The Morphology of Building Structures

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Abstract.
The structural efficiency and the architectural appearance of building forms is becoming an increasingly important field of engineering, particularly because of the present wide-spread availability of computer facilities. The realisation of complex shapes comes into reach, that would not have been possible with the traditional means. In this contribution a technique is described, where structural forms are visualised starting from a geometry based on that of regular or semi-regular polyhedra, as they form the basis of most of the building structures that are utilised nowadays. The architectural use of these forms and their influence on our man-made environment is of general importance. They can either define the overall shape of the building structure or its internal configuration. In the first case the building has a centrally symmetric appearance, consisting of a faceted envelope as in geodesic sphere subdivisions, which may be adapted or deformed to match the required space or ground floor plan. Polyhedral shapes are also often combined so that they form conglomerates, such as in space frame systems.

1. Introduction

Polyhedra can be generated by the rotation of regular polygons around the centre of the coordinate system. Related figures, that are found by derivation from these polyhedra, can be formed in a similar way by rotating planar figures that differ from ordinary polygons. An inter-active program for personal computers is being developed - which at present is meant mainly for study purposes - with the help of which geometric data as well as visual presentations can be obtained of the regular and semi-regular polyhedra not only, but also of their derivatives. This method thus allows the rotation also of 3-dimensonal figures, that may be of arbitrary shape. The rotation procedure can eventually be used repeatedly, so that quite complex configurations can be described starting from one general concept. The outcome can be gained in graphical and in numerical form. The latter data can be used as the input for further elaboration, such as in external, currently available drawing or computation programs.

2. Definition of Polyhedra

A common definition of a polyhedron is [1]:
1) It consists of plane, regular polygons with 3, 4, 5, 6, 8 or 10 edges.
2) All vertices of a polyhedron lie on one circumscribed sphere.

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3) All these vertices are identical. In a particular polyhedron the polygons are grouped around each vertex in the same number, kind and order of sequence.
4) The polygons meet in pairs at a common edge.
5) The dihedral angle at an edge is convex. In other words: the sum of the polygon face angles that meet at a vertex is always smaller than 360^{o}.

3. Regular and Semi-Regular Polyhedra

Under these conditions a group of 5 regular and 13 semi-regular, principally different polyhedra is found. There are actually 15 semi-regular solids, as two of them exist in right- and left-handed versions. All uniform polyhedra consist of one or more - maximally 3 - sets of regular polygons.

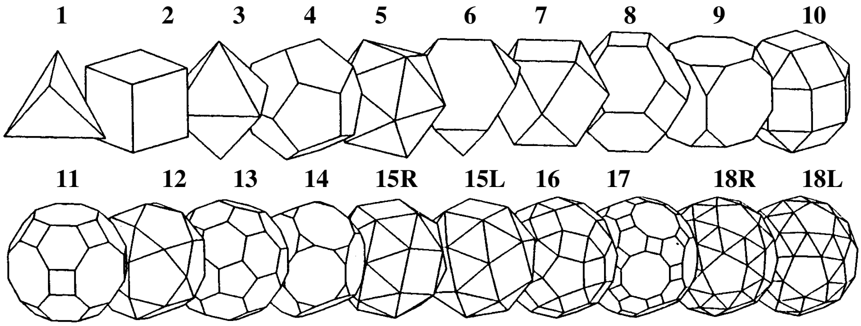


Fig. 1. Review of the regular (1 to 5) and semi-regular (6 to 18R) polyhedra

The five regular polyhedra have a direct mutual relationship: they are dual to each other in pairs. Fig. 3 shows this relation. All other polyhedra of Fig.2 also have dual or reciprocal partners [9]. This principle of duality is explained in Figs. 4 and 5.

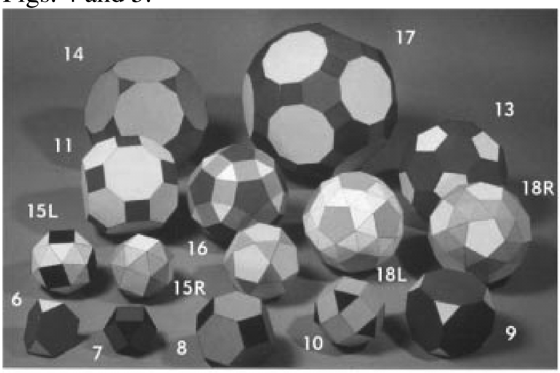


Fig. 2. Models of the 15 semi-regular polyhedra

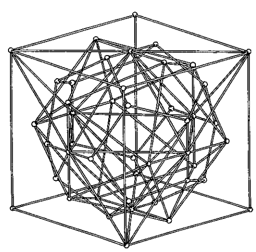
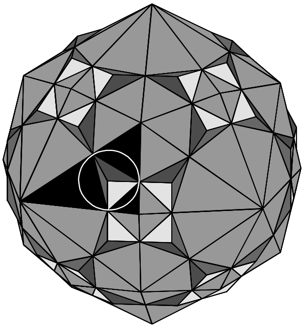


Fig. 3. The relations of the 5 regular solids



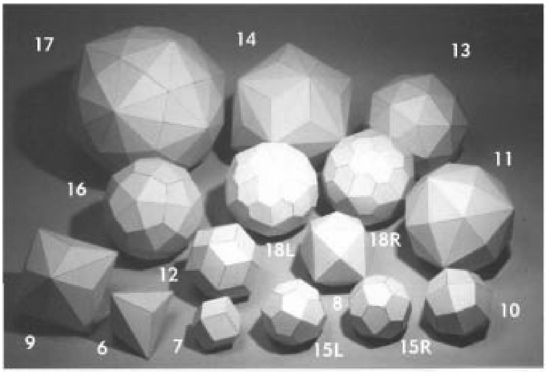


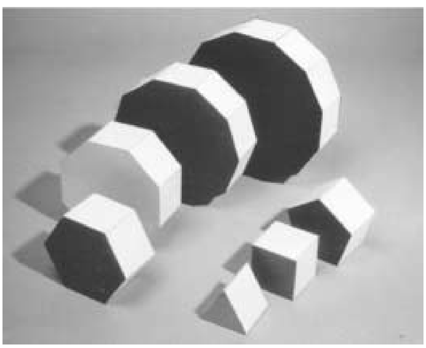
Fig. 4. The principle of duality **Fig. 5.** Models of the dual semi-regular polyhedra

4. Close-Packings

Some of the polyhedra lend themselves to being put together in tight packed formations. In this way quite complex forms can be realised. It is obvious hat cubes and rectangular prisms can be stacked most densely, but many of the other polyhedra can also be packed in certain combinations.

5. Prisms and Antiprisms

Other solids that also respond the previous definition of a polyhedron are the prisms and the antiprisms. Prisms have two parallel polygons like the lid and the bottom of a box and square side-faces; antiprisms are like the prisms but have one of the polygons slightly rotated so as to turn the side-faces into triangles.



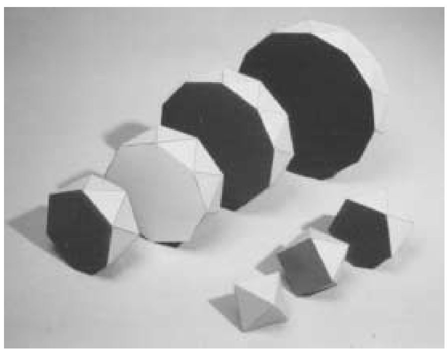


Fig. 6 and 7. Models of prisms and antiprisms

The simplest forms are the prismatic shapes. They fit usually well together and they allow the formation of many variations of close-packings. If a number of antiprisms is put together against their polygonal faces, a geometry is obtained of which the outer mantle has the appearance of a cylindrical, concertina-like folded plane. [7]

These forms can be described with the help of only few parameters, a combination of 3 angles: α, β and γ. The element in Fig. 9A represents 2 adjacent isosceles triangles.

α = half the top angle of the isosceles triangle ABC with height *a* and base length *2b*.

γ = half the dihedral angle between the 2 triangles along the basis.

φ_n = half the angle under which this basis *2b* is seen from the cylinder axis. = π/n, being *n* here the number of sides of the respective polygon.

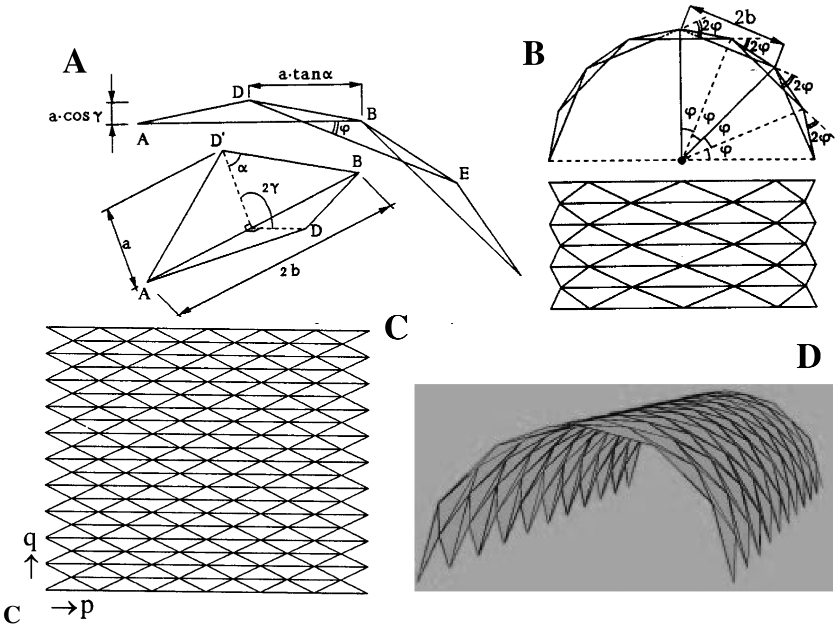


Fig. 8. Variables that define the shape of antiprismatic forms

The relation of these angles α, γ and φ_n [4]:

$$\tan\alpha = \cos\gamma \ \cot an\,(\varphi_n/2) \qquad \{1\}$$

These three parameters define together with the base length *2b* (or scale factor) the shape and the dimensions of a section in such a structure. This provides an interesting tool to describe any antiprismatic configuration. Two additional data must be given: the number of elements in transverse direction (*p*) and that in length direction (*q*).

6. Augmentation

Upon the regular faces of the polyhedra other figures can be placed that have the same basis as the respective polygon. In this way polyhedra can be 'pyramidized'. This means that shallow pyramids are put on top of the polyhedral faces, having their apexes on the circumscribed sphere of the whole figure. This can be considered as the first frequency subdivision of spheres. In 1582 Simon Stevin introduced the notion of 'augmentation' by adding pyramids, consisting of triangles and having a triangle, a square or a pentagon for its base, to the 5 regular polyhedra [2]. More recently, in 1990, D.G. Emmerich extended this idea to the semi-regular polyhedra (Fig.9). He suggested to use pyramids of 3-, 4-, 5-, 6-, 8- or 10-sided base, composed of regular polygons, and he found that 102 different combinations can be made. He called these: composite polyhedra [3].

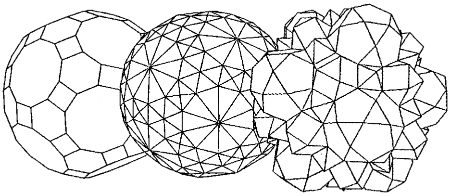


Fig. 9 A composite polyhedron (see also Fig. 21)

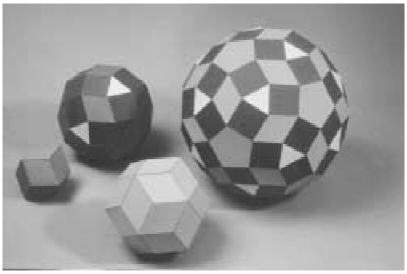


Fig. 10. Models of additions in the form of square or rhombic elements.

7. Sphere Subdivisions

For the further subdivision of spherical surfaces generally the Icosahedron – and in some cases the Tetrahedron or the Octahedron - are used as the starting point, because they consist of equilateral triangles that can be covered with a suitable pattern that is subsequently projected upon a sphere. This leads to economical kinds of subdivision up to high frequencies and with small numbers of different member lengths [8].

Fig. 11. Models of various dome subdivision methods

All other regular and semi-regular solids, and even their reciprocals as well as prisms and antiprisms can be used similarly [8]. The polygonal faces are first subdivided and then made spherical.

8. Sphere Deformation

The spherical co-ordinates can be written in a general form, so that the shape of the sphere may be modified by changing some of the variables. This leads to interesting new shapes, that all have a similar origin but that are governed by different parameters. According to H. Kenner [6] the equation of the sphere can be transformed into a set of two general expressions:

$$R_1 = E_1 / (E_1^{n1} \sin^{n1}\varphi + \cos^{n1}\varphi)^{1/n1} \qquad \{2\}$$

$$R_2 = R_1 E_2 / (E_2^{n2} \sin^{n2}\theta + R_1^{n2} \cos^{n2}\theta)^{1/n2} \qquad \{3\}$$

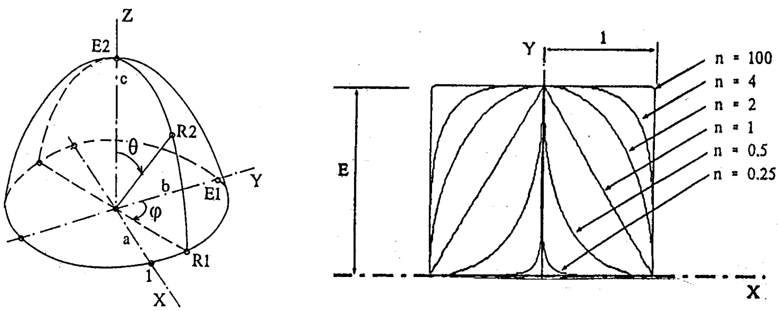


Fig. 12. Deformation process of spheres

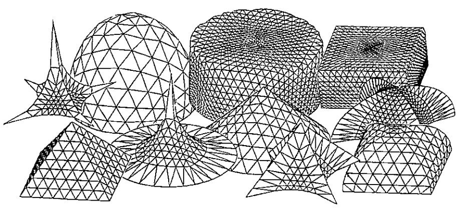


Fig. 13. Form variation of domes by the use of different variables

The variables n_1 and n_2 are the exponents of the horizontal and vertical ellipse and E_1 and E_2 are the ratios of their axes. The shape of the sphere can be altered in many ways, leading to a number of transformations. The curvature is a pure ellipse if $n = 2$, but if n is raised a form is found, which approximates the circumscribed rectangle. If n is decreased, the curvature flattens until $n = 1$ and the ellipse then has the form of a pure rhombus with straight sides, connecting the maxima on the co-ordinate axes. For $n < 1$ the curvature becomes concave and obtains a shape, reminiscing a hyperbola. For $n = 0$ the figure coincides completely with the X-, and Y-axes. By changing the value of both the horizontal and the vertical exponent the visual appearance of a hemispherical shape can be altered considerably [6, 8].

9. Stellation

Most polyhedra can be stellated. This means that their planes can be extended in space until they intersect again. This can sometimes be repeated one or more times. The Tetrahedron and the Cube are the only polyhedra that have no stellated forms, but all others do. The Octahedron has one stellated version, the Dodecahedron has three and the Icosahedron has as many as 59 stellations.

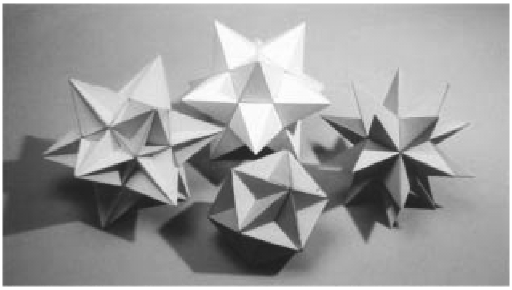


Fig. 14. Some stellated versions of the regular polyhedra

10. Polyhedra in Building

The role that polyhedra can play in the form-giving of buildings is very important, although this is not always fully acknowledged. Some possible or actual applications are referred to here briefly.

10.1. Cubic and Prismatic Shapes

Most of our present-day architectural forms are prismatic with the cube as the most generally adopted representant. Prisms are used in a vertical or in a horizontal position, in pure form or in distorted versions. This family of figures is therefore of utmost importance for building.

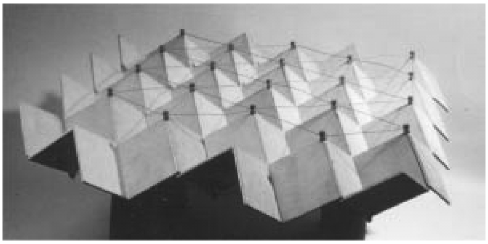


Fig. 15. Model of a space frame made of square plates in cubic arrangement

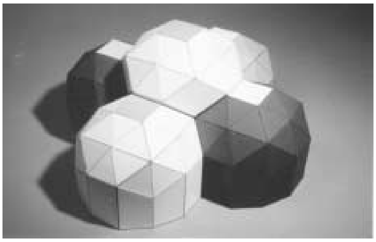


Fig. 16. Model of house, based on Rhombic Dodecahedron (honeycomb cell)

10.2. Solitary Polyhedra

Architecture can become more versatile and interesting with macro-forms, derived from one of the more complex polyhedra or from their reciprocal (dual) forms, although this has not often been done. Packings of augmented polyhedra form sometimes interesting alternatives for the traditional building shapes.

10.3. Combinations

Packings are also suitable as the basic configuration for space frames, because of their great uniformity. If these frames are based on tetrahedra or octahedra, all members are identical and meet at specific angles. Many of such structures have been built in the recent past and this has become a very important field of application. The members usually meet at joints having a spherical or a polyhedral shape.

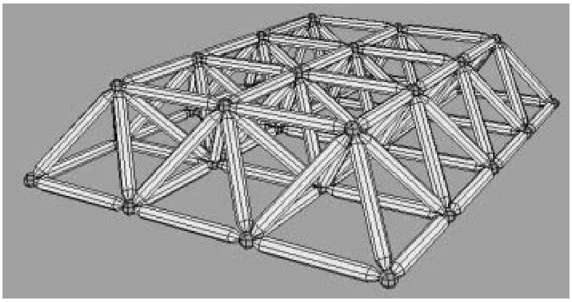
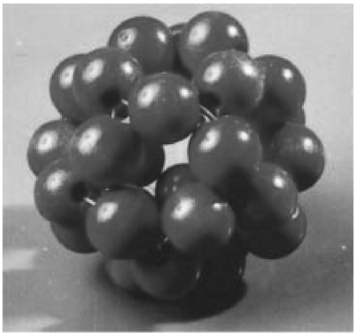


Fig. 17. Computer generated space frame made of identical struts

Fig. 18. Office building in Mali, based on three augmented rhombicuboctahedra and two tetrahedra, with a frame construction of palm wood

10.4. Domes

R.B. Fuller re-discovered the geodesic dome principle. This has proven to be of great importance for the developments in this field. Many domes have been built during the last decades, up to very large spans. A new group of materials with promising potential has been called after him, which has molecules that basically consist of 60 atoms, placed at the corners of a Truncated Icosahedron.



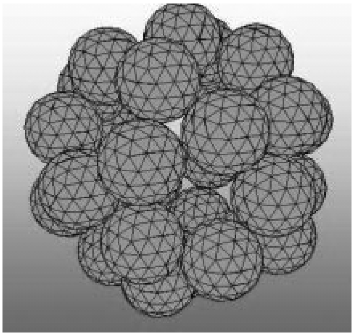


Fig. 19 and 20. Model and computer sketch of small 'Fullerene'

11. Stereographic Slide Presentation

The author will show during the conference a few applications with the help of a 3-D colour slide presentation. Two pictures with different orientation of the light wave directions are projected simultaneously on one screen. The screen must have a metal surface which maintains these two ways of polarisation, so that the two pictures can be observed with Polaroid spectacles that disentangle them again into a left and a right image. These pictures are made either analogously: with a normal photo camera on dia-positive film and with the two pictures token at a certain parallax or they are made by computer and subsequently printed and photographed or written directly onto positive film. This technique allows coloured pictures to be shown in a really three-dimensional way and gives thus a true impression of the spatial properties of the object [10]. This slide show will be done in cooperation with members of the Region West of the Dutch Association of Stereo Photography. Their assistance is gratefully acknowledged here.

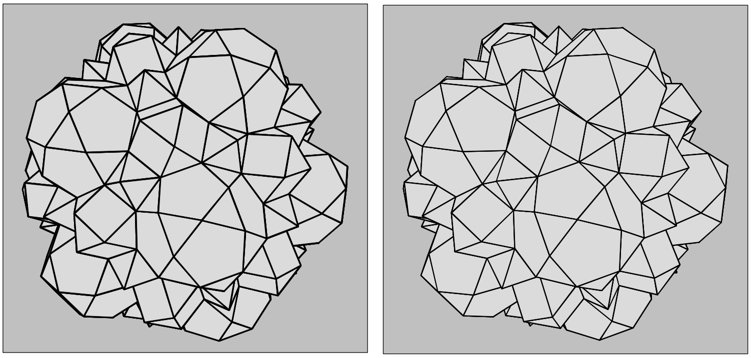


Fig. 21. Pair of stereoscopic pictures

12. References

1. Huybers, P., Polyhedra and their Reciprocals, Proc. IASS Conference on the Conceptual Design of Structures, 7-11 October, 1996, Stuttgart, 254-261.
2. Struik, D.J., The principle works of Simon Stevin, Vol. II, Swets & Seitlinger, Amsterdam, 1958.
3. Emmerich, D.G., Composite polyhedra. Int. Journal of Space Structures, 5, 1990, p. 281-296.
4. Huybers, P. and G. van der Ende: Prisms and Antiprisms, Proc. Int. IASS Conf. on Spatial, Lattice and Tension Structures, Atlanta, 24-28 april 1994, p.142-151.
5. Wenninger, M., 1979, Spherical Models, Cambridge University Press, USA
6. Kenner, H., Geodesic math and how to use it, University of California Press, London, 1976.
7. Huybers, P., Prismoidal Structures, The Mouchel Centenary Conf. on Innovation in Civil & Structural Engineering, Cambridge, p. 79-C88.
8. Huybers, P., and G. van der Ende, Polyhedral Sphere Subdivisions, Int. IASS Conf. on Spatial Structures, Milan 5-9 June, 1995, p. 189-198.
9. Huybers, P., The Polyhedral World, In: 'Beyond the Cube: The Architecture of Space frames and Polyhedra', J.F. Gabriel Ed., John Wiley and Sons, Inc., New York, 1997, p. 243-279.
10. Ferwerda, J.G., The World of 3-D, A practical Guide to Stereo Photography, 3-D Book Productions, Borger, 1987.