A Discrete Model of Oil Recovery

Germán González-Santos[1] and Cristobal Vargas-Jarillo[2]

[1] Escuela Superior de Física y Matemáticas del IPN, Unidad Profesional Adolfo López Mateos, México D.F. 07738
gsantos@esfm.ipn.mx

[2] Departamento de Matemáticas del CINVESTAV, A. P. 14-740, México D. F. 07000
cvargas@math.cinvestav.mx

Abstract. We propose the simulation of oil recovery by means of a molecular type approach. By using a finite set of particles under the interaction of a Lennard-Jones type potential we simulate the behavior of a fluid in a porous media, and we show that under certain conditions the fingering phenomena appears.

1 Introduction

In this work we propose the simulation of oil recovery by means of a molecular type approach. This means that we consider the materials to be composed of a finite number of particles, which are approximants for molecules. Porous flow is studied qualitatively under the assumption that particles of rock, oil and the flooding flow interact with each other by means of a compensating Lennard-Jones type potential. We also consider the system to be under the influence of gravity. We study miscible displacement in an oil reservoir from various sets of initial data. The velocity and the rate of injection of the ingoing particles proved to be among the most important parameters that can be adjusted to increase the rate of production. It is also noted that the fingering phenomenon is readily detected. This simulation technique has been used in [1-2] and [4] to simulate several physical systems. Details of this method applied to the study of porous flow can be founded in [3].

2 Model formulation

Consider a rectangular region R, which is a porous medium. We assume that in this region we have a resident fluid or oil. We shall introduce a different kind of fluid which, as a matter of convenience, will be called water, although it is an aqueous solution which could be a polymeric solution, surfactant solution or a brine. The physical system consists of $N = N_1 + N_2 + N_3$ particles, $P_1, P_2, ..., P_N$, with masses $m_1, m_2, ..., m_N$. The particles

$P_1, P_2, ..., P_{N_1}$,	Represent rocks,
$P_{N_1+1}, P_2, ..., P_{N_2}$,	Oil, and
$P_{N_2+1}, P_2, ..., P_N$	Incoming water

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For purposes of injection of water and production of oil, two wells are opened, one in the bottom left corner of R, for injection, and other in the diagonally opposite corner for production, see Fig. 1. The variables at time $t = k\Delta t$ are:

$\overline{r}_{i,k}$ Coordinates of the particle,
$r_{i,j,k}$ Distance between the particles P_i and P_j,
$\overline{v}_{i,k}$ Velocity of the particle,
$\overline{a}_{i,k}$ Acceleration of the particle,
$\overline{F}_{i,j,k}$ Local force exerted on P_i by P_j,
$\overline{F}^*_{i,k}$ Local force acting on P_i due to the other particles,
$\overline{f}_{i,k}$ Long range force acting on P_i (like gravity),
$\overline{F}_{i,k}$ Total force on particle P_i,
for $i = 1, 2, ..., N$ and $k = 0, 1, ...$

The local force $\overline{F}_{i,j,k}$ exerted on P_i by P_j is

$$\overline{F}_{i,j,k} = m_i m_j \left[\frac{H_{i,j}}{r_{i,j,k}^{q_{i,j}}} - \frac{G_{i,j}}{r_{i,j,k}^{p_{i,j}}} \right] \frac{\overline{r}_{j,k} - \overline{r}_{i,k}}{r_{i,j,k}}, \tag{1}$$

where the values of the parameters $H_{i,j}$, $G_{i,j}$, $q_{i,j}$ and $p_{i,j}$ depend on the particles which are interacting. The total local force $\overline{F}^*_{i,k}$ acting on particle P_i due to the other particles is given by:

$$\overline{F}^*_{i,k} = \sum_{j=1, j \neq i}^{N} \overline{F}_{i,j,k} \tag{2}$$

Therefore, the total force acting upon the particle P_i is

$$\overline{F}_{i,k} = \overline{F}^*_{i,k} + \overline{f}_{i,k}. \tag{3}$$

The aceleration of P_i is related to the force by Newton's Law

$$\overline{F}_{i,k} = m_i \overline{a}_{i,k}. \tag{4}$$

In general system (4) can not be solved analytically from given initial positions and velocities, therefore it must be solved numerically. For economy, simplicity and relatively numerical stability we use the "leap frog" formulae, which has second-order accuracy in time,

$$\begin{aligned} &\overline{v}_{i,1/2} = \overline{v}_{i,0} + \tfrac{1}{2}\overline{a}_{i,0}\Delta t \\ &\overline{v}_{i,k+1/2} = \overline{v}_{i,k-1/2} + \tfrac{1}{2}\overline{a}_{i,k}\Delta t \\ &\overline{r}_{i,k+1} = \overline{r}_{i,k} + \tfrac{1}{2}\overline{v}_{i,k-1/2}\Delta t \\ &\qquad \text{for } i = 1, 2, ..., N, \quad k = 1, 2, ... \quad . \end{aligned}$$

The number of calculations required to evaluate (1) at each iteration is $O(N^2)$. However this number is much smaller if the potential is truncated for a distance greater than r_c .

3 Boundary Conditions

We assume that the particles of the fluids loose energy when they interact with the walls of the region R, therefore it will be necessary to model the hardness of the wall relative to the reflection of the interacting fluid, and it is done by using the following damping factors acting on the velocity of the reflected particles.

$$\delta_i = 0.4 \; for \; i = N_1 + 1, ..., N_1 + N_2 \text{ , } and$$
$$\delta_i = 0.8 \; for \; i = N_1 + N_2 + 1, ..., N.$$

4 Initial conditions

The rock and oil particles, for two an three dimensions; were set up at the initial time in such a way that they satisfied an equilibrium state, as shown in Fig. 1 and 2.

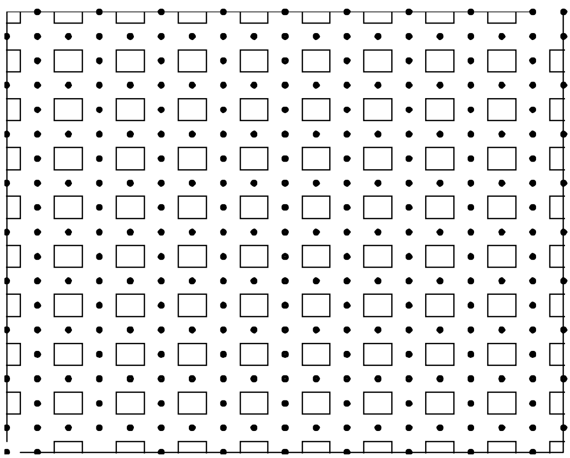

Fig. 1. Initial configuration in two dimensions

5 Numerical results in 2D

Figure 3 shows the system evolution. All the examples were run with time step $\Delta t = E - 5$ on the Sun workstation Ultra 60, the distance between particles of water before going to into the well was $d = 0.5$ and their velocity was $v = 15.0$. The gravity constant was equal to $g = 9.8$. The Lennard-Jones potentential parameters are summarized in table 1.

Figure 4 shows the advancement of water for different times, the shaded area is the region which has been traveled only by water, this means that not oil

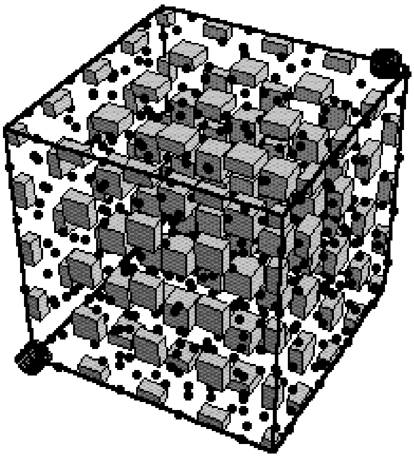

Fig. 2. Initial configuration in three dimensions

Table 1. Parameters for the numerical experiments in two dimensions. In this case $F = 0.5$

	Rock	Oil	Water
Rock	$H = 0$ $G = 0$		
Oil	$H = 1$ $G = 3$ $E = F * \sqrt{13/36}$	$H = 1$ $G = 1$ $E = F * 1.3$	
Water	$H = 1.5$ $G = 0$ $E = F * 13/36$	$H = 1$ $G = 0$ $E = F * 1.15$	$H = 1$ $G = 0$ $E = F$

particle has been in that area for some time. Figure 5 shows the number of particles of oil out and the number of particles of water out versus time. We can see from the graph that for t small, the rate of oil production is higher when v is higher. We can also observe that water comes out of the production well sooner for $v = 100$ than for $v = 15$.

6 Numerical results in 3D

The results in three dimensions are shown in Fig 6. All the examples were run with time step $\Delta t = E - 5$ on a Cluster of PC computers.

The distance between particles of water before going into the well, was $d = 0.5$ and their and velocity was $v = 5.0$. The gravity constant was equal to $g = 9.8$. The Lennard-Jones potential parameters are summarized in table 2.

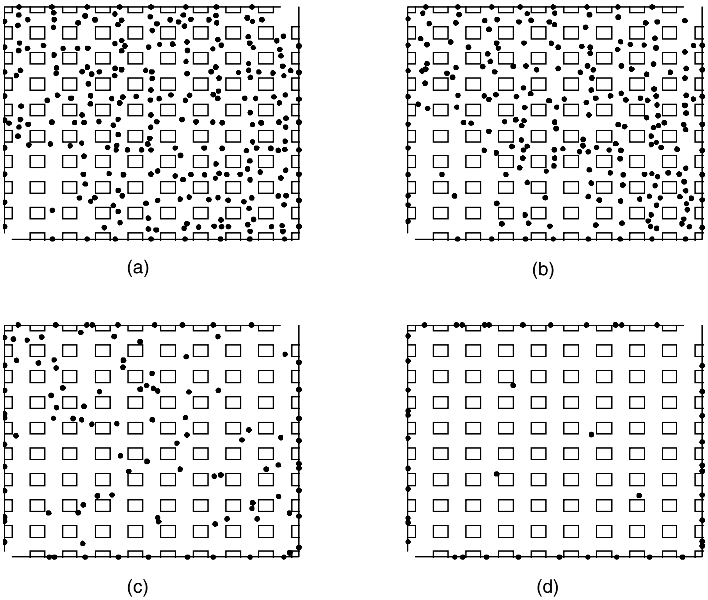

Fig. 3. Evolution of the oil particles. (a) $Time = 0.1$, (b) $Time = 0.8$, (c) $Time = 2.0$, (d) $Time = 4.8$

Figure 6 shows the effect of the oil and water production, when velocity of the water particles is increased. An increment in the velocity of the water particles produces an increment on the oil and water production.

References

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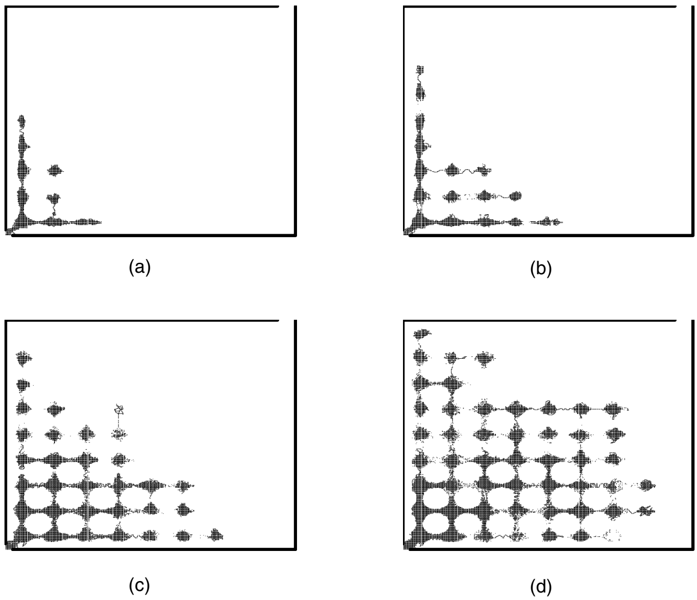

Fig. 4. Advancement of the water for $d = 1, v = 5$ at different times. (a) $Iter = 3E5$, (b) $Iter = 4.25E5$, (c) $Iter = 7.8E5$, and (d) $Iter = 1.2E6$

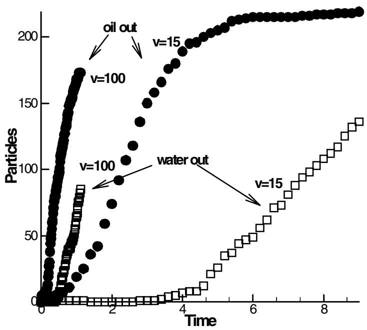

Fig. 5. Comparison of the effect of the velocity of the water particles on the oil and water production

Table 2. Parameters for the numerical experiments in three dimensions. In this case $F = 1.0$.

	Rock	Oil	Water
Rock	$H = 0$ $G = 0$		
Oil	$H = 1$ $G = 3$ $E = F * \sqrt{13/36}$	$H = 1$ $G = 1$ $E = F * 1.3$	
Water	$H = 1.5$ $G = 0$ $E = F * 13/36$	$H = 1$ $G = 0$ $E = F * 1.15$	$H = 1$ $G = 0$ $E = F$

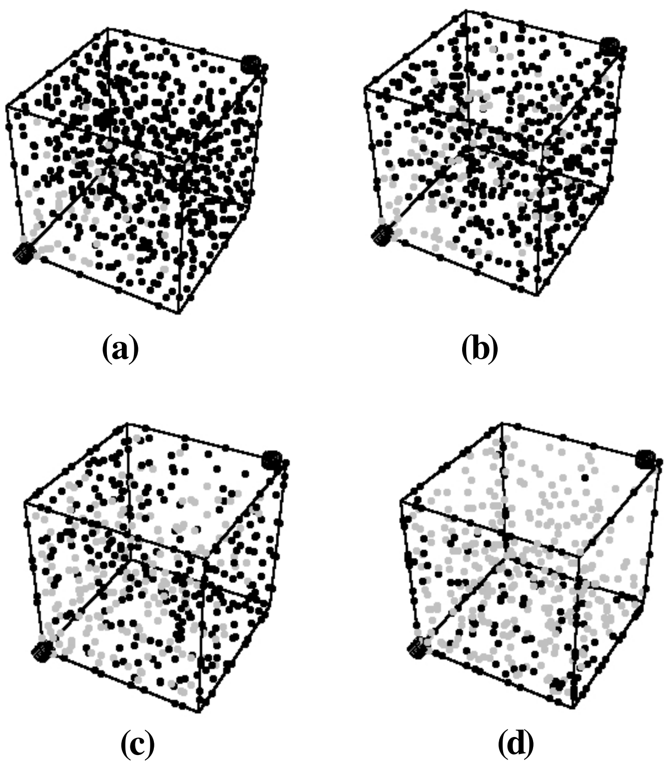

Fig. 6. Evolution of the oil and water particles at different times. (a)$Iter = 1E6$, (b)$Iter = 2E6$, (c)$Iter = 4E6$, and (d)$Iter = 1E6$

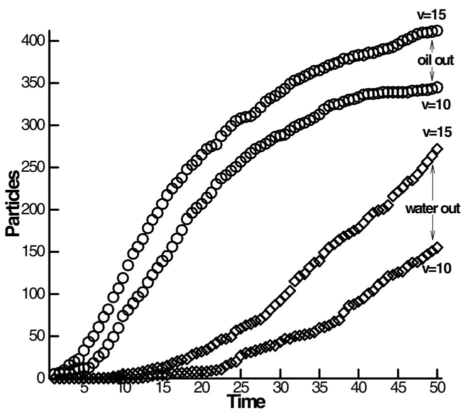

Fig. 7. Comparison of the effect of the velocity of the water particles on the oil and water production