Planar Symmetry Detection by Random Sampling and Voting Process

Atsushi Imiya, Tomoki Ueno, and Iris Fermin

Dept. of IIS, Chiba University, 1-33, Yayo-cho, Inage-ku, Chiba, 263-8522, Japan
imiya@ics.tj.chiba-u.ac.jp

Abstract. We propose a randomized method for the detection of symmetry in planar polygons without assuming the predetermination of the centroids of the objects. Using a voting process, which is the main concept of the Hough transform in image processing, we transform the geometric computation for symmetry detection which is usually based on graph theory and combinatorial optimization, to the peak detection problem in a voting space in the context of the Hough transform.

1 Introduction

In pattern recognition, the symmetry of an object is an important feature because symmetry provides references for the recognition and measurement of objects. The symmetry information enables the speeding up of the recognition process and also the reduction of the space required for storage of the object models. The symmetry properties of objects yield valuable information for image understanding and compression. For the utilization of the symmetry properties, it is necessary to determine the symmetry axes or shape orientations.

In this paper, we propose a symmetry detection method based on the random sampling and voting process. An object is said to be rotationally symmetric if the object, after being affected an transformation, becomes identical to the original object. If point set $\mathbf{V}$ is an n-fold rotationally symmetric object with $n \geq 3$, this point set $\mathbf{V}$ will be identical to itself after being rotated around the centroid through any multiple of $\frac{2\pi}{n}$. The voting process converts direct geometric and analytical computation of features from data to the peak detection problem in a voting space. The method proposed here is an extension of our randomized method for motion detection, which we proposed previously [1,2]. In our previous papers [1,2], we derived algorithm for both 2D and 3D motion estimation without the predetermination of point correspondences. If an object is rotationally symmetrical, the result of a rotation which is determined by the symmetry derives the same shape, that is, there are ambiguities in the solutions obtained from the motion detection algorithm if the object has symmetry axes and occlusion is not considered. In this paper, we use these ambiguities for the determination of the symmetries of planar polygons.

Many methods have been proposed to determine the object orientation, such as those involving principal axes [3], reflection-symmetry axes [4], and universal

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principal axes [5]. However, these methods are not suitable when the shape is rotationally symmetries. Lin [6] proposed a method for the determination of shape orientations by the fold-invariant introducing the concepts of the fold-invariant centroid (FIC) and the fold-invariant weighted mean (FIRWM). In this method, the rotational symmetry of a shape is defined as the direction of the unique half-line which begins from the centroid and passes through FIC and FIRWM. The number of folds n of a given rotational symmetry shape can be determined by string matching technique [7]. Lin *et al* [8] proposed a method for the determination the number of folds based on a simple mathematical property. Recently, Lin [9] also proposed a modification of his previous method in which the matching procedure is discarded. Additionally, we can find other approaches such as the proposed by Yip *et al* [10], who use the Hough transform method to determine the rotational symmetry of planar shapes.

The motion analysis algorithm which we proposed in references [1] and [2] detects motion parameters for planar planar and spatial motions without any assumption for the point correspondences among image frames. Therefore, if an object is rotationally symmetrical, this algorithm yields ambiguities of solutions. Using this fundamental property of the motion analysis algorithm based on the random sampling and voting process, we construct a common framework for the detection of symmetry of both planar and spatial object. Our algorithm does not require the predetermination of centroid since our motion analysis algorithm does not require centroid. Never the less, our algorithm detects the centroid after detecting symmetry of an object for both planar and spatial objects. Furthermore, the algorithm detects both rotation symmetry and reflection symmetry for planar objects, simultaneously. Moreover, the algorithm estimates the centroid of polyhedrons from surface information using symmetry. These properties are the significant advantages of our new algorithm for the detection of symmetry. In this paper, we assume that point set $\mathbf{V}$ is a polygonal set on a plane. Furthermore, we assume that points on the boundaries of these point sets are extracted and sampled with an appropriate method.

2 Symmetry and Transformation

Symmetry indicates the congruence of an object under transformations. Here we assume Euclidean transformations. The presence of an axis of symmetry in an object is considered as the existence of rotational or reflectional symmetry. In this paper, we only consider rotation symmetry and the number of axes of rotation symmetry. The order of rotation symmetry is called the folding number for planar figures.

For a set of vectors $\mathbf{V}$ in Euclidean space, we define $\boldsymbol{F}(\mathbf{V}) = \{\boldsymbol{y}|\boldsymbol{y} = \boldsymbol{F}\boldsymbol{x}, \forall \boldsymbol{x} \in \boldsymbol{V}\}$ for linear transformation $\boldsymbol{F}$. Let $\boldsymbol{U}$ be a rotation matrix such that $\boldsymbol{U}^m = \boldsymbol{I}$, for an appropriate positive integer m such that $m \geq 2$. Setting $\boldsymbol{g}$ to be the centroid of $\mathbf{V}$, we define a set of vectors $\mathbf{V}_{\boldsymbol{g}} = \{\bar{\boldsymbol{x}}|\boldsymbol{x} \in \mathbf{V}\}$ for $\bar{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{g}$. Setting $\boldsymbol{U}^k(\mathbf{V}) = \{\bar{\boldsymbol{y}}|\bar{\boldsymbol{y}} = \boldsymbol{U}^k\bar{\boldsymbol{x}}, \bar{\boldsymbol{x}} \in \mathbf{V}\}$, if $\mathbf{V} = \boldsymbol{U}^k(\mathbf{V})$, then $\mathbf{V}$ has a symmetry axis with respect to $\boldsymbol{g}$. Then, $\mathbf{V}$ is n-rotation symmetrical, and we call n the folding number of an object $\mathbf{V}$ with respect to the axis of rotation $\boldsymbol{k}$.

Let the rotation matrix on two-dimensional Euclidean plane to be

$$\boldsymbol{U} = \begin{pmatrix} \cos\frac{2\pi}{n}, & -\sin\frac{2\pi}{n} \\ \sin\frac{2\pi}{n}, & \cos\frac{2\pi}{n} \end{pmatrix} \tag{1}$$

for a positive integer n. Setting $\mathbf{S}^k = \{\bar{\boldsymbol{y}} | \bar{\boldsymbol{y}} = \boldsymbol{U}^k \bar{\boldsymbol{x}}\}$ for $k = 1, 2, \cdots, n$, if $\mathbf{S}^k = \mathbf{V}_{\boldsymbol{g}}$, then $\mathbf{V}$ is rotation symmetrical and the folding number of $\mathbf{V}$ is n.

Furthermore, for orthogonal matrix $\boldsymbol{M}$ and rotation matrix $\boldsymbol{R}$ such that

$$\boldsymbol{M} = \begin{pmatrix} 1, & 0 \\ 0, & -1 \end{pmatrix}, \ \boldsymbol{R}(\theta) = \begin{pmatrix} \cos\theta, & -\sin\theta \\ \sin\theta, & \cos\theta \end{pmatrix}, \tag{2}$$

setting a set $\mathbf{M}$ to be the result of the application of matrix $\boldsymbol{M}$ and rotation $\boldsymbol{R}(\theta)$ to $\mathbf{V}_{\boldsymbol{g}}$, that is, $\mathbf{M} = \{\bar{\boldsymbol{y}} | \bar{\boldsymbol{y}} = \boldsymbol{R}(\theta)\boldsymbol{M}\bar{\boldsymbol{x}}\}$, if the equality $\mathbf{M} = \mathbf{V}_{\boldsymbol{g}}$ is satisfied, then point set $\mathbf{V}$ is reflectionally symmetrical with respect to a line such that $(\boldsymbol{\omega}^\perp)^\top(\boldsymbol{x} - \boldsymbol{g}) = 0$, for $\boldsymbol{\omega}^\perp = (-\sin\frac{\theta}{2}, \cos\frac{\theta}{2})^\top$, which passes through $\boldsymbol{g}$ and parallel to vector $\boldsymbol{\omega} = (\cos\frac{\theta}{2}, \sin\frac{\theta}{2})^\top$.

3 Motion Analysis by Sampling and Voting

Setting $\{\boldsymbol{x}_\alpha\}_{\alpha=1}^n$ and $\{\boldsymbol{y}_\beta\}_{\beta=1}^n$ to be points on an object on Euclidean plane $\mathbf{R}^2$, which are observed at times t_1 and t_2, respectively, such that $t_1 < t_2$, we assume that for arbitrary pairs of α and β, $\boldsymbol{x}_\beta$ and $\boldsymbol{y}_\beta$ are connected by Euclidean motion If we do not know the point correspondences between frames, the motion parameters $\boldsymbol{R}$ and $\boldsymbol{t}$, which are a rotation matrix and a translation vector, respectively, are obtained as the solution which minimizes the criterion

$$E = \min_{\sigma \boldsymbol{R}, \boldsymbol{t}} |\boldsymbol{y}_{\sigma(\alpha)} - (\boldsymbol{R}\boldsymbol{x}_\alpha + \boldsymbol{t})|, \tag{3}$$

where $\sigma(\alpha)$ is a permutation over $1 \leq \alpha \leq n$ and $\boldsymbol{R}$ and $\boldsymbol{t}$ are a rotation matrix and a translation vector, respectively.

Rotation symmetry and the folding number of an object define point correspondences with respect to the rotation axes. Therefore, if we detect point correspondences, we can determine symmetry and the folding number with respect to an axis of rotation of an object. Since the random sampling and voting method for the motion analysis detect both motion parameters and point correspondences concurrently, we apply this method for the detection of symmetry of an object.

Assuming that $\boldsymbol{t} = 0$, motion analysis algorithms detect the rotation of an object, if an object is rotationally symmetrical, the result of a rotation which is determined by the folding number n derives the same shape. Therefore, if we apply the motion analysis algorithms to an object which is rotationally symmetrical, we can obtain all rotation matrices $\boldsymbol{U}^k$, that is,

$$\min_{\sigma, \boldsymbol{R}, \boldsymbol{t}} |\boldsymbol{y}_{\sigma(\alpha)} - (\boldsymbol{R}\boldsymbol{x}_\alpha + \boldsymbol{t})| = \min_{\sigma, \boldsymbol{R}, \boldsymbol{t}} |\boldsymbol{y}_{\sigma(i)} - (\boldsymbol{U}^k\boldsymbol{R}\boldsymbol{x}_\alpha + \boldsymbol{t})|, \ \ k = 1, 2, \cdots, n. \tag{4}$$

Therefore, our algorithm detects all $\boldsymbol{U}^k\boldsymbol{R}$ for $k = 1, 2, \cdots, n$, where $\boldsymbol{R}$ is the true rotation matrix. However, all estimated matrices have the same translation vector. Using this ambiguity, we detect symmetry axes and the folding numbers of spatial objects. After a sufficient number of iterations, this algorithm detects all rotation matrices $\{\boldsymbol{U}^k\}_{k=1}^n$ such that $\boldsymbol{U}^n(\mathbf{V}) = \mathbf{V}$.

Figure 1 shows two hexagons which are connected by motion equation. However, if we do not know any point correspondences as the solution of motion analysis algorithm we have

$$\boldsymbol{R} = \boldsymbol{R}(2\pi k/6)\boldsymbol{R}(2\pi/12),\ k = 1, 2, \cdots, 6. \tag{5}$$

Therefore, there exist an ambiguities of solutions for rotationally symmetrical objects. Here, the number 6 of $\boldsymbol{R}(2\pi/6)$ is the folding number of the hexagon. Therefore, the ambiguity of the solutions derives the folding number of an planar object. Furthermore, if a planar shape is reflectionally symmetrical, two shapes $\mathbf{V}$ and $\boldsymbol{M}(\mathbf{V})$ are the same shape. Therefore points on $\mathbf{V}$ satisfy the relation

$$\min_{\sigma,\boldsymbol{R},\boldsymbol{t}} |\boldsymbol{y}_{\sigma(\alpha)} - (\boldsymbol{U}^k\boldsymbol{M}\boldsymbol{x}_\alpha + \boldsymbol{t})| = 0,\ k = 1, 2, \cdots, n. \tag{6}$$

for an appropriate rotation matrix $\boldsymbol{U}$. This geometric property leads to the conclusion that by applying our motion analysis algorithms, it is possible to detect the reflection axes if we detect all peaks in the accumulator space.

Let $\{\boldsymbol{x}_i = (x_i, y_i)^\top\}_{i=1}^m$ be a set of points which is moving on a plane. For $\boldsymbol{x_i}$, and $\boldsymbol{x_k}$,

$$\boldsymbol{x}_{ik} = (x_i - x_k, y_i - y_k)^\top,\ \ \boldsymbol{x}_{ik}^\perp = (y_i - y_k, -(x_i - x_k))^\top, \tag{7}$$

which is orthogonal to $\boldsymbol{x}_{ik}$, are invariant under a Euclidean motion of a set of points. Furthermore,

$$\boldsymbol{u}_{ik} = \frac{\boldsymbol{x}_{ik}}{|\boldsymbol{x}_{ik}|},\quad \boldsymbol{u}_{ik}^\perp = \frac{\boldsymbol{x}_{ik}^\perp}{|\boldsymbol{x}_{ik}^\perp|} \tag{8}$$

form an orthogonal basis. Moreover, in the same way we define $\boldsymbol{y}_{ik}$, $\boldsymbol{y}_{ik}^\perp$, $\boldsymbol{v}_{ik}$, $\boldsymbol{v}_{ik}^\perp$ for the second image frame. These two sets of orthogonal base derive the orthogonal expansions of vectors

$$\boldsymbol{x}_{jk} = \alpha_{ik}^j, \boldsymbol{u}_{ik} + \alpha_{ik}^{j\perp}\boldsymbol{u}_{ik}^\perp,\ \ \boldsymbol{y}_{jk} = \beta_{ik}^j\boldsymbol{v}_{ik} + \beta_{ik}^{k\perp}\boldsymbol{v}_{ik}^\perp, \tag{9}$$

where

$$\alpha_{ik}^j = \boldsymbol{x}_{jk}^\top\boldsymbol{u}_{ik},\ \alpha_{ik}^{j\perp} = \boldsymbol{x}_{jk}^\top\boldsymbol{u}_{ik}^\perp,\ \beta_{ik}^j = \boldsymbol{y}_{jk}^\top\boldsymbol{v}_{ik},\ \beta_{ik}^{j\perp} = \boldsymbol{y}_{jk}^\top\boldsymbol{v}_{ik}^\perp. \tag{10}$$

Setting $\boldsymbol{x}_\gamma$ and $\boldsymbol{y}_\gamma$, $\gamma = i, j, k$, to be noncollinear triplets of points in the frames 1 and 2, respectively, if these triplet of pairs are coresponding points which satisfy the same motion equation, the equations

$$\alpha_{ik}^j = \beta_{ik}^j,\quad \alpha_{ik}^{j\perp} = \beta_{ik}^{j\perp},\quad |\boldsymbol{x}_{p\,q}| = |\boldsymbol{y}_{p\,q}|, \tag{11}$$

where $p \neq q$ for $p,\, q \in \{i, j, k\}$, hold. Moreover, using these corresponding triplets, we have

$$\boldsymbol{R} = \left(\boldsymbol{v}_{ik}, \boldsymbol{v}_{ik}^{\perp}\right)\left(\boldsymbol{u}_{ik}, \boldsymbol{u}_{ik}^{\perp}\right)^{\top}. \tag{12}$$

Although random sampling finds pairs of triplets of vectors which hold relations in eq. (11), these relations do not conclude that a pair of triplet vectors are connected by a Euclidean transform. Therefore, we solve this problem using the voting procedure, since voting procedure collects many evidences and inference the solution.

If a pair of triplets $\{\boldsymbol{x}_i, \boldsymbol{x}_j, \boldsymbol{x}_k\}$ and $\{\boldsymbol{y}_i, \boldsymbol{y}_j, \boldsymbol{y}_k\}$, is connected by the relation

$$\boldsymbol{y}_{ij} = \boldsymbol{R}(\theta)\boldsymbol{M}\boldsymbol{x}_{ij}, \tag{13}$$

the coefficients defined eq. (10) satisfy the relations

$$\alpha_{ik}^{j} = \beta_{ik}^{j}, \quad \alpha_{ik}^{j\perp} = -\beta_{ik}^{j\perp}, \quad |\boldsymbol{x}_{p\,q}| = |\boldsymbol{y}_{p\,q}|, \tag{14}$$

where $p \neq q$, for $p,\, q \in \{i, j, k\}$. Therefore, applying motion motion analysis algorithm to a pair of vector triplets $\{\boldsymbol{x}_i, \boldsymbol{x}_j, \boldsymbol{x}_k\}$ and $\{\boldsymbol{M}\boldsymbol{y}_i, \boldsymbol{M}\boldsymbol{y}_j, \boldsymbol{M}\boldsymbol{y}_k\}$, we can detect the direction of the line of reflection as $\boldsymbol{\omega}$.

As shown in Figure 2, our planar algorithm first determines a triangle in each frame, and second computes the rotation parameter between these two triangles. In Figure 2, the algorithm determines the angle between line segments $\overline{13}$ and $\overline{1'3'}$, if we assume vertices i and i' correspond for $i = 1, 2, 3$. Although there are many possibility for the combinations of triangles and point correspondences, for hexagons in Figure 2, the number of vertex-combinations which determine the rotation angle

$$\theta = \frac{2\pi}{6}k + \frac{2\pi}{12}, k = 1, 2, \cdots, 6, \tag{15}$$

larger than the number of combinations of edges which determine the angle such that

$$\theta \neq \frac{2\pi}{6}k + \frac{2\pi}{12}, k = 1, 2, \cdots, 6, \tag{16}$$

for 2×2 rotation matrix $\boldsymbol{R}(\theta)$. If we apply motion analysis to an object by sampling a pairs of simplexes, then $\theta = 2\pi k/6$ for $k = 1, 2, \cdots, 6$. Then we can determine the folding number of planar polygonal objects.

4 Detection of Symmetry

In the followings, we assume that the boundary points on an object are extracted using an appropriate methods. As we mentioned in the previous section, our motion analysis algorithm based on the random sampling and voting process

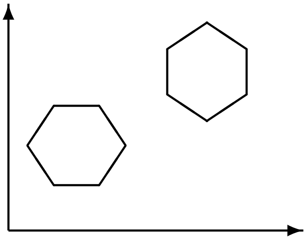

Fig. 1. Tow hexagons related by an Euclidan motion.

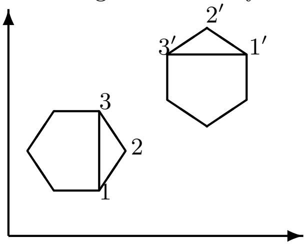

Fig. 2. Trainagls and point correspondences on two hexagons.

detects all rotation matrices $\boldsymbol{U}^k$, for $k = 1, 2, \cdots, n$ if the folding number of an object is n, that is, an object $\mathbf{V}$ is n-rotationally symmetry.

The parameter θ_c, such that $-\pi < \theta_c < \pi$, which is computed by

$$\boldsymbol{R}(\theta_c) = (\boldsymbol{v}_{12}, \boldsymbol{v}_{12}^{\perp})(\boldsymbol{u}_{12}, \boldsymbol{u}_{12}^{\perp})^{\top} \tag{17}$$

is an estimation of the rotation angle for a planar object. Therefore, our accumulator space for the detection of rotation angles is a finite linear array equivalent to the interval $[-\pi, \pi]$, which is equivalent to $[0, 2\pi]$. Setting θ_{min} to be the smallest positive values which possess a peak in this accumulator space, the folding number is $2\pi/\theta_{min}$. Therefore, the detection of the folding number is achieved by detecting peaks in accumulator space. We detect the folding number applying the cepstrum analysis in the accumulator space for the folding-number detection, since the peak distribution along cells in the accumulator space is considered as a periodic function [11]. This property derives the following algorithm for the detection of folding numbers of planar shapes.

Algorithm for the detection of the folding number

1 Compute the DFT (the Discrete Fourier Transform) of $scor(\theta)$ using the FFT (the Fast Fourier Transform).
2 Compute the power spectrum of $scor(\theta)$ from the result of Step 1, and set it as $S(n)$.
3 Compute the logarithm of the power spectrum of $S(\theta)$ from the result of Step 2, and set it $C(n)$.
4 Detect the positive peak of $C(n)$ for the smallest n and set it as n^*.
5 Adopt n^* as the folding number.

Once detecting rotation matrix $\boldsymbol{R}(\frac{2\pi}{n})$, it is possible to determine the correspondences of points on the boundary of an object for a motion

$$\boldsymbol{y} = \boldsymbol{R}(2\pi/n)\boldsymbol{x} + \boldsymbol{t}. \tag{18}$$

Therefore, we can determine $\boldsymbol{t}$ and the centroid of an object by

$$\boldsymbol{g} = (\boldsymbol{I} - \boldsymbol{R}(2\pi/n))^{-1}\boldsymbol{t}. \tag{19}$$

Therefore, for the detection of the folding number , we need not to prepare vectors whose centriod is the origin of the coordinate system.

As mentioned in the previous section, if we apply motion motion analysis algorithm to $\{\boldsymbol{x}_i, \boldsymbol{x}_j, \boldsymbol{x}_k\}$ and $\{\boldsymbol{M}\boldsymbol{y}_i, \boldsymbol{M}\boldsymbol{y}_j, \boldsymbol{M}\boldsymbol{y}_k\}$ we can detect the direction of the line of reflection as $\boldsymbol{\omega}_r$. For the minimum values in the accumulator space which is computed by

$$\boldsymbol{R}(\theta_r) = (\boldsymbol{v}_{12}, \boldsymbol{v}_{12}^{\perp})\left\{\boldsymbol{M}(\boldsymbol{u}_{12}, \boldsymbol{u}_{12}^{\perp})\right\}^{\top}, \tag{20}$$

a reflectionally symmetrical object satisfies the relation $\mathbf{V}\boldsymbol{g} = \boldsymbol{R}(\theta_r)(\boldsymbol{M}(\mathbf{V}\boldsymbol{g}))$. Therefore, the reflection axis of an object is line $(\boldsymbol{\omega}_r^{\perp})^{\top}(\boldsymbol{x} - \boldsymbol{g}) = 0$, for vector $\boldsymbol{\omega}_r^{\perp} = (-\sin\frac{\theta_r}{2}, \cos\frac{\theta_r}{2})^{\top}$. This line passes through the centroid of $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ and perpendiculat to vector $\boldsymbol{x}_i - \boldsymbol{y}_i$.

For a simple closed polygonal curve on a plane, the folding number is equivalent to or less than the number of vertices of this polygon, since matrix $\boldsymbol{R}(\frac{2\pi}{n})$ transform vertices to vertices. Therefore, the folding number satisfies the relation $2 \leq n \leq v(n)$, where $v(n)$ is the number of vertices of a polygon. Setting $\bar{n}$ to be the folding number detected by $\boldsymbol{R}(\theta_r)$, we call $\bar{n}$ the number of folding for reflection since number $\bar{n}$ determines the number of reflection axes. If an object is reflectionally symmetrical and rotationally asymmetrical $\bar{n} = 1$ and $n = 1$. Therefore, using $\bar{n}$ and n, we have the following classification criterions for planar objects.

1. If $n = 1$ or $n > v(n)$, and $\bar{n} \neq 1$ then an object is asymmetrical.
2. If $n = 1$ or $n > v(n)$, and $\bar{n} = 1$ then an object is reflectionally symmetrical.
3. If $2 \leq v(n) \leq n$, and $\bar{n} \neq 1$ then an object is rotationally symmetrical and is not reflectionally symmetrical.
4. If $2 \leq v(n) \leq n$, and $\bar{n} = n$ then an object is rotationally and reflectionally symmetrical.

5 Computational Results

For a planar object $\mathbf{V}$, let k and m be the folding number and the total number of sample points. We assume that m points are separated into k independent subset whose number of sample points is n. Therefore, k, m and n satisfy the relation $kn = m$. For this object, the total number of combinations for the selection of a pair of triplets is ${}_mC_3 \times_m C_3$. The number of combinations for the selection of a subset is ${}_kC_2$, and the number of combinations for the selection of triplets of sample points in each subset is ${}_mC_k$.

For a pair of triplets of sample-points, if thay are congruent triangles, the rotation angles between them is one of $\frac{2\pi}{q}$ for $q = 1, 2, \cdots, k$. This means that the number of combination of pair of triplets of sample- points which is combined by angles which are determined by the folding number is the same as the folding number. Therefore, setting p to be the probability that a selected triples is not collinear, the probability for the correct selection of noncollinear triplets is

$$P = p\frac{1}{k}\frac{{}_kC_2 \times_n C_3}{{}_mC_3 \times_m C_3}. \tag{21}$$

In the following, we deal with the case that $O(m) = O(n)$ and $O(k) = 10$ Therefore, assuming that $k \ll m$, we have $P^{-1} = O(m^3)$. Next, setting N and e to be the total number of iteration and the threshold for the detection of peaks, respectively, P, N, and e which is $e = O(m^s)$ satisfy the relation $NP \geq e$. Therefore, we have $N \geq O(m^{3+s})$. In the following examples, we set $N = 10^7$ for $m \cong 50$ and $s = 4$.

During the computation of θ_c and θ_r, we also compute centroid $\boldsymbol{g}$ and line $(\boldsymbol{\omega}_r^\perp)^\top(\boldsymbol{x} - \boldsymbol{g}) = 0$, simultaneously. Furthermore, using the imaging plane on which the original object is expressed as the accumulator space, we vote 1 to vector $\boldsymbol{g}$ and the line for the estimation of the centroid and the reflection axes. Using this procedure, we can superimpose the centroid and the reflection axes on the original object.

From figure 3, we show the numerical results of symmetry analysis for planar objects. Subfigures, (a), (b), (c), (d), (e), (f), (g), (h), and (i) show input data, the peaks in the accumulator space for the detection of rotation, the cepstrum of the peak-distributions for the rotation, the peaks in the accumulator space for the detection of reflection, the cepstrum of the peak-distributions for the reflection, the peaks for the detection of the centroids, the peaks for the detection of the reflection axes, the centroids, the reflection axes, and the input objects, and the reconstructed objects by folding the input using folding numbers for rotating inputs. In these figures, small square blocks in figure (a) show sample points. For the performance analysis of the algorithm, we assumed that the boundaries of objects are extracted using appropriate algorithm and that finite numbers of samples are extracted from boundaries. Here, the total number of sample points on the boundaty is 54, and the times for iteration is 10^7.

A point set $\mathbf{V}$, which is n-rotationally symmetrical, satisfies the relation

$$\mathbf{V} = \bigcup_{\alpha=1}^{n} \left(\boldsymbol{U}^\alpha(\mathbf{V}_{\boldsymbol{g}} \oplus \{\boldsymbol{g}\})\right), \tag{22}$$

where $\oplus$ expresses the Minkowski addition of two sets of points. This expression of the rotationally symmetrical object implies that if the algorithm is stable against occlusions, for partially occluded objects, we can reconstruct the complete object by first rotating the object around the estimated centroid by $2\pi k/n$

$k = 1, 2, \cdots, n$ and second superimposing all of them. In figure (i) shows reconstructed object using this property of the rotationally symmetrical object. This process is possible because our algorithm is stable against occlusions.

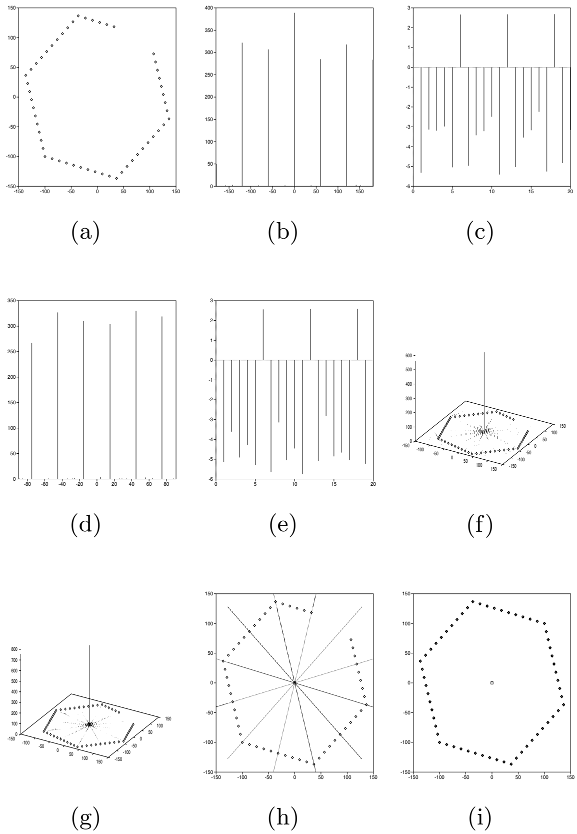

Fig. 3. Numerical examples

6 Conclusions

In this paper, we developed a randomized algorithm for the detection of rotational symmetry in planar polygons without assuming the predetermination of the centroids of the objects. Our algorithm is simple because we converted the matching problem for the detection of symmetry to peak detection in a voting space. This result showed that the voting process is a suitable approach to simplify matching problems. The numerical stability of the random sampling and voting process for motion analysis is disucussed in our previous papers [1,2,11]. We also determied the size of cells in the accumulator spaces for the detection of motion parameters using this analysis.

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