Geometric Categories, O-Minimal Structures and Control

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Abstract. The theory of subanalytic sets is an excellent tool in various analytic-geometric contexts, including geometric control theory. (See [1], for example.)
One can axiomatize the notion of "behaving like the category of subanalytic sets (in manifolds)" by introducing the notion of "analytic-geometric category". (The category of subanalytic sets is the smallest analytic-geometric category.) The objects of such a category share many of the hereditary and geometric finiteness properties of subanalytic sets. Proofs of the more difficult results of this nature, like the Whitney-stratifiability of sets and maps in such a category, often involve the use of charts to reduce to the case of subsets of $\mathbb{R}^n$. For subsets of $\mathbb{R}^n$, the theory of o-minimal structures on the real field, an abstraction of the theory of semialgebraic sets, provides an elegant and efficient setting in which to work. (See [2] and [3].)
(Some reasonable sets—like $\{ (x, x^r) : x > 0 \}$ for irrational r, $\{ (x, e^x) : x > 0 \}$, and $\{ (x, \Gamma(x)) : x > 0 \}$—are not *globally* subanalytic in $\mathbb{R}^2$. Because there are o-minimal structures on the real field which include these sets, we now have available analytic-geometric categories which include these sets "at infinity" among their objects.)
In analogy with the semilinear, semialgebraic and subanalytic settings, one considers hybrid systems whose relevant data (guards, resets, flows and so on) all belong to some o-minimal structure. It can be shown, for example, that such hybrid systems admit finite bisimulations; see [4].

References

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