Double Auction in Two-Level Markets*

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Abstract. In the general discussion of double auction, three properties are mainly considered: incentive compatibility, budget balance, and economic efficiency. In this paper, we introduce another property of double auction: semi-independence, from which we are trying to reveal the essential relation between incentive compatibility and economic efficiency.
· Babaioff and Nisan [1] studied supply chain of markets and corresponding protocols that solve the transaction and price issues in markets chain. In the second part of the paper, we extend their model to two-level markets, in which all markets in the supply chain are independent and controlled by different owners. Beyond this basic markets chain, there is a communication network (among all owners and another global manager) that instructs the transaction and price issues of the basic markets. Then we discuss incentive compatible problems of owners in the middle level of the markets in terms of semi-independence.

1 Introduction

With the rapid progresses of e-commerce over the Internet, a number of economic concepts have been integrated with computer science extensively, such as Game Theory [9,10], Mechanism Design [7,8] and Auction Theory [6]. One of the most remarkable combinations is that of electronic market, which is on the basis of double auctions.

Double auction, a classic economic concept, specifies that multiple sellers and buyers submit bids to ask for transactions for some well-defined goods [4]. For different purposes, the designed protocols should satisfy required properties over the Internet. Three properties are mainly concerned: incentive compatibility, budget balance, and economic efficiency. The first one emphasize the truthful behavior of agents (sellers and buyers), whereas the last two are macro requirements to the outcome. It's well known that the three properties can not be hold simultaneously under mild assumptions [7]. Hence we are trying to seek for the more essential relations among these properties. Specifically, we present

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another property, semi-independence, from which we study the relation between incentive compatibility and economic efficiency deeply.

Note that such electronic market, on which the double auction applies, can not reflect the information of related commodities exactly. Hence Babaioff and Nisan introduced the model of supply chain of markets [1] to solve this problem. For example, in lemonade-stand industry, a lemon market sells lemons, a squeezing market offers squeezing services, and lemonade are provided by a juice market. These three markets are composed of the supply chain of lemonade. In addition, Babaioff and Nisan introduced symmetric (pivot) protocol for exchanging information along markets. These protocols determine the supply/demand curve for each market, and allow each market to function independently.

However, markets of different goods in that model are controlled by a single person. Sometimes, a supply chain of markets may be very difficult to construct and manage for a single one. Hence we propose to study a model that allows the individual market to be fully independent. Specifically, we consider two-level markets, where each market in the original supply chain is controlled by a different owner. Moreover, there is another higher level market among owners. The transaction and price issues of the basic markets chain are determined through the interaction of owners at the upper level market.

Many appealing problems arise in such two-level markets model. For example, as participants of the markets, all owners may have their own targets and utilities. Thus we need consider the truthful behaviors of owners, *i.e.*, incentive compatibility. Observing that as a link between two-level markets, such behaviors include two directions: to agents and to manager. That is, the owner may lie to each of them or both. Therefore in this paper, we study the relation between the mechanisms of the markets chain and the truthful behavior of owners. Specifically, we show several sufficient and necessary conditions that guarantee incentive compatibility for different types of owners.

Note that the study of truthful behavior of owners is more closer to practical (electronic) markets, since intermediaries (owners) play an important role in the transactions of the markets and the performance of the whole markets is mostly determined by behaviors of these owners. As we have seen in reality, the loss of efficiency and budget deficit are mainly due to the selfish behaviors of such owners. Therefore we believe that this paper is an important step in the quest for the mechanisms that promote economic efficiency and market revenue.

In section 2, we review the basic concepts of double auction and introduce the property of semi-independence. Next, we briefly review the supply chain markets model of [1] and introduce the model of two-level markets. In section 4, we study the issue of incentive compatibility of owners in two-level markets.

2 Double Auction

2.1 The Model

In a market of a kind of goods, there are n sellers and buyers, respectively. Each seller has one indivisible goods to sell and each buyer plans to buy at most one item. Each agent (seller or buyer) has a privately known non-negative real, termed as *type*, representing the true valuation that the seller/buyer wants to charge/pay.

To win the auction, each agent submits a value to the auctioneer. Let s_i be the i-th supply bid (in the non-decreasing order) of sellers and d_j the j-th demand bid (in the non-increasing order) of buyers, *i.e.*, $0 \leq s_1 \leq s_2 \leq \cdots \leq s_n$ and $d_1 \geq d_2 \geq \cdots \geq d_n \geq 0$. We denote $S = (s_1, \ldots, s_n)$ as the *supply curve* and $D = (d_1, \ldots, d_n)$ as the *demand curve* of market. The *utility* for trading agent (*i.e.*, *winner*) is the absolute value of the difference between price and his true type. For non-trading agent, the utility is zero. We assume that all agents are self-interested, that is, they aim to maximize their own utilities. Note that to maximize the utilities, agents may not submit their types, the strategy is determined according to different double auction rules.

Double auction (DA) rule R is a mechanism that, upon receiving input: supply and demand curves S and D, specifies the quantity $q = R_q(S, D)$ of transactions to be conducted and the price $p_s = R_s(S, D)$, $p_d = R_d(S, D)$ that the trading sellers/buyers receive/pay. Note that in all DA discussed in this paper, non-trading agents receive/pay zero, and once the trade quantity q is fixed, the winners are the first q sellers (with lowest supply bids) and buyers (with highest demand bids), respectively. Moreover, we only concern *non-discriminating* DA, *i.e.*, the price paid by all buyers is same and the price paid to all sellers is same too, but the two values are not necessarily equal.

Let l, *optimal trade quantity*, be the maximal index such that $s_l \leq d_l$. Assume without loss of generality that there always exist the $(l+1)$-th supply and demand bid, s_{l+1} and d_{l+1}, such that $s_{l+1} > d_{l+1}$. Otherwise, we may add sellers with bids ∞ and buyers with bids zero[1]. Followings are some classic DA rules:

- **k-DA:** ([12,2]) $q = l$, and $p_s = p_d = k \cdot s_l + (1-k) \cdot d_l$, where $k \in [0, 1]$.
- **VCG DA:** ([11,3,5]) $q = l$, and $p_s = \min\{s_{l+1}, d_l\}$, $p_d = \max\{s_l, d_{l+1}\}$.
- **Trade Reduction (TR) DA:** ([1]) $q = l - 1$, and $p_s = s_l$, $p_d = d_l$.

2.2 Semi-independence of Double Auction

We denote $(a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$ as a_{-i} and (a_{-i}, a_i) as the tuple $(a_1, \ldots, a_n)$. In this paper, we mainly consider the following properties of DA: (i) *incentive compatibility*: The DA motivates self-interested agents to submit their types to maximize the utility values. It's easy to see that for any incentive compatible DA, there must be $p_s \geq s_q$ and $p_d \leq d_q$, where q units of goods traded. (ii) *budget balance*: The total payment of buyers should be at least the total amount given to sellers, *i.e.*, the revenue of the mechanism is non-negative. (iii) *economic efficiency*: The desired outcome should maximize the total types of all agents. That is, all sellers with types below the market clearing price should trade with all buyers whose types are above the clearing price [1].

[1] Note that for any efficient DA, the optimal trade quantity l may not be identical under different supply and demand curves.

Note that trading l units of goods maximizes efficiency if all agents submit their true types. We denote such DA that trading l units of goods as *efficient DA*. Following let's first look at another property of DA, semi-independence.

Definition 1 (Semi-Independence): Given efficient DA R, supply and demand curves S and D, let $p_s = R_s(S, D)$, $p_d = R_d(S, D)$, and $l = R_q(S, D)$ be optimal trade quantity. We say price p_s is *semi-independent* of s_i under S and D, if

- $1 \leq i \leq l$, $R_s((s_{-i}, s'_i), D) = p_s$, for $\forall$ $s'_i \in [s_{i-1}, s_i]$ (if i=1, $s'_1 \in [0, s_1]$).
- $l < i \leq n$, $R_s((s_{-i}, s'_i), D) = p_s$, for $\forall$ $s'_i \in [s_i, s_{i+1}]$ (if $i = n$, $s'_n \in [s_n, \infty)$).

That is, when s_i changes to its neighbor bid continuously, the price p_s will not change. To sellers, the price is *semi-independent* of i-th supply bid if for $\forall$ S and D, price $R_s(S, D)$ is semi-independent of the i-th supply bid s_i under S and D. If the price is semi-independent of all supply bids under any supply and demand curves, we say DA R is *semi-independent* of supply curve. Similarly, we can define R is semi-independent of demand curve.

Following lemma reveals the essential connections between incentive compatibility and economic efficiency on the basis of semi-independence.

Lemma 1 For any efficient and incentive compatible DA R, let l be the optimal trade quantity, then the price to sellers/buyers is semi-independent of i-th supply/demand bid, for all $1 \leq i \leq l$. And if the $(l+1)$-th supply/demand bid is strictly smaller/larger than the $(l+2)$-th bid, the price is not semi-independent of the $(l+1)$-th bid.

Proof. We only prove the case to sellers and supply bids, the other one is similar. We first prove the first part, that is, the price to sellers is semi-independent of i-th supply bid, for all $1 \leq i \leq l$.

For any supply and demand curves $S = (s_1, \dots, s_n)$ and D, let $p_s = R_s(S, D)$. Then we need to show that p_s is semi-independent of $s_1, \dots, s_l$. Suppose otherwise, that there exists i, $1 \leq i \leq l$, such that modifying s_i to s'_i will change p_s to p'_s, where s'_i satisfies Definition 2.1. If $p_s < p'_s$, the seller who bids s_i can increase his utility simply by submitting s'_i untruthfully. If $p_s > p'_s$, let $(s_1, \dots, s_{i-1}, s'_i, s_{i+1}, \dots, s_n)$ be true types of all sellers respectively. Then same as above, the seller who bids s'_i can increase his utility by reporting s_i. A contradiction to incentive compatibility.

It remains to prove that the price to sellers is not semi-independent of the $(l+1)$-th supply bid s_{l+1} if $s_{l+1} < s_{l+2}$. The idea is to construct a pair of supply and demand curves $S = (s_1, \dots, s_n)$ and $D = (d_1, \dots, d_n)$, *s.t.* $d_{l+1} < s_l < s_{l+1} < d_l$, to show that the price $p_s = R_s(S, D)$ is not semi-independent of s_{l+1}. It's easy to see that such pair of curves does exist.

Claim 1. Price $p'_s = R_s(S', D) = R_s(S, D)$, where $S' = (s_{-l}, s_{l+1}) = (s_1, \dots, s_{l-1}, s_{l+1},$
$s_{l+1}, s_{l+2}, \dots, s_n)$, *i.e.*, $p'_s = p_s$.

Proof of the Claim. Note that the l-th supply bid equals the $(l+1)$-th supply bid in S', since $d_{l+1} < s_{l+1} < d_l$, the number of trading goods under S' and D is also l according to the efficiency of R, *i.e.*, $R_q(S', D) = l$. Thus from above discussion, price p'_s is semi-independent of the l-th supply bid s_{l+1}. Hence, when reducing s_{l+1} to s_l (note that $s_l \in [s_{l-1}, s_{l+1}]$), the price does not change, *i.e.*, $p'_s = p_s$. □

Our following discussions are based on S' and D. Note that $R_q(S', D) = l$, and the l-th supply bid is s_{l+1} in S', thus there must be $p'_s \geq s_{l+1}$. Next we consider two cases.

Case 1. $p'_s > s_{l+1}$. In this case, let s_{l+1} be the true type of the $(l+1)$-th seller who bids s_{l+1}. Then if he submits the bid truthfully, he won't get any utility. But if he submits an sufficiently small value $\varepsilon \geq 0$, the supply curve will be $\widetilde{S} = (\varepsilon, s_1, \ldots, s_{l-1}, s_{l+1}, s_{l+2}, \ldots,$
$s_n)$ (assuming the submitted bids of other sellers don't change). According to R, he will be a trading seller since the number of trading goods is still l ($s_{l-1} \leq s_l < d_l$ and $s_{l+1} > d_{l+1}$). Following we consider the price $\widetilde{p}_s = R_s(\widetilde{S}, D)$. Because $\widetilde{p}_s$ is semi-independent of all the first l supply bids, $(\varepsilon, s_1, \ldots, s_{l-1})$, then the following supply curves sequence share the same price:

$$\begin{aligned} S &= (s_1, s_2, \ldots, s_l, s_{l+1}, \ldots, s_n) \rightarrow \\ &(\varepsilon, s_2, s_3, \ldots, s_l, s_{l+1}, \ldots, s_n) \rightarrow \\ &(\varepsilon, s_1, s_3, \ldots, s_l, s_{l+1}, \ldots, s_n) \rightarrow \cdots \rightarrow \\ (\varepsilon, s_1, s_2, \ldots, s_{l-1}, s_{l+1}, \ldots, s_n) &= \widetilde{S}. \end{aligned}$$

That is, we have $R_s(\widetilde{S}, D) = R_s(S, D)$, *i.e.*, $\widetilde{p}_s = p_s = p'_s$. Hence the seller who bids ε untruthfully will get utility $\widetilde{p}_s - s_{l+1} = p'_s - s_{l+1} > 0$, a contradiction.

Case 2. $p'_s = s_{l+1}$. It's easy to see there exists $\delta > 0$ such that $d_{l+1} < s_{l+1} + \delta < d_l$ and $s_{l+1} + \delta < s_{l+2}$. Thus when the $(l+1)$-th supply bid changes from s_{l+1} to $s_{l+1} + \delta$, if price p'_s is changed, then the lemma follows. Otherwise, we have

$$R_s((s_1, \ldots, s_{l-1}, s_{l+1}, s_{l+1} + \delta, s_{l+2}, \ldots, s_n), D) = p'_s.$$

Similar as claim 1, we can ensure the same price p'_s even increasing the l-th supply bid from s_{l+1} to $s_{l+1} + \delta$. That is,

$$R_s((s_1, \ldots, s_{l-1}, s_{l+1} + \delta, s_{l+1} + \delta, s_{l+2}, \ldots, s_n), D) = p'_s.$$

Let $s_{l+1} + \delta$ be the true type of the l-th seller who bids $s_{l+1} + \delta$. Then although he is a trading seller, but the utility of him is negative, *i.e.*, $p'_s - (s_{l+1} + \delta) < 0$, which contradicts the property of incentive compatibility.

Therefore, we have constructed specified supply and demand curves such that the price is not semi-independent of the $(l+1)$-th supply bid, hence the lemma follows. □

Note that if $s_{l+1} = s_{l+2}$, the lemma also works for the first index i, where $l + 1 < i < n$ and $s_{l+1} = \cdots = s_i < s_{i+1}$, such that the price to sellers is semi-independent of s_i. If $s_{l+1} = \cdots = s_n$, then the price to sellers is semi-independent of s_n.

3 Supply Chain of Markets

Supply chain of markets is a sequence of markets, where the first one M^0 provides resources, the last one M^t consumes the final desired goods and all middle markets $M^1, \ldots, M^{t-1}$ convert the previous goods to the following one sequentially. Without loss of generality, assume the number of agent in each market is equal to n.

3.1 Symmetric Protocol

Note that only the *agents* (buyers) in the last consume market submit demand bids, whereas other *agents* (sellers) in other markets submit supply bids. For any market M^i, denote $S^i = (s_1^i, \ldots, s_n^i)$ as the *supply curve* and $D^i = (d_1^i, \ldots, d_n^i)$ the *demand curve* of M^i, where $0 \leq s_1^i \leq \ldots \leq s_n^i$ and $d_1^i \geq \ldots \geq d_n^i \geq 0$. Here $S^0, S^1, \ldots, S^{t-1}, D^t$ are composed of the submitted bids of agents of each market respectively, other curves are computed in terms of the following symmetric protocol by Babaioff and Nisan [1].

Symmetric Protocol:

Input: Supply/demand curves $S^0, S^1, \ldots, S^{t-1}, D^t$.

Algorithm:

(1) $s_j^t = \sum_{i=0}^{t-1} s_j^i$, for $1 \leq j \leq n$.
(2) $e_j = d_j^t - s_j^t$.
(3) $d_j^i = s_j^i + e_j$, for $0 \leq i \leq t-1$, $1 \leq j \leq n$.

Output: Demand/supply curves $D^0, D^1, \ldots, D^{t-1}, S^t$.

We stress here that above definition is just in terms of its mathematical sense, the original one (exchanging information along the sequence of markets) is referred to [1]. One of the most attractive properties of symmetric protocol is the following lemma.

Lemma 2 The supply curve S^i of market M^i is independent of its demand curve D^i under symmetric protocol, for all $0 \leq i \leq t$.

Proof. We only prove the lemma for supply market M^0, others are similar. Let $E = (e_1, \ldots, e_n)$, According to symmetric protocol, the demand curve of M^0 is:

$$D^0 = S^0 + E = S^0 + (D^t - \sum_{i=0}^{t-1} S^i) = D^t - \sum_{i=1}^{t-1} S^i$$

which implies that S^0 is independent of D^0. □

3.2 Two-Level Markets

Babaioff and Nisan [1] studied the supply chain of markets based on that the auctioneer, who creates the markets chain, conducts all affairs among them. Sometimes, however, it may be very difficult to deal with such a huge markets network for a single one.

Therefore, we consider the following *two-level markets* model, in which all basic markets in original supply chain are independent and controlled by different *owners*, rather than a single one. We denote the owner of market M^i as O^i. Among all owners and another (global) *manager*, the previous auctioneer, there is a communication network (market) that instruct the transaction and price issues of all basic markets. That is, all owners submit supply/demand curves to the manager, who specifies affairs of basic markets in terms of the global mechanism. Formally,

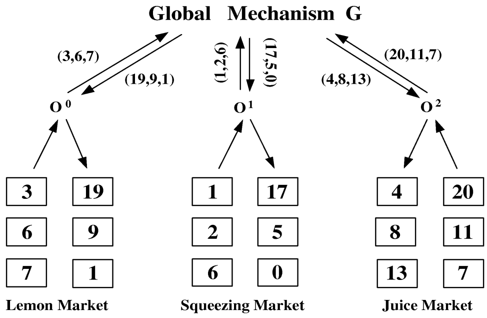
Definition 2 (Global Mechanism) A *global mechanism* G of manager, upon receiving supply/demand curves of markets owners, specify the following two issues:

1. Computing demand/supply curve for each market in terms of symmetric protocol.
2. Deciding the number of transactions to be conducted and the price that the sellers/buyers should receive/pay among all markets.

In this paper we only consider the case that take DA (with symmetric protocol) as global mechanism, *i.e.*, *global DA mechanism*. That is, the number of transactions and price in all markets are determined independently by (the unique) DA. Hence, all discussed properties of DA also apply in the global case.

Example 1 (Lemonade stand industry [1]) Assume there are three communicating electronic markets that produce lemonade: a lemon market M^0, sells lemons; a squeezing market M^1, offers squeezing services; a juice market M^2, from which buyers buy lemonade. There're three agents in each market with valuations showed in the left columns of each market in Graph 1. Then all participants work as follows:

1. Each agent submits his supply/demand bid to the corresponding owner. (Assume all agents know the protocol of global mechanism G at first).
2. Each owner submits the received bids (curve) to the manager.
3. The manager performs the global mechanism G, and returns the results to owners.
4. Each owner returns the above results to agents in his market.
5. Each market acts independently in terms of mechanism G and corresponding curves, then agents get their awards from the owner. For example, if we use global VCG DA, two juices are sold in juice market at price $8 = \max\{8, 7\}$ each. □



Graph 1: Operation of Lemonade Stand Industry

Note all agents participate step 1, 4, 5, whereas the manager participates step 2, 3. That is, agents and manager do not communicate directly. For simplicity, following we denote M as the supply market M^0 and omit the index 0 of all bids and curves of M.

4 Selfish Owners

Observing that besides performing the protocol of two-level markets, each owner may has his own goal and to optimize it, he may not execute the protocol correctly. Thus we need take their reactions into consideration. In this paper, we assume the *utility* of each owner is the currency he obtained from the market. Hence in this case, the function of the selfish owner is, on one hand, controlling the market, on the other hand, maximizing his utility. Following we only consider owner O of supply market M. Other owners of the conversion markets and demand market are similar.

Example 2 (Selfish owner O of lemon market under global VCG DA) Consider owner O of lemon market showed in above section, the supply bids that he received are $(3, 6, 7)$. If O submits the supply curve truthfully, the manager will return corresponding demand curve $(19, 9, 1)$ to him. According to VCG DA, the first two sellers are traded at price 7 each, and O will pay them 14, which is obtained from the manager. Then we regard that O does not get any utility. However, if O submits $(3, 6, 8)$ untruthfully, the manager will handout 16 to him but at the inner lemon market, O only needs to pay 14 to the two trading sellers. Thus O gets 2 dollars successfully from his lying. (Note that agents do not know what the supply curve that O submits to the manager). □

Remark 1. In fact, the trick of O is that the manager only knows the submitted bid 8, whereas agents only know the true bid 7. Similarly, O may get utility by lying to agents. Moreover, this phenomenon also arises in global k-DA and TR DA.

Remark 2. For non-incentive compatible DA, the submitted bids of agents may not be their true types. But here, we only consider the conduct of owners, *i.e.*, whether he can get positive utility from his lying.

Remark 3. Note that when O submits the supply curve untruthfully, the number of transactions may be different from that when he submits the curve truthfully. Thus, to avoid from being detected by the manager and agents, all owners should be restricted to *quantity consistent* (QC): if the number of transactions is q when all owners truthfully submit their received curves, the quantity should also be q even if any owner lies.

Therefore it's reasonable to consider the following property of global mechanism (here, to distinguish the definition of incentive compatibility of agents, we use the term *immaculate* to describe the truthful behavior of owners):

Definition 3 (Immaculate) In two-level markets, a global mechanism is said to be *immaculate* if for any market owner O^i, $0 \leq i \leq t$, and supply/demand curves submitted/returned by other owners, O^i won't get any positive utility under the restriction of QC, no matter what the supply/demand curve he submits/returns.

4.1 Both-Side Untruthful

In Example 2, O may get positive utility simply by deceiving all agents in M that the demand curve was $(19, 6, 1)$ instead of the true one $(19, 9, 1)$. Thus O (who obtains 14 from the manager) gets 2 dollars (he only pays 12 to the first two sellers). That is, owners may lie to both the manager and agents in his market, *i.e.*, both-side untruthful.

Definition 4 (Pseudo-Constant DA Families) DA rule R belongs to *pseudo-constant DA families* if trading quantity is the unique variable of price function to trading sellers. That is, for any supply and demand bids, there exists a function f, *s.t.* the price to all trading sellers is $f(q)$, where q is the trading quantity according to R.

It's easy to see that for any global DA mechanism that is contained in pseudo-constant DA families, the owner won't get any utility under QC constraint.

Theorem 1 Any global DA mechanism R that does not belong to pseudo-constant DA families is not immaculate when the owner is both-side untruthful.

Proof. Because R is not contained in pseudo-constant DA families, there exist two pairs of supply/demand curves (S, D) and (S', D'), such that $q = R_q(S, D) = R_q(S', D')$, and $R_s(S, D) \neq R_s(S', D')$. Let $p_s = R_s(S, D)$ and $p'_s = R_s(S', D')$.

Assume without loss of generality that $p_s < p'_s$. Thus we can regard S as the true bids of sellers, whereas owner O submits S' to the manager. Assume the demand curve of market M is D' according to other markets' curves. Note that under symmetric protocol, there always exist the submitted curves of other markets owners such that the demand curve of M is D' (Lemma 2). Hence in terms of S' and D', the manager decides q units of goods to be trade and the price to trading sellers is p'_s. When getting all payments, $q \cdot p'_s$, O returns D to all agents in M. In such situation, O only needs to pay trading sellers p_s rather other the true price p'_s (since $p_s = R_s(S, D)$). Therefore, O defalcates $q \cdot (p'_s - p_s) > 0$ from his lying. □

4.2 One-Side Untruthful – To Manager

Although pseudo-constant DA families avoid corruptions of owners, such mechanisms perform infeasibly because they can not reflect the supply curves efficiently. That is, the price may deviate from the values of bids arbitrarily, since it only depends on the number of transactions. Therefore in this section, we restrict the power of owners that assume they are always truthful to agents in their own markets (we denote such owners as *type* 1). That is, the demand/supply curve that the owner returns to agents is always the truth. Hence, the utilities of owners are only from the manager, and all trading agents indeed get their original payment.

Lemma 3 If all owners are type 1 and efficient global DA mechanism R is not semi-independent of supply curve, then R is not immaculate.

Proof. Trivially, there exist supply/demand curves S and D and i, $1 \leq i \leq n$, such that p_s is not semi-independent of s_i, where $p_s = R_s(S, D)$. Note that Lemma 2 implies that the demand curve D is independent of $S = (s_{-i}, s_i)$. Let l be optimal trade quantity.

Since p_s is not semi-independent of s_i, there exists $S' = (s_{-i}, s'_i)$ such that $p'_s \neq p_s$, where $p'_s = R_s(S', D)$, and s'_i satisfies Definition 1. Following we consider two possible cases. If $p_s < p'_s$, let S be submitted bids of agents, then O can get $l \cdot (p'_s - p_s)$ utilities by submitting S' untruthfully to the manager (note that untruthfulness of the owner is restricted to QC constraint). If $p'_s < p_s$, let S' be submitted bids of agents. Similar as above, O can get $l \cdot (p_s - p'_s)$ utilities by submitting S to the manager.

Thus O may always get positive utility as long as he does not truthfully submit supply curve. Therefore the global DA mechanism R is not immaculate. □

Whereas if p_s is semi-independent of all supply bids, intuitively, no matter what the supply curve that O submits, he won't get any utility from the manager. Formally,

Lemma 4 If all owners are type 1 and efficient global DA mechanism R is semi-independent of supply curve, then R is immaculate.

Proof. Assume submitted bids of agents in M are $s_1, \ldots, s_n$ and $s_1 \leq s_2 \leq \cdots \leq s_n$. Let $D = (d_1, \ldots, d_n)$ be demand curve of market M, which is computed in terms of symmetric protocol. Let $p_s = R(S, D)$. To prove R is immaculate, we only need to show that for any supply curve $S' = (s'_1, \ldots, s'_n)$ that O submits, $p'_s = p_s$, where $p'_s = R_s(S', D)$. Note that QC constraint implies that $s'_l \leq d_l$ and $s'_{l+1} > d_{l+1}$. Let $\widetilde{S} = (s'_1, \ldots, s'_l, s_{l+1}, \ldots, s_n)$ and $\widetilde{p}_s = R_s(\widetilde{S}, D)$. We observe the following two facts.

Claim 1. $\widetilde{p}_s = p_s$.

Proof of the Claim. We use mathematical induction on the number k that the last $n-k$ elements of S and $\widetilde{S}$ are all identical, *i.e.*,

$$(s'_{k+1}, \ldots, s'_l, s_{l+1}, \ldots, s_n) = (s_{k+1}, \ldots, s_l, s_{l+1}, \ldots, s_n),$$

where $0 \leq k \leq l$. The cases of $k = 0, 1$ are trivial. Suppose the claim is correct when $k \leq j - 1$, $2 \leq j < l$. Then if $k = j$, we have

$$(s'_{j+1}, \ldots, s'_l, s_{l+1}, \ldots, s_n) = (s_{j+1}, \ldots, s_l, s_{l+1}, \ldots, s_n) \ \ \& \ \ s'_j \neq s_j.$$

Assume without loss of generality that $s'_j < s_j$ (the other part is similar as above). We consider two cases about the relations between s'_j and $(s_1, \ldots, s_j)$ as follows.

Case 1. There exists i, $1 \leq i \leq j-1$, such that $s_i \leq s'_j < s_{i+1}$. Then the following supply curves sequence share the same price:

$$\begin{aligned} S \quad = \quad & (s_1, \ldots, s_i, s_{i+1}, \ldots, s_j, s_{j+1}, \ldots, s_n) \to \\ & (s_1, \ldots, s_i, s'_j, s_{i+2}, \ldots, s_j, s_{j+1}, \ldots, s_n) \to \\ & (s_1, \ldots, s_i, s'_j, s'_j, s_{i+3}, \ldots, s_j, s_{j+1}, \ldots, s_n) \to \cdots \to \\ & (s_1, \ldots, s_i, s'_j, \ldots, s'_j, s_{j+1}, \ldots s_n) \to \\ & (s'_1, \ldots, s'_i, s'_{i+1}, \ldots, s'_j, s_{j+1}, \ldots, s_n) = \widetilde{S}. \end{aligned}$$

Note that the last step is by induction hypothesis and others are by semi-independence.

Case 2. $s'_j < s_1 \leq \cdots \leq s_j$. Similar as above case, supply curve S and $\widetilde{S}$ share the same price. Therefore, by induction, we have $\widetilde{p}_s = p_s$. □

Claim 2. $p'_s = \widetilde{p}_s$.
Proof of the Claim. (sketch) Similar as the proof of Claim 1, but here the induction is on the number k that the first $n - k$ elements of S' and $\widetilde{S}$ are identical, where $0 \leq k \leq n - l$. We omit details here. □

Therefore the payment p'_s, $\widetilde{p}_s$ and p_s, on submitted supply curves S', $\widetilde{S}$ and S respectively, are all identical, *i.e.*, $p'_s = \widetilde{p}_s = p_s$. Thus the lemma follows. □

Theorem 2 When all owners are type 1, efficient global DA mechanism R is immaculate if and only if it's semi-independent of supply curve.

4.3 One-Side Untruthful – To Agents

Contrary to above subsection, we may assume all owners are always truthful to manager (*e.g.*, they may be appointed by the manager) and they can return deliberately constructed demand/supply curves to agents to get utility (we denote such owners as *type* 2). Note that the utilities of owners in this case are from their lying to agents. Similarly, we have the following theorem.

Theorem 3 When all owners are type 2, efficient global DA mechanism R is immaculate if and only if it's semi-independent of demand curve.

From above discussion and Lemma 1, we have the following corollary.

Corollary 1 If global DA mechanism R is efficient and incentive compatible, then it's not immaculate, regardless the types of agents.

Thus global VCG mechanism is not immaculate. Note that although global k-DA and TR mechanisms do not satisfy the conditions of above corollary (efficiency and incentive compatibility), they are not immaculate mechanisms too. But as to global TR mechanism, we can modify it to immaculate one simply by exchanging the price functions to trading sellers and buyers.

5 Conclusion and Further Research

A more general model in practice, two-level markets, is studied in this paper. Note that our discussions are based on symmetric protocol, but all results also apply to other protocols that share Lemma 2.

In addition, we study incentive compatible problem for selfish owners who aim to maximize their own utilities. Contrary to this direction, we may consider another one that the goal of each owner is to optimize some common values, such as the total utility of trading agents and income/outcome of the agents in his market. Actually, this work (on selfless owners) is under investment right now.

As we have seen in this paper and many other works, the implementation of incentive compatibility is on the cost of decreasing the revenue of the markets. Thus, whether there exists a weaker feasible notion of incentive compatibility (such as approximation or average case) is a very meaningful direction in the future work.

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