Parallel Computing Method of Valuing for Multi-asset European Option*

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Abstract. A critical problem in Finance Engineering is to value the option and other derivatives securities correctly. The Monte Carlo method (MC) is an important one in the computation for the valuation of multi-asset European option. But its convergence rate is very slow. So various quasi Monte Carlo methods and there relative parallel computing method are becoming an important approach to the valuing of multi-asset European option. In this paper, we use a number-theoretic method, which is a H-W method, to generate identical distributed point set in order to compute the value of the multi-asset European option. It turns out to be very effective, and the time of computing is greatly shortened. Comparing with other methods, the method computes less points and it is especially suitable for high dimension problem.

1 Introduction

The benefit and the risk of derivatives tools are not only influenced by the self relationship between demands and services, but also rely on the balance of demand and serve of underlying asset. The key problem of the financial project is how to estimate the value of option and other portfolio properly. Black and Scholes concluded the precise formula to estimate the value of European call option and put option[1]. But now the problem of estimating the value of European option, which is relying on several underlying asset price, is not solved preferably[1]. At the present time, there are three methods to estimate the value of option[2]: formulas, deterministic numerical methods and Monte Carlo simulation. MC method is an important one for estimating the value of European option. Random numbers are used on lots of disparate routes for sampling, and

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the variables of underlying asset in the world of riskless follow these tracks. We can calculate the benefit of each route and such benefit discounts due to riskless rate. This lose and benefit are discount according to riskless rate. Then we use the arithmetical average value of all benefit after discount as the evaluation of option. Compared with other methods, MC method is effective if there are several variables, because MC method leads to an approximately linear increase in the computation time when the number of variables grows, while most of other methods lead to an exponential increase. In the MC method, samples must be done on the route of every variable in each simulation. For example, in a simulation, N samples are obtained from multi-dimension standard normal school. Then on one of simulative routes for the estimating the value of option relying on n variables, there are N samples needed. To estimate accurately, both the number of execution of simulation and the number of samples N are large. Because the convergence rate of MC is quite slow, when the number of variables n is also large, some methods, such as tens and hundreds, $\mathbf{O}(n^{-1/2})$ are needed to deal with N sample routes, and the compute load goes beyond the capacity of a single computer. Therefore Parallel computing method and quasi Monte Carlo method are used widely to solve the problems[2][3][4][5][6].

Many researches have been done aboard. Paskov brought forward the method of generating quasi MC sequence of Soblo and Halton to estimate the value of European option relying on several variables, and they compared the performance of these two methods[7]; Pagageorgiou and Traub selected a quasi MC method, which is faster than MC method and uses less sample points, to solve a problem of European option relying on 360 variables[8].In addition, Acworth compared several MC methods and quasi MC methods detailedly, and concluded that quasi MC method is better than normal MC method[4]. In [3], a quasi MC method called (t,m,s)-net method is selected to estimate the value of European option of underlying asset. In this paper, we introduce NTM and select H-W (HUA Luogeng - WANG Yuan) method to generate consistent distributed point set to estimate the value of European option of several underlying asset, and make out satisfying result in little time.

2 Model of Estimating The Value of European Option of Multiple Assets

We consider how to evaluate the value of European option with multiple statuses. The value of its derivatives securities relies on the N statuses variable, such as the price and elapsed time τ of the discount of ventual asset. Suppose S_i $(i = 1, 2, \cdots, n)$ as the value of underlying asset_I and $V(S_1, S_2, \cdots, S_n, \tau)$ as the value of derivatives securities. According to the Black-Scholes equation, we can acquire the patiel diferebtial coefficients equation of estimating the derivatives securities value of European option of several variables[1]:

$$\frac{\partial V}{\partial \tau} = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\sigma_i\sigma_j S_i S_j \frac{\partial^2 V}{\partial S_i \partial S_j} + r\sum_{i=1}^{n}\frac{\partial V}{\partial S_i} - rV, 0 < S_1, \cdots, S_n < \infty, \tau \in (0,T) \tag{1}$$

and ρ_{ij}, $(i, j = 1, 2, \cdots, n)$ is the relative coefficient, a known constant. $\sigma_i > 0$ is volatility of underlying asset. $r > 0$ is riskless rate. Both σ_i and r are constants. Suppose $S = (S_1, S_2, \cdots, S_n)^T$ as the asset price vector at current time t. $V(S, \tau)$ is meant to represent $V(S_1, S_2, \cdots, S_n, \tau)$ and S_T is meant to represent the asset price vector at expire time T. The boundary condition is:

$$V(S_T, 0) = \max(X - \max(S_1, \cdots, S_n), 0)$$

Here X is the strike price. For European call option, the boundary condition $V(S_T, 0)$ is

$$\begin{cases} C_{\max}(S,0)=\max(\max(S_1,\cdots,S_n) - X, 0) \\ C_{\min}(S,0)=\max(\min(S_1,\cdots,S_n) - X, 0) \end{cases} \tag{2}$$

And for European put option, the boundary condition $V(S_T, 0)$ is

$$\begin{cases} P_{\max}(S,0)=\max(X - \max(S_1,\cdots,S_n), 0) \\ P_{\min}(S,0)=\max(X - \min(S_1,\cdots,S_n), 0) \end{cases} \tag{3}$$

To estimate the value of European option of several status variables, the equation(1) can be induced to multi-integral problem[5]:

$$V(S,\tau) = e^{-r\tau}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} V(S,0)\Psi(S_T; S, \tau)dS_T \tag{4}$$

Here:

$$\Psi(S_T; S, \tau) = \frac{1}{(2\pi\tau)^{\frac{n}{2}}\sqrt{det\ R}\tilde{\sigma}\tilde{S_T}}\exp(-\frac{1}{2}W_T^T R^{-1} W_T) \tag{5}$$

is the transform density function of several variables, where

$$W_T = \left(\frac{\ln S_{T_1} - \hat{S}_1}{\sigma_1\sqrt{\tau}}, \cdots, \frac{\ln S_{T_n} - \hat{S}_n}{\sigma_n\sqrt{\tau}}\right) \tag{6}$$

$$\hat{S}_i = \ln S_i + \left(r - \frac{\sigma_i}{2}\right)\tau, i = 1, 2, \cdots, n,\ \tilde{\sigma} = \prod_{i=1}^{n}\sigma_i, \tilde{S}_i = \prod_{i=1}^{n} S_{T_i} \tag{7}$$

$R = (\rho_{ij})_{n\times n}, \rho_{ii} = 1$ and when $i \neq j, \rho_{ij} \in (0,1)$

3 The Parallel Strategy and Algorithm

3.1 NTM Method

NTM is the derivation of numeric theory and proximate analysis. In fact it is also a kind of quasi MC method. The key problem of computing approximately the multi-integral on S- dimension unit cube C^s using NTM method is how to obtain the symmetrically distributed points set on C^s. Assume $P_n = \{c_k^{(n)}, k = 1, \cdots, n\}$ as a points set on C^s. If it is a NT-nets on C^s, in the other word, it has low difference[3], $I(f)$ can be approached by :

$$I(f, P_n) = \frac{1}{n} \sum_{k=1}^{n} f(c_k^{(n)}) \tag{8}$$

Therefore, how to conclude the best quadrature formula is equivalence to how to find the best consistent distributed point set. In the reference[9], Korobov put forward the method to find the best consistent distributed point set, and the error rate is $\mathbf{O}(n^{-1}(\log n)^s)$, Considered at the point of view of approximation, the result of Korobov method is a existence theorem, so it is difficult to solve real problems using this method. Therefore HUA Luogeng and WANG Yuan (called H-W method) brought up a method that obtains net point aggregation using partition round region[9], which is called H-W method, and the error rate is $\mathbf{O}(n^{-\frac{1}{2}-\frac{1}{2(s-1)}+\varepsilon})$, H-W method obtains symmetrically distributed points set by this way:

$$\gamma = \left\{ \left(\left\{ 2\cos\frac{2\pi}{p} \right\}, \left\{ 2\cos\frac{4\pi}{p} \right\}, \cdots, \left\{ 2\cos\frac{2\pi n}{p} \right\} \right), k = 1, 2, \cdots \right\} \tag{9}$$

Here, p is a prime number and $p \geq 2n + 3$, $\{x\}$ is meant to represent the fraction part of x. By the means of $\gamma_i(1 \leq i \leq s$ rational number approach defined at (9), H-W method brought forward a method obtaining net point aggregation, which is called partition round region method[3]. Here we use the algorithm of parallel computing, combining the method of numeric theory, to resolve the high dimension integral problem in estimate the value of European option.

3.2 Method Comparison

For the European option of several assets, the number of assets is normally to be tens, or even hundreds. Therefore the multi-integral to compute is tens-integral, or even hundreds-integral. At present the method to compute multi-integral is approximate enumeration, and the quality of solution relies on the large numbers of points set. As the scale of problem and the quality of solution increase, the computing time increases. Sometimes, because the increase of computing dimension or the scale of problem often overwhelms the time restriction easily, the method would loose its feasibility. For example, sometimes we change the multi-integral to overlapped integral of single integral on [0,1], then apply the formula on single integral in turns. But this traditional method is not feasible sometimes. For example, Simpson formula, the error rate is $\mathbf{O}(n^{-2/s})$, and the convergent rate is $\mathbf{O}(n^{-1/2})$, When s is some of large, the number of approximate computing point increases quickly. Another MC method of formatting quadrature formula is to transform the analytic problem to a probability problem with the same solution,

$$I(f,n) = \frac{1}{n}\sum_{i=1}^{n} f(x_i)$$

then study this probability problem by statistical simulation. The error rate of this method is $\mathbf{O}(n^{-1/2})$ and is better than $\mathbf{O}(n^{-1/s})$ The convergent rate is irrespective to the number of dimensions, but it is very slow, just $\mathbf{O}\sqrt{\ln(ln(n))/n}$. The efficiency of MC method is not nice, and only when n is very large, $I(f,n)$ can obtain the satisfied approached result. So in order to increase the convergent rate and reduce the scale of computing, a lot of quasi MC methods emerge as the times require[3][4]. Especially as the NTM develops, the method of computing multi-integral develops quickly. The points set of C^s obtained by NTM is more even, less point number and less computation than by MC method.

3.3 Parallel Computing of NTM

When we use parallel method to compute equation (4), we first make the problem discrete, then divide the compute task into several parts averagely and distribute them to corresponding processor to do. Equation (4) changes to:

$$V(S,\tau) = e^{-rt}\int_0^{\infty}\cdots\int_0^{\infty} Q(S_T;S,\tau)dS_T \tag{10}$$

where

$$Q(S_T;S,\tau) = V(S_T,0)\Psi(S_T;S,\tau) = V(S_T,0)\frac{\exp\left(-\frac{1}{2}W_T^T R^{-1} W_T\right)}{(2\pi\tau)^{\frac{n}{2}}\sqrt{det\ R}\tilde{\sigma}\tilde{S_T}} \tag{11}$$

$$V(S_T,0) = \max(X - \max(S_{T_1},\cdots,S_{T_n}),0) \tag{12}$$

Suppose $\{\theta_j\} = \{(\theta_1,\cdots,\theta_n)\}, \Delta\tau = T/M, \Delta S = a/N$, (T is the expire time, a is strike price) After making discrete,equation (8) changes to

$$V_{i,k} = V(i\Delta S, k\Delta t) = V(i_1\Delta S,\cdots,i_n\Delta S,k\Delta\tau) = \frac{\exp(rk\Delta\tau)}{N}\sum_{j=1}^{n} Q(a\theta_j;i\Delta S,k\Delta\tau) \tag{13}$$

Here N is the number of sample points in each status. The value of derivatives securities at some time for different asset can be obtained by equation (13). The cluster can deal with the problem of dividing the compute grid point easily and apply on parallel compute of equation (13).

4 Experiment Result and Conclusion

We use MPI environment in cluster system. When the number of dimension is certain, the grid point can be generated ahead, be stored in a file and be read out when needed. But when the number of dimension is some of large, the file generated is very large, so it must be divided into several parts in the parallel environment of cluster system. Therefore each processor generates NT-net grid points parallel and deals with the computation of data generated by itself. After each processor finished the computation of itself, we collect the result. In the process of parallel computation, there is nearly not data communication. We take the computation of estimating the value of 50 assets option for an example, the parameter is selected as [3]. The computation result is also similar with this paper. Table 1 lists the speedup using different number of processors on "Tsinghua TongFang Explorer 108 Cluster System".

Table 1. Speedup in different number of processors

Number of Processor	1	2	4	6	8	12
Speedup(S_p)	/	1.89	3.67	5.34	6.83	9.96

At present, when the number of assets, which is relied on by European option, is very large, such as tens or hundreds, if we need to get a precise result, the number of execution and the number of sample N are some of large. Common MC methods can not match the time restriction. In this paper, NTM is selected. H-W method generates consistent distributed points set to estimate the value of European option of several underlying assets, and obtains satisfied result, with advanced algorithm and short computing time. We conclude that the method is suited for high dimension computation.

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