Non-interactive Public-Key Cryptography[1]

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Abstract. An identity-based non-interactive public key distribution system is presented that is based on a novel trapdoor one-way function allowing a trusted authority to compute the discrete logarithm of a given number modulo a publicly known composite number m while this is infeasible for an adversary not knowing the factorization of m. Without interaction with a key distribution center or with the recipient of a given message a user can generate a mutual secure cipher key based solely on the recipient's identity and his own secret key and send the message, encrypted with the generated cipher key using a conventional cipher, over an insecure channel to the recipient. Unlike in previously proposed identity-based systems, no public keys, certificates for public keys or other information need to be exchanged and thus the system is suitable for many applications such as electronic mail that do not allow for interaction.

1. Introduction

Public-key dis'ribution systems and public-key cryptosystems suffer from the following well-known authentication problem. In order to prevent an adversary from fraudulently impersonating another user, it must be possible to verify that a received public key belongs to the user it is claimed to belong to. A commonly used solution to this authentication problem is the certification of public keys by a trusted authority which, after checking a user's identity, signs the concatenation of his name and public key using a digital signature scheme. Systems based on either the RSA [23] or the ElGamal [3] signature schemes have been proposed [5, 6].

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Shamir [25] suggested as a simple but ingenious method for solving the authentication problem in public-key cryptography to let each user's public key be his (publicly-known) identification information. Because it must be infeasible for users to compute the secret key corresponding to a given identity (including their own), the secret keys must be computed by a trusted authority who knows some secret trapdoor information. The security of such an identity-based system depends on the trusted authority in a more crucial way than the security of a public-key certification system because in the former the trusted authority knows all secret keys.

Because a user's identity can be assumed to be publicly known (the identity can be defined as that part of the identification information that *is* publicly known), the public keys of an identity-based public-key cryptosystem need not be transmitted. Therefore an identity-based system can be used in a completely non-interactive manner.

A simple way to set up an identity-based public-key cryptosystem would seem to be to use the RSA-system with a universal modulus where each user's public encryption exponent is his (odd, and relatively prime to $\varphi(m)$) identity and in which a trusted authority knowing the factorization of the modulus computes the secret decryption exponents for users. However, this system is insecure because knowledge of a matching (secret/public) key pair allows to easily factor the modulus.

While Shamir presented an identity-based signature scheme, he left and proposed as an open problem to find an identity-based public-key cryptosystem or public-key distribution system [25]. In the context of signature schemes, however, an identity-based system is less advantageous than it would be in the context of public-key cryptosystems (which can be made non-interactive) because in a signature scheme, public keys can be certified by a trusted authority and a user's certified public key can be disclosed together with the signature, thus requiring no additional protocol steps for the transmission of the public keys.

Many previously proposed systems [5, 7, 19, 20, 27] have been called identity-based public-key distribution systems because they make use of Shamir's idea for self-authentication of public keys. However, none of these (with the exception of the quite impractical and also insecure version of a scheme discussed in [27]) is an identity-based system in Shamir's sense because the public key is a function not only of the identity but also of some random number selected either by the user or by the trusted authority. As a consequence, these systems are bound to be interactive. A major achievement of this paper is that it presents the first truly identity-based public-key distribution system. It should be mentioned that the key predistribution system of Matsumoto and Imai [13], which is based on a completely different approach, also achieves non-interactive key distribution.

The original Diffie-Hellman public key distribution system [2] with a prime modulus p cannot be used as an identity-based system in Shamir's sense because if the scheme is secure, that is when discrete logarithms modulo p are infeasible to compute, it is infeasible even for a trusted authority to compute the secret key corresponding to a given public key, i.e., a given identity. This comment applies to any public-key distribution system based on a one-way function without trapdoor. One of the major achievement of this paper is that a method for building a *trapdoor into the modular exponentiation one-way function* is proposed which allows a trusted authority to feasibly compute discrete logarithms whereas this is nevertheless completely infeasible for an

adversary using present technology and algorithmic knowledge. This allows a trusted authority to set up a non-interactive public-key distribution system. Non-interactiveness is crucial in some applications (e.g. electronic mail, some military applications) and in some other applications allows at least to simplify the protocols. The computational effort that the trusted authority must spend is considerable but the key distribution protocol is efficient.

In Section 2, the preferred version of our system is presented. A security and feasibility analysis is given in Section 3 and some alternative implementation approaches are discussed in Section 4. The final section summarizes some conclusions.

2. A Non-interactive Public Key Distribution System

From a protocol viewpoint, the difference between a public-key distribution system and a public-key cryptosystem is that in the former, both parties must receive the other party's public key whereas in the latter, only the sending party must receive the public key of the receiving party. Therefore, a public-key distribution system, when combined with a conventional symmetric cryptosystem used for encryption, cannot be used as a public-key cryptosystem. In contrast, a non-interactive public-key distribution system *can* be used as a public-key cryptosystem by sending as one message the sender's identity and the enciphered plaintext, where the cipher key is computed from the receiver's identity and the sender's secret key and where some agreed conventional cipher is used for encryption of the message.

Our non-interactive public key distribution system is based on a variant of the Diffie-Hellman system with composite modulus m. By choosing the prime factors of m appropriately such that discrete logarithms modulo each prime factor can feasibly be computed but such that computing discrete logarithms modulo m is nevertheless infeasible, a trusted authority can set up a public key distribution system based on exponentiation modulo m.

Two different ways of generating such a modulus m are presented below and in Section 4, respectively. To use a composite modulus $m = pq$ with p and q prime in the Diffie-Hellman scheme has previously been proposed by Shmuely [26] and McCurley [15] in order to exhibit a system which to break requires the ability both to factor m and to compute discrete logarithms modulo p and q.

Our approach to identity-based public key distribution differs in a crucial way from previous approaches [5, 6, 7, 19, 20, 27] in that the public key consists entirely of public identity information (e.g. name, address, physical description), but does not depend on an additional random number selected either by the user or the trusted authority. This is the reason why our system can be used in a truly non-interactive manner. Clearly, the type and amount of information about a user that can be assumed to be publicly known depends on the application, but note that in most applications, at least part of the identification information is indeed publicly known. For instance, the receiver's address, which must be known in every communication system in order to send a message, can serve as his public key.

One problem that arises in the proposed system is that the multiplicative group Z_m^* is cyclic if and only if m is either 2, 4, a power of an odd prime or twice the power of an odd prime. When m is the product of distinct odd primes there hence exists no element that generates the

entire group Z_m^*. Thus not every identity number that corresponds to some valid identification information is guaranteed to have a discrete logarithm with respect to some universal base α. This problem could be solved by adding the smallest offset to every identity number that makes the new number have a discrete logarithm. However, the resulting system would have to be interactive since the offsets must be exchanged between the users. Two different solutions to this problem are presented below and in Section 4, respectively. Both are computationally more efficient (for the trusted authority) than the offset method and at the same time allow to preserve the advantage of non-interactiveness of our scheme.

Let $m = p_1 \cdot p_2 \cdots p_r$ where the primes $p_1, \ldots, p_r$ are in the following assumed to be odd and distinct. The maximal order of an element of the multiplicative group Z_m^* is given by $\lambda(m) = \mathrm{lcm}(p_1 - 1, \ldots, p_r - 1)$, which is at most 2^{-r+1} times the group order $\varphi(m)$. $\lambda(m)$ is strictly less than $\varphi(m)/2^{r-1}$ unless the numbers $(p_1 - 1)/2, \ldots, (p_r - 1)/2$ are pairwise relatively prime. Let α be an element of Z_m^* that is primitive in each of the prime fields $GF(p_1), \ldots, GF(p_r)$, i.e., such that for $1 \leq i \leq r$, $p_i - 1$ is the smallest exponent t_i for which $\alpha^{t_i} \equiv 1 \pmod{p_i}$. Then α has maximal order $\lambda(m)$ in Z_m^*. The discrete logarithm of a number y modulo m to the base α is defined as the smallest non-negative integer x such that $\alpha^x \equiv y \pmod{m}$ (if such an x exists) and can, when the complete factorization of m is given, be obtained by computing for $i = 1, \ldots, r$ the discrete logarithm x_i of y to the base α modulo p_i, i.e., by computing x_i satisfying $\alpha^{x_i} \equiv y \pmod{p_i}$, and solving the system

$$\begin{aligned} x &\equiv x_1 \pmod{p_1 - 1}, \\ &\cdot \\ &\cdot \\ x &\equiv x_r \pmod{p_r - 1} \end{aligned}$$

of r congruences for x by the Chinese remainder technique. As mentioned above, this system need not have a solution because the numbers $p_1 - 1, \ldots, p_r - 1$ are not pairwise relatively prime. In particular, the system has no solution unless either all x_i are odd or all x_i are even.

The following Lemma is a special case of a more general result proved in the journal version of this paper. It suggests an easy to compute publicly-known function that transforms, without use of the secret trapdoor, any identity number into a modified identity number that is guaranteed to have a discrete logarithm.

Lemma. *Let m and α be as defined above where the numbers $(p_i - 1)/2, \ldots, (p_r - 1)/2$ are pairwise relatively prime. Then every square modulo m has a discrete logarithm modulo m to the base α.*

A complete description of the preferred version of the proposed non-interactive public key distribution system follows. The following three paragraphs describe the system set up by a trusted authority, the user registration phase and the user communication phase, respectively.

To set up the system we suggest that a trusted authority choose the primes p_i such that the numbers $(p_i - 1)/2$ are odd and pairwise relatively prime [14]. Preferably, $(p_i - 1)/2$ are chosen to be primes themselves. The primes p_i are chosen small enough such that computing discrete logarithms modulo each prime is feasible (though not trivial) using for instance the algorithm of [1] but such that factoring the product, even with the best known method for

finding relatively small prime factors [10] of a number, is completely infeasible. The trusted authority then computes the product

$$m = p_1 \cdot p_2 \cdots p_r$$

of the selected primes, determines an element α of Z_m^* that is primitive in every of the prime fields $GF(p_i)$ and publishes m and α as system parameters. We refer to Section 3 for an analysis of the security versus the feasibility for different sizes of parameters. To choose 3 to 4 primes of between 60 and 70 decimal digits seems at present to be appropriate, but these figures can vary according to future progress in computer technology and number-theoretic algorithms. An alternative approach to making the discrete logarithm problem feasible other than by choosing the prime factors of m sufficiently small is described in Section 4.

When a user A wants to join the system she visits the trusted authority, presents her identification information ID_A together with an appropriate proof of her identity (e.g. a passport) and receives the secret key s_A corresponding to ID_A. The secret key s_A is computed by the trusted authority as the discrete logarithm of ${ID_A}^2$ modulo m to the base α:

$$s_A \equiv \log_\alpha({ID_A}^2) \pmod m.$$

Due to the squaring of ID_A, s_A is guaranteed to exist as a consequence of the above lemma.

In order to send a message M securely to a user B without interaction, user A establishes the mutual secure cipher key K_{AB} shared with user B by computing

$$K_{AB} \equiv (ID_B)^{2s_A} \pmod m.$$

Note that $K_{AB} \equiv \alpha^{s_A s_B} \pmod m$. She then uses a conventional symmetric cryptosystem (e.g. DES) to encipher the message M using the cipher key K_{AB}, which results in the ciphertext C. User A then sends C together with her identity number ID_A to user B. In order to decipher the received ciphertext C, user B proceeds symmetrically and computes

$$K_{BA} \equiv (ID_A)^{2s_B} \equiv \alpha^{s_B s_A} \equiv K_{AB} \pmod m.$$

He then deciphers C using the conventional cryptosystem with the secret key K_{AB}, which results in the plaintext message M.

Note that the trusted authority is only required for the initial system set up and for user registration, but not in the user communication phase described above. In fact, the trusted authority could close itself down if no additional users need to be registered, thereby irreversibly erasing the factorization of m.

In the described system the secret key shared by two users is the same when the protocol is repeated several times. This in many applications undesirable property can easily be removed without losing non-interactiveness by having user A choose a random number R and use $f(K_{AB}, R)$ as the mutual cipher key, where f is a cryptographically secure hash function. R is sent to B together with the ciphertext C. In order to prevent an adversary knowing a previously cipher key from impersonating at a later time, a time stamp can be used as an additional argument of the hash function. It is possible to build a dynamic key distribution system using no

hash function, that is provably as hard (on the average) to break against a disruptive adversary as factoring the modulus [28].

Although in the proposed trapdoor one-way function the trapdoor is the factorization of the modulus as in the RSA trapdoor one-way function [23], the two functions are nevertheless entirely different. In the RSA function, the argument is the base and the exponent e is a constant whereas in our exponentiation trapdoor one-way function the argument is the exponent and the base α is a constant. Accordingly, the inverse operations are the extraction of the e-th root and the discrete logarithm to the base α, respectively, and are infeasible to compute without knowledge of the trapdoor.

3. Security and Feasibility Analysis

The following fact has previously been observed but is not widely known nor published. A proof is given in the journal version of this paper.

Proposition. *Let m be the product of distinct odd primes $p_1, \ldots, p_r$ and let α be primitive in each of the prime fields $GF(p_i)$ for $1 \leq i \leq r$. Then computing discrete logarithms modulo m to the base α is at least as difficult as factoring m completely.*

The function

$$L_x(a,b) \;=\; e^{b(\log x)^a(\log\log x)^{1-a}}$$

is commonly used to express the conjectured asymptotic running time of number-theoretic algorithms. The fastest known algorithm for computing discrete logarithms in $GF(p)$ [1] has asymptotic running time $L_p(1/2,1)$. The largest primes for which this algorithm is at present feasible with massively parallel computation have between 90 and 100 decimal digits. For primes of up to 65-70 decimal digits the algorithm is feasible on a small to medium size computer. An important feature of this algorithm is that most of the running time is spent in a precomputation phase that is independent of actual elements for which the logarithm is to be computed. After the precomputation, individual logarithms can be computed much faster in asymptotic running time $L_p(1/2,1/2)$. The algorithm is well suited for a parallel implementation.

The largest general integers that can at present feasibly be factored using massively parallel computation have on the order of 110 decimal digits [9]. The factoring algorithm with the best conjectured asymptotic running time $L_m(1/3,c)$ for some constant $c < 2$ is the number field sieve [11], but for the size of general integers m that can be factored within reasonable time a variant of the quadratic sieve with asymptotic running time $L_m(1/2,1)$ is more efficient [9]. The running time of both these algorithms is independent of the size of the factor that is found. The best known algorithm for finding factors of moderate size is the elliptic curve algorithm [10] which is with massively parallel computation successful for factors with up to 40 decimal digits [8, 18]. Its asymptotic running time is $L_p(1/2,\sqrt{2})$ where p is the factor to be found. It is the ratio $L_p(1/2,\sqrt{2})/L_p(1/2,1) = L_p(1/2,\sqrt{2}-1)$ of the running times of the elliptic curve factoring algorithm and the discrete logarithm algorithm [1] that provides a range for the size of the primes for which our public-key distribution system is both practical and secure.

It seems at present to be appropriate to choose 3 to 4 prime factors of between 60 and

70 decimal digits. To factor such a modulus is for all presently known factoring algorithms completely infeasible. The largest factor that has been found by the elliptic curve algorithm has 38 decimal digits [8]. Odlyzko [18] estimated that with the same computational effort that was spent on the factorization of the 106-digit number of [12], one could compute discrete logarithms for 92-digit prime moduli. To find a 70 digit factor with the elliptic curve factoring algorithm takes about $L_{10^{70}}(1/2, \sqrt{2})/L_{10^{38}}(1/2, \sqrt{2}) \approx 270.000$ times more time than to find a 38 digit factor. On the other hand, computing discrete logarithms for a 70-digit prime modulus is about $L_{10^{92}}(1/2, 1)/L_{10^{70}}(1/2, 1) \approx 157$ times faster than for a 92-digit prime modulus. An asymptotic analysis of the work factor of our system is given in the journal version of the paper.

4. Alternative Implementations

There exists a discrete logarithm algorithm for $GF(p)$ due to Pohlig and Hellman [21] whose running time is proportional to the square root of the largest prime factor of $p-1$, if the factorization of $p-1$ is known. Hence the primes p_i can be chosen such that $(p_i-1)/2$ is the product of some primes of a certain relatively small size. Unfortunately, there also exists a special purpose factoring algorithm due to Pollard [22] that is particularly efficient for finding prime factors p for which $p-1$ has only relatively small prime factors. However, the running time of Pollard's algorithm is proportional to the largest prime factor of $p-1$ rather than its square root. Therefore there may exist a range for the size of the largest prime factors of p_i-1 for which a system based on this idea is both practical and secure. A possible choice could be to let m be the product of 2 primes p_1 and p_2 of about 100 decimal digits each, where $(p_1-1)/2$ and $(p_2-1)/2$ both are the product of several 13- to 15-digit primes.

When the computational effort spent by the trusted authority is increased by a factor k, this forces an adversary to increase his computational effort by a factor k^2. Thus when k-fold faster computer hardware becomes available this system's security can also be increased by a factor of k. This system is asymptotically superior to the system of Section 2 for which the work factor could be increased only by a factor $k^{\sqrt{2}-1} = k^{.414}$.

The lemma in the previous section suggests a way to derive an identity number from a user's identity such that the discrete logarithm of this number modulo m to the base α is guaranteed to exists. In the case where m is the product of only two prime factors there exists an alternative though less practical approach which is mentioned here for the sake of completeness. Let $m = p_1p_2$ and let $\gcd(p_1-1, p_2-1) = 2$. The (without knowledge of the factorization of m) easily computable Jacobi symbol $(x|m)$ is equal to 1 if and only if x is a quadratic residue either for both $GF(p_1)$ and $GF(p_2)$ or for none of them. Equivalently, $(x|m) = 1$ if and only if the discrete logarithms in $GF(p_1)$ and $GF(p_2)$ are congruent modulo 2, i.e., if and only if x possesses a discrete logarithm modulo m. Hence a user's identity number can be defined as the smallest integer x greater or equal to the number representing his name and such that $(x|m) = 1$. No interaction is required for transmitting the offset since it can easily be determined without knowing the factorization of m.

5. Conclusions

A remarkable property of the presented systems is that not only the cryptanalyst, but also the trusted authority must spend time super-polynomial in the input size. However, because the system is used for an appropriate fixed size of parameters, the trusted authority's computation is nevertheless feasible. Progress in computer technology can be exploited to increase the security of the system.

There may exist other approaches than those presented to making the discrete logarithm problem feasible only when given the factorization of the modulus. Any progress in the discrete logarithm problem not leading to a comparable progress in the factorization problem, especially when applicable to primes of a certain special form, has the potential of leading to an improvement of the presented system. An interesting open question is whether it is possible to construct primes p of a special form containing a trapdoor such that computing discrete logarithms modulo p is feasible if and only if the trapdoor is known.

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