Comparing Intensity Transformations and Their Invariants in the Context of Color Pattern Recognition

Florica Mindru[1], Theo Moons[2], and Luc Van Gool[1,3]

[1] Katholieke Universiteit Leuven, ESAT – PSI, Leuven, Belgium
[2] Katholieke Universiteit Brussel, Brussel, Belgium
[3] Swiss Federal Institute of Technology, ETH – BIWI, Zürich, Switzerland

Abstract. In this paper we compare different ways of representing the photometric changes in image intensities caused by changes in illumination and viewpoint, aiming at a balance between goodness-of-fit and low complexity. We derive invariant features based on generalized color moment invariants – that can deal with geometric and photometric changes of a planar pattern – corresponding to the chosen photometric models. The geometric changes correspond to a perspective skew. We compare the photometric models also in terms of the invariants' discriminative power and classification performance in a pattern recognition system.

1 Introduction

Color constancy and viewpoint invariance are crucial properties of the human visual system, which motivated and inspired a great amount of work in computer vision. Substantial efforts have been targeted towards geometric invariants for contours under changing viewpoint [18,19]. These invariants do not exploit the pattern within the contours. In parallel, an extensive literature on photometric invariants of surface patterns has emerged. Much attention has been paid to the illumination independent characterization of the color distribution of the pattern [7,9,10,12,22,24,28], but there tends to be little emphasis on the spatial distribution of the colors within the pattern, or on their deformations under changing viewpoint.

In [25] and [16] it was proposed to use moment invariants, that combine invariance under geometric and photometric changes for planar patterns. This work could be considered a generalization of the work by Reiss [21]. In particular, in [16] invariants were built as rational expressions of *generalized color moments* rather than the traditional moments. Consider a color pattern, represented as a vector-valued function I defined on an image region Ω that assigns to each image point $(x, y) \in \Omega$ the 3-vector of RGB values $I(x, y) = (\,R(x, y)\,,\, G(x, y)\,,\, B(x, y)\,)$. Then the generalized color moment M_{pq}^{abc} of the pattern is given by

$$M_{pq}^{abc} = \iint_{\Omega} x^p y^q \; [R(x,y)]^a \, [G(x,y)]^b \, [B(x,y)]^c \; dxdy \; . \tag{1}$$

A. Heyden et al. (Eds.): ECCV 2002, LNCS 2353, pp. 448–460, 2002.

M_{pq}^{abc} is said to be a (generalized color) moment of *order* $p+q$ and *degree* $a+b+c$. In the sequel they will sometimes be called "moments" for short. These moments characterize the shape information (x, y) and the color information (R, G, B) in a more uniform manner, as powers of each of these are used. From a practical point of view, the advantage is that even for low orders and degrees, a sufficiently large set of such moments can be extracted to build invariants. In [16] only generalized color moments up to the first order and the second degree were considered. Also in this paper we will stick to these primary features to build invariants from because of their higher reliability.

Apart from the choice of basic features to use, another important issue to decide on when extracting invariants, are the exact transformation groups under which the invariance should hold. In [16] moment invariants were extracted and implemented that combine invariance under affine deformations (for the geometric part of the changes) and a scaling of the color bands combined with offsets (for the photometric part of the changes; scale factors and offsets were different for the different bands). The offset allows to better model the combined effect of diffuse and specular reflection [29] and has been found to give better performance [21]. In this paper, we investigate alternative choices for the geometric and photometric transformations. In particular, since we assume planar patterns, as geometric transformations we may consider

1. affine transformations
2. perspective transformations

This amounts to extracting invariants under plane affine or projective transformations, resp. (invariance under perspectivities implies invariance under projectivities, which form the encompassing group). For the photometric changes, we consider invariance under

'Type SO': scaling and an offset

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = \begin{pmatrix} s_R & 0 & 0 \\ 0 & s_G & 0 \\ 0 & 0 & s_B \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix} + \begin{pmatrix} o_R \\ o_G \\ o_B \end{pmatrix}$$

'Type AFF': affine

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = \begin{pmatrix} a_{RR} & a_{RG} & a_{RB} \\ a_{GR} & a_{GG} & a_{GB} \\ a_{BR} & a_{BG} & a_{BB} \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix} + \begin{pmatrix} o_R \\ o_G \\ o_B \end{pmatrix}$$

which have been among the most popular models in the literature and have proven most appropriate based on the considerations described in sections 2 and 3. In [16] the first choice in either case has been considered. Here we also consider other model combinations that could allow the invariants to be used on a larger range of data. The final goal is to achieve invariant features that can cope with changes in illumination and viewpoint, and which are effective under a broad range of lighting conditions without the need of additional photometric adjustments. We therefore compare the performance of the two photometric choices for the case of indoor and outdoor images, respectively. For achieving viewpoint invariance, we focus on the more complete perspective model of geometric deformations. Moment invariants can only deal with the photometric part of the changes, however, because there are no moment invariants for the projectivities [27]. Hence, these deformations are dealt with through a prior normalization step. Note that we still restrict our analysis to planar (parts of) scenes.

The structure of the paper is as follows. Section 2 compares the quality of fit of the two photometric models (Type SO and Type AFF) to actual changes as observed in indoor and outdoor images. Section 3 derives the moment invariants for both types. Section 4 discusses the outcome of a recognition experiment based on these invariants. Finally, section 5 concludes the paper.

2 Modeling Photometric Changes

Previous publications are mixed about their conclusions concerning intensity changes and achieving illumination invariance, but many seem to agree on the appropriateness of linear models for the case of planar surfaces. Investigations on photometric models were carried out in different contexts, like for instance, colour constancy algorithms ([6,8,1]), color image retrieval ([10]), color object recognition algorithms ([3,5,20,15]), and recovering lighting and reflectance properties of real scenes ([4]). The choice of a particular model also depends on the application context. On the one hand, good results have been reported based on rather simple photometric models [6,15,10] in the same vein as our Type SO transformations. On the other hand, some experiments suggested the need for more complicated, Type AFF-like ones [5,23], especially in the case of outdoor images. Part of the reasons that account for the difference is that the handled type of scenes and illumination conditions were quite different. Also, some experiments were carried out indoor, others use outdoor imagery.

A first question to be investigated is whether the Type AFF photometric transformation model is really better than other simpler transformations, like Type SO. It goes without saying that Type AFF will be able to yield a better fit to the photometric changes that are observed in real scenes, as it simply has more degrees of freedom. The relevance of such improvement has to be demonstrated before one would actually embark on the extraction of invariants for such case, as these will undoubtedly be more complicated than those for simpler photometric models. This in turn could mean that the resulting invariants are less reliable and therefore useful under fewer rather than more conditions.

In this paper we compare different types of linear transformations for the global intensity changes on planar surfaces when observed under variable viewpoint and illumination conditions. Two main cases are here considered, namely indoor images under internal changes of light and outdoor imagery under variable viewing angles and illumination. A way of comparing the possible photometric models is by means of model selection procedures. Selecting a particular model also depends on the influence of the model on the invariant features computation and performance.

For the case of outdoor imagery we performed a series of model selection tests on the following type of images. Several views of several instances of billboards (advertisement panels) were taken under different viewing angles and different illumination (natural light), without gamma correction. An overview of the 16 billboard types is given in Figure 2. The diversity among the images of the same type is illustrated by Figure 1. As can be seen, there is quite some variation

Fig. 1. Examples of images in the database of outdoor images illustrating the degree of variation in both viewpoints and illumination conditions, for 3 types of advertisement panels.

in both viewpoint and illumination conditions. About 150 pairs of billboard images belonging to the same type were considered for the photometric model selection procedure. Because linear transformation models are considered, the natural choice for the parameter estimation method is linear regression. A series of nested models ranging from diagonal to 3D affine transformations were fitted (in the least-squares sense) between the (R, G, B)-values of corresponding pixels. It turned out that in many cases the diagonal entries in the affine transformation are significantly larger than the off-diagonal entries of the linear component of the transformation. In most cases the linear component of the transformation can be approximated by one depending on fewer parameters. Unfortunately, these simplifications largely depend on the actual pattern. In particular, the subgroups that would correspond to these cases would pertain to different pairs of color bands for the different patterns. In the end, only those choices that are symmetric in the treatment of the color bands can be considered for cases where the methods are supposed to work independently from the particular pattern at hand. On the other hand, offsets were always found to be significant. This leaves us with the Type SO and Type AFF transformations. These two models were then further compared on the basis of model selection criteria.

Model selection criteria aim at choosing among several possible models the one that strikes the best balance between goodness-of-fit and low complexity. In order to quantify differences in model complexity, one typically considers the number of model parameters. This still leaves some leeway and the models are compared on the basis of several model selection criteria. These include statistical tests on the relevance of the additional parameters in the affine model (by confidence intervals, as in [11]), hypothesis testing, adjusted R^2, Mallows' $C(p)$-statistic, Bayesians information criterion, and Akaike's information criterion. Excellent references about model selection criteria are the seminal work by Kanatani [14] and the review by Torr [26]. In most cases, especially the ones taken under substantially different conditions, the full 3D affine transformations (Type AFF) were selected as statistically the best explaining model. Even though the best describing model may differ from one pairwise sample to the other, the same conclusion is reached for a particular test sample by all the model selection criteria, that is the criteria systematically agree on the outcome.

The previous analysis dealt with our outdoor images only. For the case of images of planar surfaces under artificial lighting and under indoor conditions, Gros [11] already compared our two models (and more) by means of model selection procedures. His conclusions were rather ambivalent about the statistical relevance of the two models (based on confidence intervals for the model parameters), but there was a slight preference for the Type SO model as a good compromise between complexity and accuracy.

In summary, the tests point in the direction that it is worthwhile to consider the more complex affine model (Type AFF) and the corresponding invariants. Such decision would come at a price, however. The invariants for Type AFF will be more complicated than for Type SO, and hence may be more prone to noise. Also, there will be fewer invariants for Type AFF than for Type SO that can be extracted from a given set of moments. Thus, it does not follow from the previois tests that Type AFF invariants will also give a better result in terms of recognition score. It is this overall performance that is of real interest. In the following section we derive these invariants and we will be in a position to compare their complexity and recognition performance.

3 Moment Invariants

Our goal is to build moment invariants, i.e. rational expressions of the generalized color moments (1), that do not change under the selected photometric transformations. Moreover, we prefer to only use those moments that are of a simple enough structure to be robust under noise. That means that high orders and high degrees should be avoided. Invariants involving generalized color moments up to the first order and the second degree are considered, thus the resulting invariants are functions of the generalized color moments M_{00}^{abc}, M_{10}^{abc} and M_{01}^{abc} with $(a, b, c) \in \{ (0,0,0), (1,0,0), (0,1,0), (0,0,1), (2,0,0), (0,2,0), (0,0,2), (1,1,0), (1,0,1), (0,1,1) \}$.

For the photometric part of the transformations, a complete basis of 21 invariants was extracted for Type SO and a subset of 11 out of 14 basis invariants for Type AFF. The remaining 3 invariants for Type AFF could not be extracted yet at the time of writing. This is mainly a theoretical issue, as the already extracted set still allows good recognition, as will be shown in the experimental section. An overview of the invariants is given in Table 1. The invariants can be classified according to 3 degrees of freedom: the order, the degree and the number of color bands of the moments involved. To allow maximal flexibility for the user in the choice of the color bands and to ensure the highest possible robustness of the invariants in the classification, the following 2 criteria were used to construct a basis for the moment invariants: (1) Keep the number of color bands as low as possible; and (2) include as much as possible low-order invariants. The invariants' derivation is obtained by Lie group theoretical methods ([17]).

Among the 21 invariants for Type SO, three invariants are of the type S_{11} (see Table 1) when applied to the three color bands, and similarly three invariants are of the type S^1_{12}. Six more one-band invariants are of the type S^2_{12} with $pq \in \{01, 10\}$ when applied to the three color bands. Similarly, when applied to the 3 possible combinations of 2 out of the 3 color bands, formula D^1_{12} provides 3 invariants and D^2_{12} provides 6 invariants when $pq \in \{01, 10\}$.

Of the 11 invariants for Type AFF, two are the invariants T^1_{12}, T^2_{12} and the rest are of the type $T^3_{12}(i, j)$ with $pq \in \{00, 01, 10\}$ and $ij \in \{11, 12, 22\}$.

As to the comparison of the complexity of the invariants for the two Types of photometric transformations, a first observation is that the simplest invariants for Type SO yield rational functions of the second degree, whereas this degree is raised to six in the case of Type AFF. A second observation is that the Type SO invariants can be based completely on the combination of moments that only use two color bands, whereas the Type AFF invariants need to call on all three bands simultaneously. Last but not least, it is obvious from inspection that the Type AFF invariants have a more complicated structure.

The next section investigates the practical aspect of the invariant features in terms of discriminant power and robustness to noise. The question to answer is whether the (theoretically) higher robustness of the Type AFF invariants in terms of the changes that they can withstand outweigh their far greater complexity.

4 Experimental Comparison

In order to compare the recognition performance of the two Types of photometric moment invariants and investigate their applicability on different settings, experiments were run on two main sorts of images. One kind of data consists of digital color images of outdoor advertisement panels and the second database contains indoor images of scenes under different illuminations. For both data sets the following steps were taken:

1. Extract the data from all images (with manual delineation of planar parts, if necessary), followed by the extraction of its Type SO and Type AFF moment invariants.
2. Statistical analysis of the overall sample population, and extraction of the 5 main canonical variables following a MANOVA [13] (i.e. 5 linear combinations of the moment invariants, separately for Type SO and Type AFF

Table 1. Moment invariants from both sets involving 1, 2 or 3 color bands; S_{cd} stands for 1-band invariants, D_{cd} for 2-bands invariants, and T_{cd} for 3-bands invariants of order c, and degree d, respectively. M_{pq}^{i} stands for either M_{pq}^{i00}, M_{pq}^{0i0} or M_{pq}^{00i}, depending on which color band is used; M_{pq}^{ij} stands for either M_{pq}^{ij0}, M_{pq}^{i0j} or M_{pq}^{0ij}, depending on which 2 of the 3 color bands are used.

Type SO

$$S_{11} = \frac{M_{00}^{0} M_{10}^{1} - M_{10}^{0} M_{00}^{1}}{M_{00}^{0} M_{01}^{1} - M_{01}^{0} M_{00}^{1}} \qquad S_{12}^{1} = \frac{M_{pq}^{0} M_{pq}^{2} - M_{pq}^{1} M_{pq}^{1}}{(M_{00}^{0} M_{pq}^{1} - M_{pq}^{0} M_{00}^{1})(M_{00}^{0} M_{pq}^{1} - M_{pq}^{0} M_{00}^{1})}$$

$$S_{12}^{2} = \frac{M_{pq}^{0} M_{pq}^{2} - (M_{pq}^{1})^2}{(M_{00}^{0} M_{pq}^{1} - M_{pq}^{0} M_{00}^{1})^2}$$

$$D_{12}^{1} = \frac{M_{00}^{00} M_{00}^{11} - M_{00}^{10} M_{00}^{01}}{(M_{00}^{00} M_{10}^{10} - M_{10}^{00} M_{00}^{10})^2} \qquad D_{12}^{2} = \frac{M_{pq}^{00} M_{pq}^{11} - M_{pq}^{10} M_{pq}^{01}}{(M_{00}^{00} M_{pq}^{10} - M_{pq}^{00} M_{00}^{10})(M_{00}^{00} M_{pq}^{01} - M_{pq}^{00} M_{00}^{01})}$$

Type AFF

$$k_r^1 = M_{10}^{100} M_{00}^{000} - M_{00}^{100} M_{p10}^{000} \qquad k_r^2 = M_{01}^{100} M_{00}^{000} - M_{00}^{100} M_{01}^{000}$$

$$k_g^1 = M_{10}^{010} M_{00}^{000} - M_{00}^{010} M_{10}^{000} \qquad k_g^2 = M_{01}^{010} M_{00}^{000} - M_{00}^{010} M_{01}^{000}$$

$$k_b^1 = M_{10}^{001} M_{00}^{000} - M_{00}^{001} M_{10}^{000} \qquad k_b^2 = M_{01}^{001} M_{00}^{000} - M_{00}^{001} M_{01}^{000}$$

$$J_{pq} = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} =$$

$$= \begin{pmatrix} M_{pq}^{200} M_{pq}^{000} - (M_{pq}^{100})^2 & M_{pq}^{110} M_{pq}^{000} - M_{pq}^{100} M_{pq}^{010} & M_{pq}^{101} M_{pq}^{000} - M_{pq}^{100} M_{pq}^{001} \\ M_{pq}^{110} M_{pq}^{000} - M_{pq}^{100} M_{pq}^{010} & M_{pq}^{020} M_{pq}^{000} - (M_{pq}^{010})^2 & M_{pq}^{011} M_{pq}^{000} - M_{pq}^{010} M_{pq}^{001} \\ M_{pq}^{101} M_{pq}^{000} - M_{pq}^{100} M_{pq}^{001} & M_{pq}^{011} M_{pq}^{000} - M_{pq}^{010} M_{pq}^{001} & M_{pq}^{002} M_{pq}^{000} - (M_{pq}^{001})^2 \end{pmatrix}$$

$$U_1^j = \begin{pmatrix} k_r^j & d_{12} & d_{13} \\ k_g^j & d_{22} & d_{23} \\ k_b^j & d_{32} & d_{33} \end{pmatrix} \qquad U_2^j = \begin{pmatrix} d_{11} & k_r^j & d_{13} \\ d_{21} & k_g^j & d_{23} \\ d_{31} & k_b^j & d_{33} \end{pmatrix} \qquad U_3^j = \begin{pmatrix} d_{11} & d_{12} & k_r^j \\ d_{21} & d_{22} & k_g^j \\ d_{31} & d_{32} & k_b^j \end{pmatrix}$$

$$T_{12}^{1} = \frac{|J_{10}|}{|J_{00}|} \qquad T_{12}^{2} = \frac{|J_{01}|}{|J_{00}|}$$

$$T_{12}^{3}(i,j) = \frac{k_r^j |U_1^i| + k_g^j |U_2^i| + k_b^j |U_3^i|}{|J_{pq}|}$$

moment invariants). The canonical variables try to maximize the separation between the classes.

3. Recognition following a leave-one-out strategy: each time one panel is singled out and all others are used as a training set. The panel is then assigned to a class based on nearest-neighbour classification. This was repeated for all panels of the same data set.

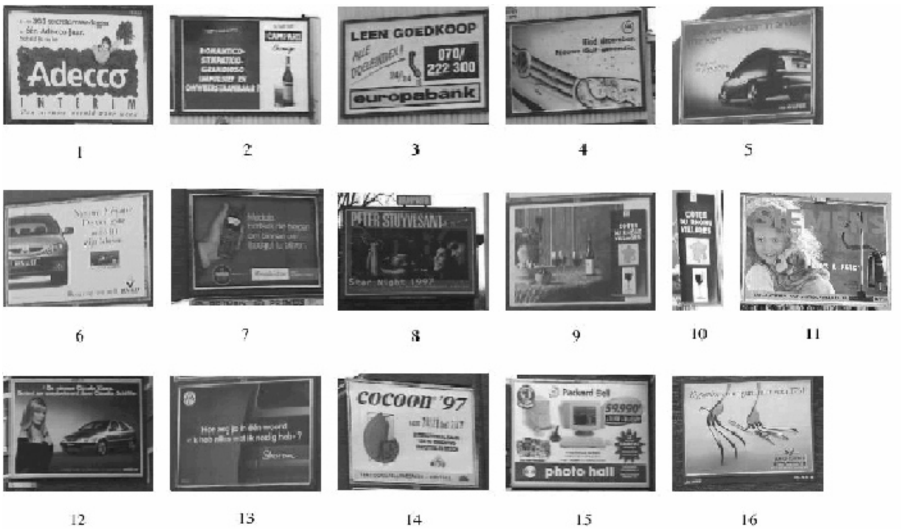

Fig. 2. Outdoor images - the 16 classes of different patterns that were used in the classification system.

4.1 Outdoor Images – Natural Light

In the case of the outdoor advertisement panels, the goal was to recognize to which of the 16 classes (Figure 2) a panel belongs. For each of the classes between 10 to 18 images were taken, under a quite large variety of viewing conditions (Figure 1). Samples of the same class also included images of several, physically different panels. In this paper we chose to consider perspective deformations for the geometric transformation model. These deformations are dealt with through a prior normalization step. This was easy in the case of our experiments, based on the advertisement panels which have a simple rectangular shape. The four corners of the panels were brought to four canonical positions in order to achieve the geometric normalization. Examples of resulting geometrically normalized images are shown in Figure 3. The normalized shape more or less corresponds to a rectangle with an aspect ratio one would get in a head-on view.

The cluster separation obtained with the first three canonical variables is illustrated in the left side plot of Figure 4 for Type SO and in the right side

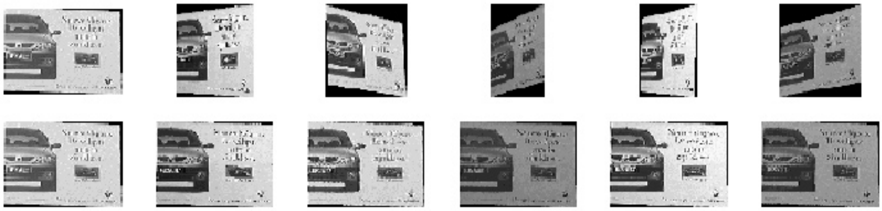

Fig. 3. Different samples of a pattern (up) and their geometrically normalized version (down).

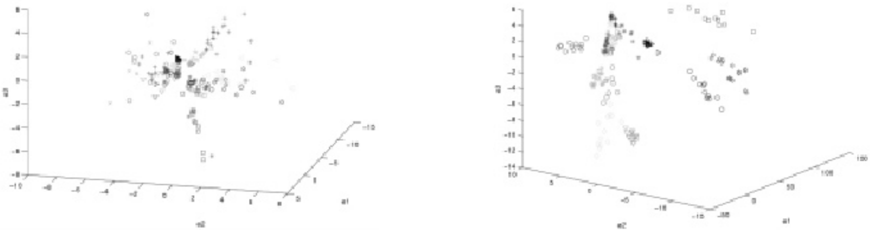

Fig. 4. Outdoor images - the first 3 canonical variables for the feature vector based on invariants of Type SO (left) and Type AFF (right).

plot for Type AFF. Visual inspection of these figures already suggests that the Type AFF canonical variables yield a better separability between classes than those of Type SO. This is also borne out by the actual recognition rates: the recognition performance using the Type SO canonical variables is only 78.6%, whereas the recognition rate for the Type AFF canonical variables reaches 93.9%. Still, at closer inspection of the Type SO invariants, it turns out that the denominators involved in all the invariants are numerically unstable because they actually represent a determinant of a nearly singular matrix. This instability explains part of the bad classification. One way of correcting the numerical instability is by computing a new set of invariants as functions of the basis invariants presented in Table 1, such that the unstable denominators get simplified. With 9 possible resulting invariants the recognition performance reaches 94.6%.

As experiments by other authors have already shown, one has to be careful in extrapolating results of one particular experiment to others. Nevertheless, this experiment was carried out under natural, outdoor conditions and constituted a quite critical test to the effectiveness of these invariants. The outcome shows equivalent performance of Type AFF and Type SO invariants, and hence also suggests that the simplicity of Type SO invariants provides similar performance to the more accurate, but also more complex Type AFF invariants.

4.2 Indoor Images – Artificial Light

We have examined the performance of the two Types of invariants on outdoor images, but we are also interested in comparing them for the case of indoor images, where transformations of Type SO are reported to be a rather good approximation of the intensity changes.

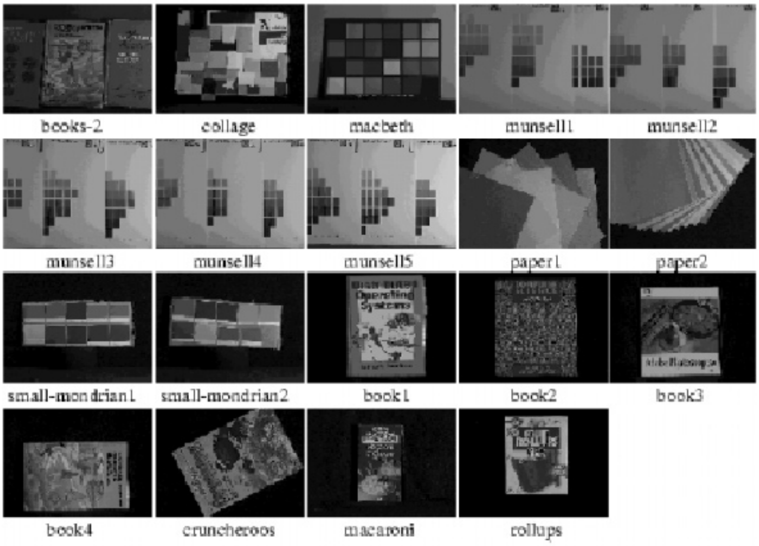

Fig. 5. Indoor images - the 19 classes of different patterns that were used in the classification system.

For this, we investigated the recognition performance of the invariant features when applied to the set of indoor images with planar surfaces contained in the database made publicly available at http://www.cs.sfu.ca/~colour/data. This data was collected by Lindsay Martin under the guidance of Kobus Barnard, as part of investigations into computational colour constancy algorithms, but its use can be extended to other colour based applications. The images are in TIFF format and have been linearized (camera offset removed, and low intensity values linearized). A detailed presentation of the data is available in [2]. Several preprocessing steps were taken to improve the data. First, some fixed pattern noise were removed. Second, images were corrected for a spatially varying chromaticity shift due to the camera optics. Finally, the images were mapped into a more linear space as described in [2]. This included removing the sizable camera black signal. The resulting images are such that pixel intensity is essentially proportional to scene radiance.

The database consists of images of objects viewed on black background, under 11 different illuminants. The images from the set of images with minimal specularities containing planar surfaces were selected for our tests. That brought us to the collection of the following objects (Figure 5): books-2, collage, macbeth, munsell1, munsell2, munsell3, munsell4, munsell5, paper1, paper2, sml-mondrian1, sml-mondrian2, book1, book2, book3, book4, cruncheroos, macaroni, rollups, thus 19 in total. When required, viewpoint invariance was achieved by a normalization step, as described in the previous section. A first observation about

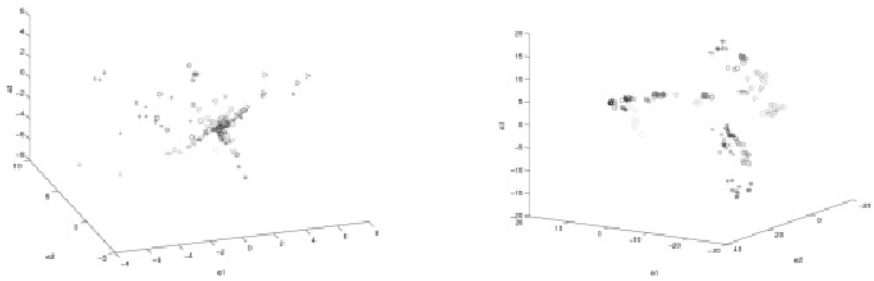

Fig. 6. Indoor images - the first 3 canonical variables for the feature vector based on invariants of Type SO (left) and Type AFF (right).

this data set is that the 11 illuminants have different colours, and it affects the objects appearance in a different way than the natural light did in the previous data set. A question to answer by using these data is therefore whether the invariant features are sensible or not to this kind of illumination change. Second, we notice that the data contains 5 very similar patterns (labeled 'munsell'), which provides a rather hard test on the discriminative power of the invariant features.

The cluster separation obtained with the first five canonical variables based on invariants of Type AFF proves again better than that obtained for the invariants of Type SO, as illustrated by Figure 6 and by the recognition rates. Only 8 samples are misclassified out of the total 209 samples in the data set when using the Type AFF invariants, that is 96.2% correct classification, whereas the invariants of Type SO yield only 46% correct classification. When using the 9 new numerically stable invariant functions, the recognition performance significantly improves again, reaching a rate of 99.5% correct classifications. This proves that the basis invariants are not to be used directly for recognition, but combined into new invariant functions that remove the numerically unstable determinants. Further investigations into the best way of combining the basis invariants are currently under consideration.

5 Conclusions

In this paper, we have investigated and compared two ways of describing the photometric transformations that patterns undergo under varying viewing and illumination conditions. The simplest one has a different scaling factor and offset for each of the three color bands (R,G,B). The more complex one considers a complete affine transformation between these bands.

The goal was to obtain invariant features better suited to the range of transformations one needs to handle for real scene images. A more complex affine photometric model was used as compared to our previous work. Perspective skews were dealt with through a prior normalization.

It turns out that the more complex photometric transformations are a better model to describe the photometric changes in outdoor images (including considerations that take model complexity into account), but similar recognition of planar structures can be obtained when using invariants under the two photometric models, for both indoor and outdoor images.

Acknowledgments. The authors acknowledge support from the IUAP4/24 project 'Intelligent Mechatronic Systems' (Belgian DWTC) and the GOA project 'Variability in Human Shape and Speech VHS+' (Research Council K.U.Leuven).

References

1. K. Barnard, G.D. Finlayson, B.V. Funt, Color Constancy for scenes with varying illumination, *Proceedings of the 4th European Conference in Computer Vision*, 1996, pp. 3 – 15.
2. K. Barnard, L. Martin, B. Funt, A. Coath, A data set for color research, accepted for publication in *Color Research and Application, 2001.*
3. D. Berwick and S. W. Lee, A Chromaticity Space for Specularity, Illumination Color- and Illumination Pose-Invariant 3-D Object Recognition, *Proceedings International Conference on Computer Vision*, 1998, pp. 165 – 170.
4. P. E. Debevec, J. Malik, Recovering High Dynamic Range Radiance Maps from Photographs, *SIGGRAPH'97*, August 1997.
5. Mark.S. Drew, Jie. Wei, Ze-Nian Li, "On Illumination Invariance in Color Object Recognition", Technical report 1997, School of Computing Science, Simon Fraser University, Vancouver, Canada.
6. G. Finlayson, M.S. Drew and B. Funt, "Color constancy: Generalized diagonal transforms suffice", Journal of the Optical society of America A, 11(11):3011-3019, 1994.
7. D. Forsyth, A novel algorithm for color constancy, *Int. Journal of Computer Vision*, Vol. 5 (1990), pp. 5 – 36.
8. G. Finlayson, "Color constancy in diagonal chromaticity space", *Proc. ICCV 1995* pp. 218-223.
9. B. Funt and G. Finlayson, Color constant color indexing, *IEEE Trans. PAMI*, Vol. 17 (1995), pp. 522 – 529.
10. T. Gevers, and A. W. M. Smeulders, A comparative study of several color models for color image invariant retrieval, *Proc. Intern. Workshop on Image Database and Multimedia Search*, 1996, pp. 17 – 23.
11. P. Gros Color illumination models for image matching and indexing, *Proceedings International Conference on Pattern recognition*, 2000, Vol. 3.

12. G. Healey and D. Slater, Global color constancy: recognition of objects by use of illumination invariant properties of color distributions, *J. Opt. Soc. Am. A*, Vol. 11 (1994), pp. 3003 – 3010.
13. R. A. Johnson and D. W. Wichern, *Applied multivariate statistical analysis,* Prentice-Hall, 1992.
14. Y. Kanazawa and K. Kanatani Stabilizing Image Mosaicing by Model Selection *Lecture Notes in Computer Science 2018*, 2000, pp. 35 – 52
15. J. Matas, D. Koubaroulis and J. Kittler, Colour Image Retrieval and Object Recognition Using the Multimodal Neighbourhood Signature, *Proceedings of the 6th European Conference in Computer Vision* Dublin, Ireland, 2000.
16. F. Mindru, T. Moons and L. Van Gool, Recognizing color patterns irrespective of viewpoint and illumination, *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR '99)*, Fort Collins (Colorado), June 1999, pp. 368 – 373.
17. T. Moons, E. Pauwels, L. Van Gool, and A. Oosterlinck, Foundations of semi-differential invariants, *International Journal Computer Vision*, Vol. 14 (1995), pp. 25 – 47.
18. J. L. Mundy, and A. Zisserman A (eds.), *Geometric invariance in computer vision*, MIT Press, 1992.
19. J. L. Mundy, A. Zisserman, and D. Forsyth (eds.). *Applications of invariance in computer vision*, LNCS 825, Springer, 1994.
20. S.K. Nayar, R.M. Bolle, Reflectance Based Object Recognition, *International Journal of Computer Vision*, 17(3):219–240, 1996.
21. T. Reiss, *Recognizing planar objects using invariant image features*, LNCS 676, Springer, 1993.
22. D. Slater and G. Healey, The illumination-invariant recognition of 3D objects using local color invariants, *IEEE Trans. PAMI*, Vol. 18 (1996), pp. 206 – 210.
23. D. Slater, G. Healey, What Is the Spectral Dimensionality of Illumination Functions in Outdoor Scenes?, *Proceedings IEEE Conference on Computer Vision and Pattern Recognition*, 1998, pp. 105 – 110.
24. M. Swain and D. Ballard, Color indexing, *Int. Journal of Computer Vision*, Vol. 7 (1991), pp. 11 – 32.
25. Van Gool L, Moons T, and Ungureanu D. Geometric/photometric invariants for planar intensity patterns, In Proceedings European Conference on Computer Vision. Springer, 1996, pp. 642 – 651.
26. P.H.S. Torr, Model Selection for Two View Geometry: A Review *submitted to International Journal of Computer Vision*, 2001.
27. L. Van Gool, T. Moons, E. Pauwels, A. Oosterlinck, Vision and Lie's approach to invariance , *Image and vision computing*, vol. 13, no. 4, pp. 259 – 277, 1995, Elsevier Science B.V.
28. L. Wang and G. Healey, Using Zernike Moments for the Illumination and Geometry Invariant Classification of Multispectral Texture", *IEEE Trans. Image Processing*, Vol. 7 (1998), pp. 196 – 203.
29. Wolff L. On the relative brightness of specular and diffuse reflection, In Proceedings of the IEEE conference on Computer Vision and Pattern Recognition, IEEE Press, 1994, pp.369 – 376.