{k \le i \le l}(\overline{c}_{i,j} \Rightarrow b_{i,j+h_i}) \ \wedge \bigwedge_{\max\{k,l\} \le i}(\overline{c}_{i,j} \Rightarrow d_{i,j+h_i})\right).$$

Remark 2. In the case of hv-convex polyominoes one more set of clauses was included describing the "Connectivity" (more exactly, the 4-connectivity) of F as

$$Con = \bigwedge_{i,j}(a_{ij} \Rightarrow \overline{d}_{i+1,j+1} \ \wedge \ b_{ij} \Rightarrow \overline{c}_{i+1,j-1}). \tag{12}$$

Then the reconstruction algorithm can be given as

Algorithm 1 for reconstructing hv-convex 8-connected sets

Input. Two vectors, H and V. (It is supposed that $H \in \mathbb{N}^m$ and $V \in \mathbb{N}^n$, $\sum_i h_i = \sum_j v_j$, $1 \le h_i \le n$ for all $i = 1,\ldots,m$, $1 \le v_j \le m$ for all $j = 1,\ldots,n$, and $m \le n$.)

Step 1. for $k, l = 1, \ldots, m$
{
if $F_{k,l}(H, V)$ is satisfiable then output $F = \overline{A \cup B \cup C \cup D}$ and halt.
}

Output. print "No solution".

Theorem 1. $F_{k,l}(H, V)$ *is satisfiable if and only if there is an* $F \in (c_8, h, v)$ *having projections* H *and* V *and it is anchored at* (k, l).

Proof. See [8].

Theorem 2. *Algorithm 1 solves the reconstruction problem for 8-connected hv-convex sets in time* $O(mn \cdot \min\{m^2, n^2\})$.

Proof. See [8].

6   Reconstruction of directed h-convex sets

The class of directed hv-convex polyominoes was studied by Del Lungo *et al.* in [11].

In this section we consider a more general class, the class of directed h-convex sets, (d, h). The following reconstruction algorithm is a straight consequence of the definition of the class.

Algorithm 2 for reconstructing directed h-convex sets

Input. Two vectors, H and V. (It is supposed that $H \in \mathbb{N}^m$ and $V \in \mathbb{N}^n$, $\sum_i h_i = \sum_j v_j$, $1 \leq h_i \leq n$ for all $i = 1, \ldots, m$, $1 \leq v_j \leq m$ for all $j = 1, \ldots, n$, and $m \leq n$.)

Step 1. Let $B = (0)_{m \times n}$ be the initial binary matrix representing the discrete set to be reconstructed.
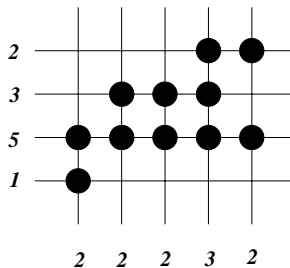

Fig. 4. Reconstruction of a directed h-convex set

Step 2. for $j = 1, \ldots, n$
 {
 if $v_j < \sum_{i=1}^m b_{ij}$ then halt (no solution);
 otherwise let i' is the maximal row index where $\sum_{l=1}^{j-1} b_{i'l} < h_{i'}$ (if there is
 no such row then halt (no solution)) and start $v_j - \sum_{i=1}^m b_{ij}$ number of rows
 (in rows $i', i'+1, \ldots$) of 1's from the column j consecutively.
 }

Theorem 3. *If F is a directed h-convex set then it is reconstructed by Algorithm 2.*

Proof. It follows from the definition of directed h-convex sets.

It is also clear that there is at most one directed h-convex set with given projections. Finally we can remark that similar algorithm and theorem can be given for directed v-convex sets.

Acknowledgments

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