Maximum Weight Triangulation and Graph Drawing*

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Introduction

Triangulation of a set of points is a fundamental structure in computational geometry. According to the authors' best knowledge, there is not much research done on *maximum weight triangulation*, $MaxWT$. From the theoretical viewpoint, MaxWT and its counterpart, the minimum weight triangulation, attract equally interest. The graph drawings as MaxWT are investigated.

• The weight of a triangulation $T(P)$ is given by

$$\omega(T(P)) = \sum_{\overline{p_i p_j} \epsilon T(P)} \omega(\overline{p_i p_j}),$$

where $\omega(\overline{p_i p_j})$ is the Euclidean length of line segment $\overline{p_i p_j}$.

• A *maximum weight triangulation* of P ($MaxWT(P)$) is defined as for all possible $T(P)$, $\omega(MaxWT(P)) = max\{\omega(T(P))\}$.

Main Results

(A) O(n^2) time algorithm for MaxWT of an inscribed n-gon.
(B) O(n) time algorithm for MaxWT of a regular n-gon.
(C) O(n^2) time 0.5-approximation algorithm for MaxWT of a general convex n-gon.
(D) Linear-time algorithm for maximum drawing of Caterpillar graphs.
(E) Forbidden (non-maximum weight drawable) graphs on any convex point set.

(A) Inscribed Polygon Case

FACT 1: If P is a convex polygon, then each interior angle of any fly triangle of the $MaxWT(P)$ must be no less than $\frac{\pi}{4}$.

FACT 2: If P is a convex polygon, then no interior angle of any fly triangle of $MaxWT(P)$ is larger than $\frac{\pi}{2}$.

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FACT 3: If P is an inscribed polygon. Then $MaxWT(P)$ cannot contain any fly triangle. That is, the internal edges of $MaxWT(P)$ form a tree.

Recurrence Formula

• $W_{i,j}$——The weight of convex subpolygon of the inscribed polygon P with vertics $(i, i+1, ..., j)$ for $i, j \epsilon [0, n]$.

$$W_{i,j} = \begin{cases} 0 & \text{if } j = (i+1)_{modulo.n}; \\ max\{W_{i,j-1}, W_{i+1,j}\} + \omega(\overline{ij}) & \text{otherwise} \end{cases}$$

• O(n^2) time algorithm.

(B) Regular Polygon Case

• Any inner-spanning tree of a regular n-gon P is maximum and it together with the boundary edges of P form a $MaxWT(P)$.

• An inner-spanning tree of a regular n-gon P can be found in linear time.

(C) A 0.5-Approximation Algorithm for MaxWT(P)

• Let $\triangle abc$ be a fly triangle of $MaxWT(P)$. The removal of a fly triangle $\triangle abc$ will divide P into three components, each associates with an edge of $\triangle abc$. $\triangle abc$ is called an *ear-fly triangle* if at most one of its three components contains other fly triangles.

• An (n^2) time approximate algorithm which guarantees $\frac{\omega(ApT(P))}{\omega(MaxWT(P))} \geq \frac{1}{2}$.

(D) Maximum Weight Drawing of Caterpillar Graph

• *caterpillar* is a tree such that all internal nodes connect to at most 2 non-leaf nodes.

• O(n) time algorithm.

(E) Forbidden Graphs for Maximum Weight Drawing

• A graph G is *outerplanar* if it has a planar embedding such that all its nodes lie on a single face; an outerplanar graph is *maximal* if no edge can be added to the planar embedding without crossing.

• If $G(V, E)$ is a maximal outerplanar graph containing a simple cycle C with four nonconsecutive nodes which form two triangles sharing a common edge, then G cannot have a maximum weight drawing.

Concluding Remarks

• Does MaxWT(P) can be found in o(n^3) time?

• Does every maxmal planar graph admit a maximum weight drawing?