On the Motion Control of a Nonholonomic Soccer Playing Robot

Giovanni Indiveri

GMD-AiS, German National Research Center on Information Technologies,
Institute for Autonomous Intelligent Systems,
Schloß Birlinghoven, 53754 Sankt Augustin, Germany
giovanni.indiveri@gmd.de
http://ais.gmd.de/~giovanni/

Abstract. A nonlinear control law to steer the unicycle model to a static or dynamic target pose is presented. If the target is static the control signals are smooth in their arguments and the solution guarantees exponential convergence of the distance and orientation errors to zero. The major advantages of the proposed approach are that there is no need for path planning and, in principle, there is no need for global self-localization either.

1 Introduction

The most simple kinematic model of an underactuated wheeled robot is given by the so called unicycle model, namely:

$$\left.\begin{aligned} \dot{x} &= u\cos\phi \\ \dot{y} &= u\sin\phi \\ \omega &= \dot{\phi} \end{aligned}\right\} \tag{1}$$

where x, y are the cartesian coordinates of the center of mass of the robot, ϕ is its orientation with respect to the x axis and u, ω are its linear and angular velocities. The major difficulties in designing motion control laws for underactuated mobile robots are captured by the above (1) simple kinematic model as it is subject to Brocketts Theorem [1]. For a detailed description of nonholonomic car-like system motion control issues refer to [2] [3] [4].

This paper is organized as follows: in section (2) the addressed problem is formally stated, in section (3) a solution for the most simple case (still ball) is described, in section (4) the more general solution is outlined. Section (5) contains some simulation examples and conclusions are drawn in section (6).

2 Problem Statement

Given the unicycle model (1) design a feedback law for the linear and angular velocities u and ω such that asymptotically the $\mathbf{q} = (x, y)^T$ position of the

A. Birk, S. Coradeschi, and S. Tadokoro (Eds.): RoboCup 2001, LNAI 2377, pp. 387–392, 2002.

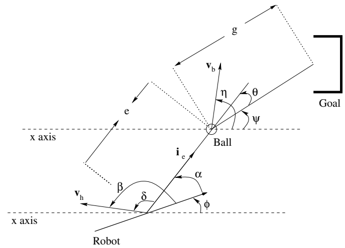

Fig. 1. The model

robot is driven to a given target one $\mathbf{q}_b = (x_b, y_b)^T$ (the ball) along a possibly time-varying reference direction $\psi(t)$. The control law will be designed adopting a polar-like representation of the unicycle kinematic model (1), namely with reference to figure (1) the following variables are introduced: $\mathbf{q} = (x, y)^T$: absolute cartesian robot position; $\mathbf{q}_b = (x_b, y_b)^T$: absolute cartesian ball position; $\mathbf{V}_b$: ball velocity vector; $V_b = \|\mathbf{V}_b\|$: norm of the ball velocity vector (assumed constant); η : absolute direction (assumed constant) of $\mathbf{V}_b$; $e = \|\mathbf{q}_b - \mathbf{q}\|$: robot - ball distance; $\mathbf{i}_e = \frac{\mathbf{q}_b - \mathbf{q}}{\|\mathbf{q}_b - \mathbf{q}\|}$: unit vector pointing from the robot to the ball; $\mathbf{e} = \mathbf{i}_e\, e$: robot - ball "distance" vector; $\mathbf{j}_e = \mathbf{k} \wedge \mathbf{i}_e$: unit vector normal to $\mathbf{i}_e$ and to the z axis unit vector $\mathbf{k}$; g : ball - target (by example, the center of the goal) distance; $\mathbf{i}_u$: unit vector parallel to the robot heading; u : robot linear velocity; $\mathbf{u} = \mathbf{i}_u\, u$: robot linear velocity vector; $\omega = \dot{\phi}$: robot angular velocity; $\psi(t)$: desired asymptotic robot heading. Time varying if $V_b \neq 0$; $\alpha = \widehat{\mathbf{i}_u, \mathbf{i}_e}$: angle between $\mathbf{i}_u$ and $\mathbf{i}_e$; other relevant angular variables defined similarly to α and depicted in figure (1) are ϕ, θ, β and δ. Notice that α and θ are not defined when $e = 0$ and ψ is not defined when $g = 0$. The relevant dynamic equations for the above defined variables may be deduced starting from the cartesian model (1) yielding:

$$\dot{e} = -u \cos\alpha + V_b \cos(\eta - \theta - \psi) \tag{2}$$
$$\dot{\phi} = \omega \tag{3}$$
$$\dot{\alpha} = -\omega + \frac{u}{e} \sin\alpha + \frac{V_b}{e} \sin(\eta - \alpha - \phi) \tag{4}$$
$$\dot{\theta} = \frac{u}{e} \sin\alpha + \frac{V_b}{e} \sin(\eta - \alpha - \phi) + \frac{V_b}{e} \sin(\eta - \psi) \tag{5}$$
$$\dot{\psi} = -\frac{V_b}{g} \sin(\eta - \psi) \tag{6}$$
$$\dot{g} = -V_b \cos(\eta - \psi) \tag{7}$$
$$\beta = \delta + \alpha \quad ; \quad \beta \equiv \widehat{\mathbf{i}_u, \mathbf{v}_h} \tag{8}$$
$$\phi + \alpha = \theta + \psi \tag{9}$$
$$\dot{\eta} = \dot{V}_b = 0 \text{ (assumption)} \tag{10}$$

The above allow to give a more formal statement of the problem, namely find u and ω such that $\lim_{t\to\infty}(e,\alpha,\theta)^T = (0,0,0)^T$. It should be noticed that given the equations (2-6) the above problem statement is well posed if and only if $V_b = 0$. It will be then shown that in the case $V_b \neq 0$ the same design method adopted for the case $V_b = 0$ allows to compute bounded control laws that drive the vehicle in a neighborhood of $(e,\alpha,\theta)^T = (0,0,0)^T$ although not exactly in $(0,0,0)^T$. The singularities occurring when $e = 0$ may be avoided by a proper choice of the control inputs as will be discussed in the following sections.

3 Still Ball Case

The controller designed for this case is based on the one reported in [5]. The idea is quite simple and originates from the following observation: if one wanted to drive a fully actuated, i.e. not subject to any nonholonomic constraint, ideal point to a fixed ($V_b = 0$) target it would be sufficient to impose to this point the velocity:

$$\mathbf{v}_h = \gamma_e\, e\, \mathbf{i}_e + \gamma_\theta\, \theta\, e\, \mathbf{j}_e \quad : \quad \gamma_e, \gamma_\theta > 0 \tag{11}$$

which is given by the superposition of a linear velocity $\gamma_e\, e\, \mathbf{i}_e$ that would drive e exponentially to zero and an angular one $\dot\theta\, \mathbf{k} = -\gamma_\theta\, \theta\, \mathbf{k}$ that would drive θ exponentially to zero. As outlined in [5], to cope with the unicycle nonholonomic constraint and guarantee asymptotic convergence of e, α and θ to zero the linear velocity may be chosen to be

$$u = \mathbf{i}_u^T\, \mathbf{v}_h = \sqrt{\gamma_e^2 + \gamma_\theta^2\, \theta^2}\ \ e \cos\beta. \tag{12}$$

and the angular one such that $\dot\beta = -\gamma_\beta\, \beta : \beta = \widehat{\mathbf{i}_u, \mathbf{v}_h}$, yielding:

$$\omega = \gamma_\beta\, \beta + \left(\frac{\gamma_e\, \gamma_\theta}{\gamma_e^2 + \gamma_\theta^2\, \theta^2} + 1\right) \sqrt{\gamma_e^2 + \gamma_\theta^2\, \theta^2}\ \ \cos\beta \sin\alpha \tag{13}$$

The $\cos\beta$ term in (12) and (13) is responsible for possible backward driving, but may be replaced by any smooth $f(\beta) \geq 0 \;\; : \;\; f(0) = 0$ without affecting the convergence properties [5]. Interestingly this design may be extended to $3D$ [6].

4 General Case

In order to extend the above design to the general case $V_b \neq 0$ first of all the field $\mathbf{v}_h$ must be re-designed: in particular one may chose $\mathbf{v}_h$ to be the superposition of the previously given $\mathbf{v}_h$ (11) plus a velocity $\mathbf{v}_r$ that, if implemented alone, would allow the robot to view e and θ as constants. With this idea in mind it follows that

$$\mathbf{v}_h = \gamma_e\, e \mathbf{i}_e + \gamma_\theta\, \theta\, e\, \mathbf{j}_e + \mathbf{v}_r \tag{14}$$

$$\mathbf{v}_r = \mathbf{V}_b + \mathbf{j}_e\, \frac{V_b\, e}{g} \sin(\eta - \psi) = \tag{15}$$

$$= \mathbf{i}_e\, V_b\, \cos(\eta - \theta - \psi) + \mathbf{j}_e \left(V_b\, \sin(\eta - \theta - \psi) + \frac{V_b\, e}{g} \sin(\eta - \psi) \right) \tag{16}$$

By direct calculation it follows that:

$$v_{hx} \equiv \mathbf{v}_h^T\,\mathbf{i}_e \quad ; \quad v_{hy} \equiv \mathbf{v}_h^T\,\mathbf{j}_e \quad \Rightarrow \quad \dot{\delta} = \frac{v_{hx}\,\dot{v}_{hy} - v_{hy}\,\dot{v}_{hx}}{\|\mathbf{v}_h\|^2} \tag{17}$$

$$\dot{v}_{hx} = \gamma_e\,\dot{e} + V_b\,\sin(\eta - \theta - \psi)(\dot{\theta} + \dot{\psi}) \tag{18}$$

$$\begin{aligned}\dot{v}_{hy} = {} & [\gamma_\theta\,e - V_b\,\cos(\eta - \theta - \psi)]\,\dot{\theta} + \left(\gamma_\theta\,\theta + \frac{V_b}{g}\sin(\eta - \psi)\right)\,\dot{e} \\ & -V_b\left(\frac{e}{g}\,\cos(\eta - \psi) + \cos(\eta - \theta - \psi)\right)\,\dot{\psi} - \frac{V_b\,e}{g^2}\,\sin(\eta - \psi)\,\dot{g}\end{aligned} \tag{19}$$

where the terms $\dot{e}, \dot{\theta}, \dot{\psi}$ and $\dot{g}$ are given by equations (2), (5), (6) and (7). Notice that $\dot{\delta}$ given above will not depend explicitly from ω. Wanting to impose once again the closed loop dynamics $\dot{\beta} = -\gamma_\beta\,\beta$ on β equation (8) is differentiated with respect to time yielding:

$$\dot{\beta} = \dot{\delta} + \dot{\alpha} = \dot{\delta} - \omega + \frac{u}{e}\sin\alpha + \frac{V_b}{e}\sin(\eta - \alpha - \phi) \quad \Rightarrow \tag{20}$$

$$\omega = \gamma_\beta\,\beta + \dot{\delta} + \frac{u}{e}\sin\alpha + \frac{V_b}{e}\sin(\eta - \alpha - \phi) \tag{21}$$

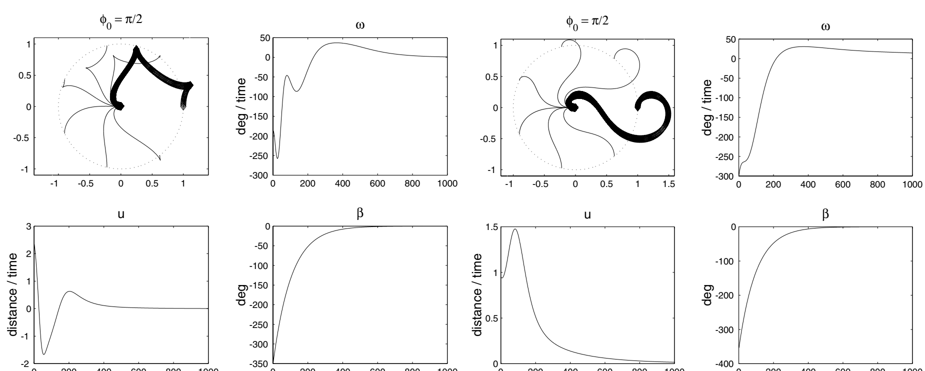

Fig. 2. Static ball ($V_b = 0$) case: paths resulting from different initial positions on a unit circle and initial orientation $\phi_0 = \pi/2$. Gains $\gamma_\theta = 0.3, \gamma_e = 0.1, \gamma_\beta = 1$.

where $\dot{\delta}$ is given by equations (17-19). As expected and anticipated in section (2), the steering input is singular in $e = 0$: indeed while the singularity could be “compensated” in the case $V_b = 0$ by a proper choice of u, i.e. $u \sim e$ in a neighborhood of $e = 0$, when $V_b \neq 0$ there is no way of compensating the terms proportional to V_b/e with a bounded control action. Notice that such terms are also present in the closed loop expression for $\dot{\delta}$. Nevertheless the steering law given by equation (21) may be still adopted in practice and indeed it is

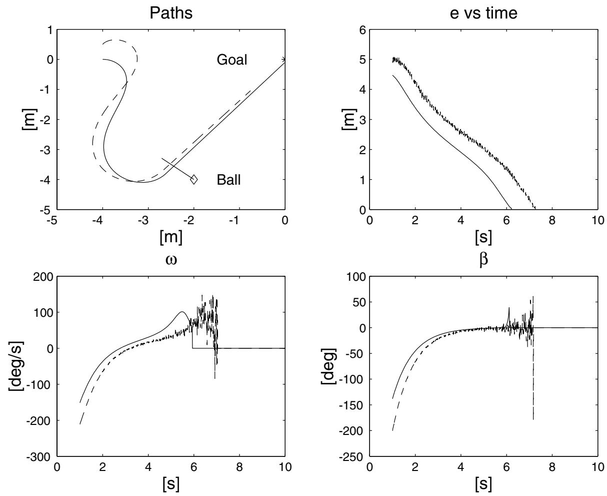

Fig. 3. $V_b \neq 0$ case. Refer to the text for details.

successfully implemented on the GMD RoboCup robots. A possible and simple implementation is to keep the linear velocity u always constant and to clip ω to zero once $e \leq \varepsilon$ for some threshold value $\varepsilon > 0$. As long as $e > \varepsilon > 0$ equation (21) can be implemented without problems and it will steer the vehicle parallel to $\mathbf{v}_h$ given by equations (14) and (16); once in a ε-neighborhood of $e = 0$ the angular velocity is kept null and the linear velocity u is kept constant. The net effect will be to hit the ball along a direction "close" to ψ. The deviation in the kicking direction with respect to the desired value ψ will be a function of the relative speed of the vehicle and the ball, of the angle η and of ε.

5 Simulation Examples

The control laws (12) and (13) relative to the static ball case ($V_b = 0$) have been simulated for several starting poses on a unit circle. Results are shown in figure (2): the first four plots (first two columns from the left) refer to the implementation of the control laws (12) and (13). As expected the linear velocity may be negative (backward drive) resulting in cusps, i.e. points of the plane where $\omega \neq 0$ and $u = 0$. The signals u, ω and β plotted refer to the path drawn with a thicker line. To avoid driving backwards and thus the cusps, the $\cos\beta$ term in (12) and (13) has been replaced with $f = 1$ giving rise to the results shown in the other two columns. Two different simulations of the general case ($V_b \neq 0$) are reported in figure (3). In both cases the robots linear velocity u was kept constant and equal to $u = 1\,m/s$. The ball had velocity $V_b = 0.1\,m/s$ and direction $\eta = 3\pi/4\,rad = 135\,deg$ starting from the initial x, y position $-2\,m, -4\,m$. The goal was positioned in the origin $(0, 0)$ in both cases. The steering law given by equation (21) has been implemented in the region $e > \varepsilon$ with $\varepsilon = 0.3\,m$. As soon as $e \leq \varepsilon$ the robots angular velocity ω was clipped to zero. The control gains where $\gamma_\beta = 1, \gamma_\theta = 0.8, \gamma_e = 1/2$ in both cases whereas the initial robots pose was $(-4\,m, 0, 0)$ for the solid line case and $(-4\,m, 1/2\,m, \pi/3\,rad)$ for the dashed line one. The solid line results refer to the ideal noise free case, while the dashed line results refer to the case in which additive gaussian noise of standard deviation $\sigma_{V_b} = 0.0333\,m/s$ and $\sigma_b = 0.05\,m$ where added respectively on the robots V_b estimate and ball position. This last noise affects in particular g, ψ, α, θ and their derivatives. For the sake of clarity, wanting to compare the two simulations the path driven by the ball is plotted as if there was no impact. The fact that indeed the impact takes place is clearly revealed by the plot of e versus time. The variables e and β are computed and plotted versus time only up to the impact and then set to zero. The high frequency action of ω in the vicinity of the target in the dashed line simulation is to be related to the terms divided by e in equation (21).

6 Conclusions

A nonlinear control law to steer the unicycle model to a desired pose has been presented. If the target is static the control signals are smooth in their arguments and the solution guarantees exponential convergence of the distance and orientation errors to zero. In the case of a moving target a solution has been presented that guarantees exponential convergence of the β angle to a neighborhood of $\beta = 0$.

References

1. Brockett, Millmann, and Sussmann, Eds., *Differential Geometric Control Theory*, chapter Asymptotic Stability and Feedback Stabilization, by Brockett, R. W., pp. 181–191, Birkhauser, Boston, USA, 1983.
2. C. Canudas de Wit, H. Khennouf, C. Samson, and O. J. Sørdalen, “Nonlinear Control Design for Mobile Robots”, in “Recent Trends in Mobile Robotics”, Yuan F. Zheng editor, vol. 11, World Scientific Series in Robotics and Automated Systems, pp. 121–156, 1993.
3. J.-P. Laumond, Ed., *Robot Motion Planning and Control*, chapter Feedback control of a nonholonomic car-like robot, by A. DeLuca, G. Oriolo, C. Samson, Springer-Verlag, ISBN 3-540-76219-1, available from http://www.laas.fr/~jpl/book.html, 1998.
4. I. Kolmanovsky and N. H. McClamroch, “Developments in nonholonomic control problems,” *IEEE Control Systems*, pp. 20–36, December 1995.
5. M. Aicardi, G. Cannata, G. Casalino, and G. Indiveri, “On the stabilization of the unicycle model projecting a holonomic solution,” in *8th Int. Symposium on Robotics with Applications, ISORA 2000*, Maui, Hawaii, USA, June 11-16 2000.
6. M. Aicardi, G. Cannata, G. Casalino, and G. Indiveri, “Guidance of 3D underwater non-holonomic vehicle via projection on holonomic solutions,” in *Symposium on Underwater Robotic Technology SURT 2000, World Automation Congress WAC 2000*, Maui, Hawaii, USA, June 11-16 2000.