4 Number Counts and the Background Radiation

At last, we can get down to the business of working out the source counts and background radiation from a cosmological distribution of discrete sources or from the diffuse intergalactic medium.

4.1 Number Counts of Discrete Sources

Consider first of all a uniform distribution of a single class of source with space density N_0 and luminosity $L(\nu)$. We can consider $L(\nu)$ to be the spectral energy distribution of this class of source. From the considerations of sections 2.5.4 and 2.5.5, the flux density of such a source at frequency ν_0 is

$$S(\nu_0) = \frac{L(\nu_1)}{4\pi D^2(1+z)} \tag{2.47}$$

where $\nu_1 = \nu_0(1+z)$ and the number of sources per steradian in the interval of comoving coordinate distance $\mathrm{d}r$ is

$$\mathrm{d}N = N_0 D^2 \mathrm{d}r \tag{2.53}$$

The *integral source counts*, $N(\geq S)$, are defined to be the numbers of sources per steradian with flux densities greater than or equal to some limiting value S at frequency ν_0. For a complete sample of sources with lower limiting flux density S, a class of sources with identical spectral energy distributions $L(\nu)$ can be observed to a limiting comoving distance coordinate r_{max} which is the solution of the expression (2.47). Then, if the sources are uniformly distributed in space, that is, the sources have a constant *comoving space density* N_0, the integral number counts are

$$N(\geq S) = \int_0^{r_{\mathrm{max}}} N_0 D^2 \mathrm{d}r \tag{4.1}$$

For any given cosmological model, $R(t)$ is defined and so there is a known relation between r and redshift z. The only point to beware of is that the above formalism assumes that S and $\mathrm{d}N(r)$ are monotonic functions of redshift and a little care has to be taken if they are not (see, for example, Chapter 11).

In the limit of small redshifts, $z \to 0$, $D \to r$ and so

$$N(\geq S) = N_0 \int_0^{r_{\mathrm{max}}} r^2 \mathrm{d}r = \frac{1}{3} N_0 r_{\mathrm{max}}^3$$

Since, in this limit, $S = L/4\pi r_{\mathrm{max}}^2$, it follows that

$$N(\geq S) = \frac{1}{3} N_0 \left(\frac{L}{4\pi}\right)^{3/2} S^{-3/2} \tag{4.2}$$

Now, we need to integrate over all luminosity classes of source. This is found by integrating over the *luminosity function* of the sources which is defined such that $N_0(L)\mathrm{d}L$ is the comoving space density of sources with luminosities in the luminosity range L to $L+\mathrm{d}L$. We therefore have to integrate (4.1) over all luminosities, that is, we need to take the double integral

$$N(\geq S) = \int_L \int_0^{r_{\max}(L)} N_0(L) D^2 \mathrm{d}r \mathrm{d}L \tag{4.3}$$

In the small redshift limit, $z \to 0$, we find

$$N(\geq S) = \frac{1}{3} S^{-3/2} \int_L N_0(L) \left(\frac{L}{4\pi}\right)^{3/2} \mathrm{d}L \tag{4.4}$$

This is the famous 'three-halves' power law, $N(\geq S) \propto S^{-3/2}$ and is often known as the *Euclidean source counts.* Notice that the Euclidean source counts are independent of the form of the luminosity function of the sources since it only appears inside the integral in (4.4). This result can be written in alternative ways. Optical counts of galaxies are normally expressed in terms of the number of galaxies brighter than a given limiting magnitude m. Since $m = \text{constant} - 2.5 \log_{10} S$, it follows that

$$N(\leq m) \propto 10^{0.6m} \tag{4.5}$$

As described by Dr. Sandage, Hubble realised that this relation provided a test of the homogeneity of the distribution of galaxies in the Universe (see Section 2.2.1 and Fig. 2.4).

One of the problems with the expression (4.3) is the fact that, in counting *all* the sources brighter than each limiting flux density S, the numbers counted are not independent at different flux densities and, to avoid this, it is preferable to work in terms of *differential source counts* which are defined to be the number of sources in the flux density interval S to $S + \mathrm{d}S$. For the Euclidean source counts, this can be found by differentiating the expression (4.4). Then,

$$\mathrm{d}N_0(S) \propto S^{-5/2} \mathrm{d}S \tag{4.6}$$

The use of integral versus differential source counts became an issue when they extended to low flux densities at which the counts began to converge (see Jauncey 1975).

The various forms of the Euclidean source count can be regarded as *null hypotheses* since such counts are expected at small redshifts in all cosmological models. In consequence, it is often helpful to normalise the observed or theoretical counts to the Euclidean prediction, that is, if $\Delta N(S)$ is the observed or predicted numbers of sources in $\mathrm{d}S$, it is often convenient to work in terms of *normalised, differential source counts* which are defined as $\Delta N(S)/\Delta N_0(S)$ where $\Delta N_0(S)$ is the Euclidean prediction.

In real world models, the source counts deviate from the Euclidean prediction because D and r are not linear functions of redshift z. As an illustration, it is useful to work out the normalised differential source count for a single luminosity class of source with a power-law spectral index $S(\nu) \propto \nu^{-\alpha}$. For the standard Friedman world models, it is can be shown that

$$\frac{\Delta N(S)}{\Delta N_0(S)} = \frac{2c(1+z)^{-\frac{3}{2}(1+\alpha)}}{H_0(\Omega_0 z+1)^{1/2}\left[D(1+\alpha)+2(1+z)\frac{\mathrm{d}D}{\mathrm{d}z}\right]} \tag{4.7}$$

In the case of the critical model, $\Omega_0 = 1$, this expression reduces to

$$\frac{\Delta N(S)}{\Delta N_0(S)} = \frac{(1+z)^{-\frac{3}{2}(1+\alpha)}}{[(1+\alpha)(1+z)^{1/2}-\alpha]} \tag{4.8}$$

The expression (4.7) is plotted in Fig. 4.1 for world models having $\Omega_0 = 0$, 1 and 2 assuming a spectral index $\alpha = 0.75$. The redshifts at which the number counts have different values of $\Delta N/\Delta N_0$ are indicated on each curve. These curves make the important point that, for a uniform distribution of sources, the slope of the differential counts departs from the Euclidean value at remarkably small redshifts. For example, for the critical model $\Omega_0 = 1$, the slope of the differential source counts at a redshift of 0.5 is -2.08, corresponding to an integral source count with slope -1.08. Even at quite small redshifts, say $0.2 - 0.3$, the departures from the -1.5 law are significant. Notice that this calculation has been undertaken for a simple power-law spectrum but the result is generally true unless the source spectra are highly inverted, an example of which is discussed in Chapter 11.

The above calculation has been carried out for a single luminosity class of source and, to find more realistic predictions, these counts have to be convolved with the luminosity function of the sources. It is apparent, however, that, in general, any population of sources which extends to redshifts $z \sim 1$ is expected to have a source count which has slope significantly less steep than -1.5.

Finally, we have to take account of the effects of cosmological evolution of the population of sources. It is simplest to regard the evolutionary changes as changes to the luminosity function of the sources with cosmic epoch. These can be written as a modification of the *comoving space density* of sources as a function of cosmic epoch. Formally, we can write

$$N(L,z) = N_0(L) f(L,z,\alpha,\text{type},\ldots) \tag{4.9}$$

where the *evolution function* f takes the constant value 1 if the source distribution does not change with cosmic epoch. The evolution function can be made as complicated as is necessary. For example, to work out the optical counts of galaxies, separate luminosity functions have to be evaluated for each class of galaxy and then, in place of simple power law spectra, the typical spectrum of each class of galaxy has to be used. This is often achieved

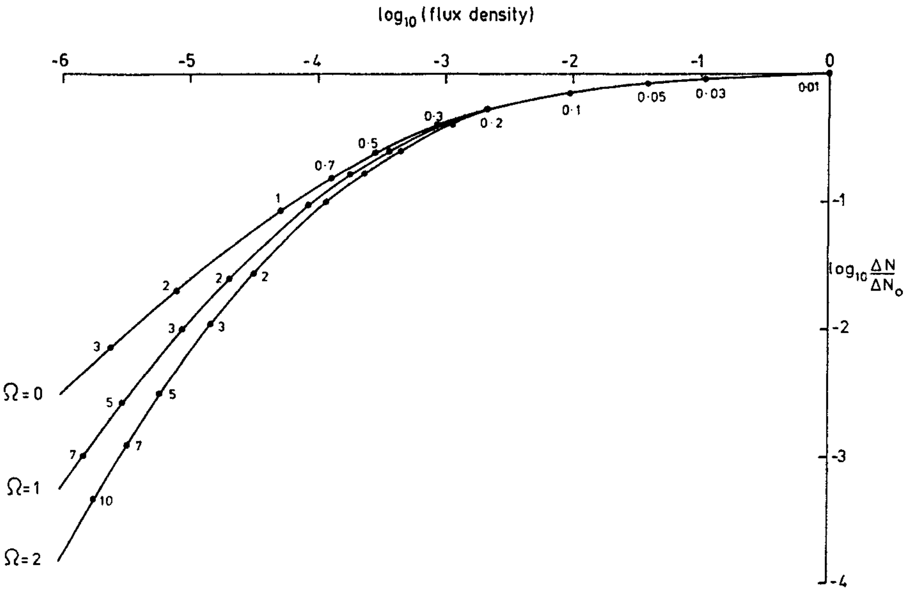


Fig. 4.1 The predicted normalised differential counts of sources for a single luminosity class of source with a power-law spectrum $S \propto \nu^{-0.75}$. The redshifts at which the sources are observed are indicated on each of the curves.

by working out the *K-corrections* for the different classes of galaxy which contribute to the counts (see the expression 2.50). In addition, evolutionary changes of the spectral energy distributions of galaxies with cosmic epoch can be built into the above evolution function. Formally, the integral source counts for any form of evolution function can be written

$$N(\geq S_0) = \int_L \int_{S \geq S_0} N_0(L) f(L, z, \alpha, \text{type}, \ldots) D^2 \mathrm{d}r \mathrm{d}L \qquad (4.10)$$

This formalism is exactly the same as that recommended by Drs. Kron and Sandage. An obvious concern is the great variety in the properties of galaxies. In the extreme case in which one wished to take account of the individual properties of *every* galaxy, it is possible to work out the space density corresponding to each of them by selecting a complete flux density limited sample of objects and then evaluating the volume of space $V_{\max}$ within which each object could have been observed and still remain within the sample. The local space density of such a object is $V_{\max}^{-1}$ and the spectral energy distribution of the galaxy can be used to work out K-corrections.

Notice also that, if it is known that cosmological evolutionary effects are influencing the counts, these effects have to be taken into account when evaluating the local space density of sources from magnitude or flux density limited samples.

4.2 The Background Radiation

At last, we are able to work out the background intensity due to a cosmological distribution of discrete sources. For the sake of illustration, we will assume that the sources have power-law spectra of the form $S \propto \nu^{-\alpha}$ and then the flux density-luminosity relation becomes

$$S(\nu_0) = \frac{L(\nu_0)}{4\pi D^2(1+z)^{1+\alpha}} \tag{4.11}$$

Now, the numbers of sources per steradian in an increment of comoving coordinate distance $\mathrm{d}r$ in the case of a uniform distribution of sources is

$$\mathrm{d}N = N_0 D^2 \mathrm{d}r \tag{4.12}$$

Therefore, background intensity $I(\nu_0)$ due to this uniform distribution of sources is

$$\begin{aligned} I(\nu_0) &= \int S(\nu_0)\mathrm{d}N \\ &= \int_0^\infty \frac{L(\nu_0)}{4\pi D^2(1+z)^{1+\alpha}} N_0 D^2 \mathrm{d}r \\ &= \frac{L(\nu_0)N_0}{4\pi} \int_0^\infty (1+z)^{-(1+\alpha)} \mathrm{d}r \end{aligned} \tag{4.13}$$

For the Friedman world models, we find from the expression (3.18)

$$\mathrm{d}r = \frac{c\mathrm{d}z}{H_0(\Omega_0 z + 1)^{1/2}(1+z)} \tag{3.18}$$

and so we obtain the important result

$$I(\nu_0) = \frac{c}{H_0}\frac{L(\nu_0)N_0}{4\pi} \int_0^\infty \frac{\mathrm{d}z}{(\Omega_0 z + 1)^{1/2}(1+z)^{2+\alpha}} \tag{4.14}$$

This result can be compared with the Newtonian version of the same calculation which, from the small redshift limit, $z \to 0$, $r = cz/H_0$, becomes

$$I(\nu_0) = \frac{L(\nu_0)N_0}{4\pi} \int_0^\infty \mathrm{d}r \tag{4.15}$$

This is the naive version of what is referred to as *Olbers' paradox*, namely, that, in a isotropic, infinite, stationary Euclidean Universe, the background radiation diverges. This naive sum has not taken account of the finite sizes of the sources and eventually we have to take into account of the overlapping of their images. Nor does the argument take account of thermodynamics since in such an infinite static Universe eventually all the matter comes into thermodynamic equilibrium at the same temperature.

Unlike the Olbers' sum, it is apparent that the integral over redshift in (4.14) converges provided $\alpha > -1.5$. Any realistic spectrum must eventually turn over at a high enough frequency and so a finite integral is always obtained. It is useful to work out the background intensity for two typical world models. For the cases $\Omega_0 = 0$ and $\Omega_0 = 1$, we find

$$\begin{aligned} \Omega_0 = 0 \qquad & I(\nu_0) = \frac{c}{(1+\alpha)H_0} \frac{L(\nu_0)N_0}{4\pi} \\ \Omega_0 = 1 \qquad & I(\nu_0) = \frac{c}{(1.5+\alpha)H_0} \frac{L(\nu_0)N_0}{4\pi} \end{aligned} \tag{4.16}$$

Thus, it can be seen that, for typical values of α, to order of magnitude, the background intensity is just that originating within a typical cosmological distance (c/H_0), that is,

$$I(\nu_0) \sim \frac{c}{H_0} \frac{L(\nu_0)N_0}{4\pi} \tag{4.17}$$

A combination of factors leads to the convergence of the integral for the background intensity. Inspection of the integral (4.13) shows that part of the convergence is due the redshift factor $(1+z)^{-(1+\alpha)}$ which is associated with the redshifting of the emitted spectrum of the sources. The second is the dependence of r upon redshift z. This relation is only linear at small redshifts, $z \ll 1$. In the case of the critical model, $r = (2c/H_0)[1-(1+z)^{-1/2}]$ which converges to the value $2c/H_0$ as $z \to \infty$. This convergence is associated with the fact that the Friedman models of the Universe have a finite age and consequently there is a finite maximum distance from which electromagnetic waves can reach the Earth.

Let us look in a little more detail at the origin of the background radiation in the uniform models. We take as an example the critical model $\Omega_0 = 1$ with $\alpha = 1$. Then, the background intensity out to redshift z is

$$I(\nu_0) = \frac{2c}{5H_0} L(\nu_0)N_0 \left[1-(1+z)^{-5/2}\right] \tag{4.18}$$

From this it is easy to show that half of the background intensity originates at redshifts $z \leq 0.31$. A similar calculation for the case of the empty world model, $\Omega_0 = 0$, shows that half the intensity comes from redshifts less than 0.42. So much for the cosmological significance of the background radiation! I am sure it must come as a disappointment to the organisers of a school entitled 'The Deep Universe' that the background radiation mostly originates at small redshifts. What is more to the point is the fact that, because half of the background is expected to originate at redshifts less than about 0.5, the principal contributors to the background radiation are not difficult to identify nowadays, provided their positions are accurately known. If the main sources of the background are associated with galaxies, there should be no difficulty in discovering the principal contributors to the background radiation, provided the sources are uniformly distributed in space. This statement is not correct if the properties of the sources have evolved strongly with cosmic epoch and we take up that topic now.

4.3 The Effects of Evolution – The Case of the Radio Background Emission

Just as in the case of the source counts, we can write the expression for intensity of the background radiation if the properties of the sources evolve with cosmic epoch in terms of the evolution function. By the same type of analysis as in the case of the source counts (expression 4.10), we find

$$I(\nu_0) = \frac{c}{H_0}\frac{L(\nu_0)N_0}{4\pi}\int_0^\infty \frac{f(L,z,\text{type},\ldots)\mathrm{d}z}{(\Omega_0 z+1)^{1/2}(1+z)^{2+\alpha}} \tag{4.19}$$

The simplest example of the effects of evolution upon the background radiation and the source counts is the radio background emission. This is a well known story (see, for example, Wall 1990, Peacock 1993). The counts of radio sources and optically selected quasars show an excess of faint sources as compared with the expectations of uniform world models (Fig. 4.2). At high flux densities, the source count is roughly $N(\geq S) \propto S^{-1.8}$ which represents an excess of faint sources even compared with the Euclidean prediction. It is now known that the radio galaxies and quasars which display the excess of faint sources have redshifts about 1 and so the difference between the uniform models and the observations is very significant indeed. This is illustrated by the comparison of the expected counts of sources with the observations in Fig. 4.2. Notice that, at the faintest flux densities, there is a flattening of the source counts and these are probably low luminosity radio sources associated with starburst galaxies (Rowan-Robinson et al 1993).

The interpretation of radio source counts has been the subject of many studies, the most complete analysis of a very large body of high quality data being due to Dunlop and Peacock (1990). They employed the free-form modelling techniques developed by Peacock and Gull (1981) and Peacock (1985) to determine best-fits models for the evolution of the radio source populations. An example of the results of the modelling procedures is shown in Fig. 4.3 which shows how the *comoving* luminosity functions of the radio galaxies and quasars change with redshift. It appears as though the luminosity function is shifted to higher luminosities out to redshifts of about 2 and then begins to shift back again. According to these analyses, it looks as though there is a 'cut-off' to the evolving source distribution beyond redshifts of 2 but this is not well established because of the statistical difficulty of finding sufficient sources at large redshifts (Peacock 1993). The volume elements decrease rapidly with increasing redshift and so it becomes progressively more and more difficult to find large redshift sources even if there is no cut-off.

In his most recent analyses of the observations, Peacock has concluded that the simplest description of the forms of evolution necessary to account

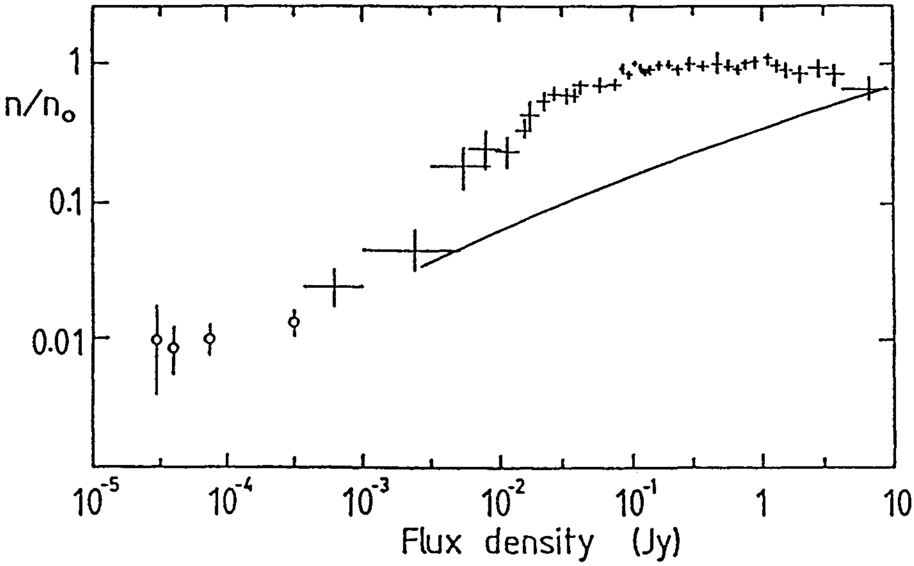


Fig. 4.2 Comparison of the counts of radio sources at 5 GHz with the expectations of uniform world models based upon the radio luminosity function of radio sources determined from complete high flux density samples (After Wall 1990).

for both the radio source counts and the optical counts of quasars (Boyle 1993) have the same form. Both sets of data can be satisfactorily described by 'luminosity evolution' models in which it is assumed that the luminosities of the sources change in the following manner with redshift:

$$\begin{aligned} L(z) &= L_0(1+z)^3 \qquad 0 < z < 2 \\ L(z) &= 27L_0 \qquad z > 2 \end{aligned} \tag{4.20}$$

Notice that this represents very strong evolution of the source population between redshifts 0 and 2.

Let us illustrate by a simple calculation how such evolution can strongly influence the intensity of the background radiation. It is a simple calculation to work out the integrated background emission from a population of sources which locally has luminosity L_0 and space density N_0 with and without this form of evolution. From the expressions (4.13) and (3.18), the integrated background intensity is

$$I(\nu_0) = \frac{c}{H_0}\frac{N_0}{4\pi}\int_0^\infty \frac{L(z)\mathrm{d}z}{(1+z)^{7/2}} \tag{4.21}$$

where we have assumed that the spectral index of the sources α is 1 and that $\Omega_0 = 1$.

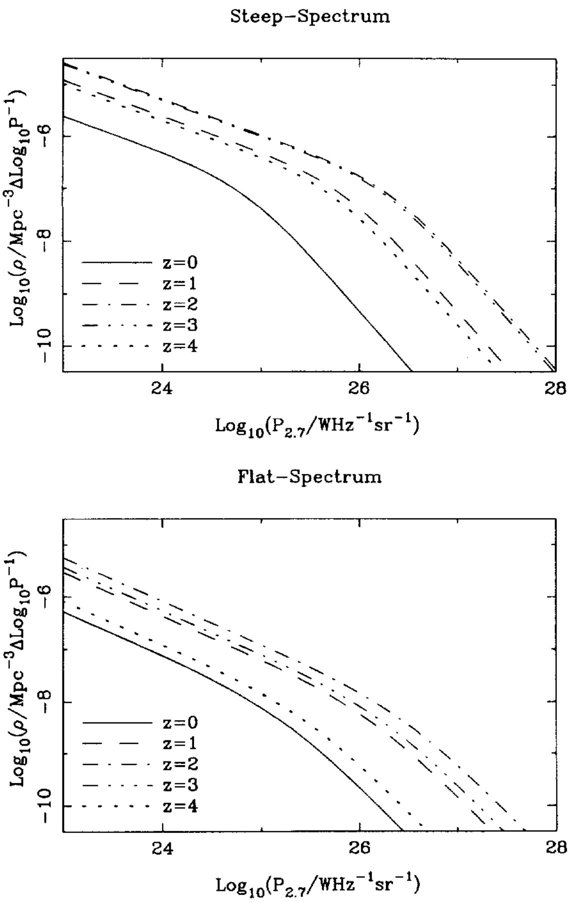


Fig. 4.3 Illustrating the evolution of the luminosity function of extragalactic radio sources with steep and flat radio spectra with redshift (or with cosmic epoch). Note that these luminosity functions are presented by per unit *comoving volume* so that the changes in the functions are over and above the changes in number density associated with the expansion of the Universe (Dunlop and Peacock 1990)

In the *no evolution* case, $L(z) = L_0$ and the background intensity is

$$I(\nu_0) = \frac{2}{5}\frac{c}{H_0}\left(\frac{N_0 L_0}{4\pi}\right) \tag{4.22}$$

In contrast, in the *evolution* case, adopting the variation of luminosity with redshift given by the expressions (4.20), the background intensity is

$$I(\nu_0) = \frac{12\sqrt{3}}{5}\frac{c}{H_0}\left(\frac{N_0 L_0}{4\pi}\right) \tag{4.23}$$

Inclusion of the effects of cosmological evolution into the calculation results in a background $6\sqrt{3} \sim 10$ times greater than without evolution. This simple calculation is entirely consistent with the discussion presented in Section 1.4.1 on the extragalactic radio background emission in which it was stated that the background due to strong radio sources would amount to only $1-2$ K if there were no evolution but amounts to about $16-19$ K when the effects of evolution are taken into account.

Thus, in this case, the background emission *does* come from the 'deep Universe', the bulk of the background originating at redshifts of the order 2. Note, however, that this only occurs because of the *very* strong effects of cosmological evolution. Inspection of Fig. 4.3 shows that, although the luminosities of the sources only increase by about 27 between redshift 0 and 2, the comoving space density of high luminosity sources increases by about a factor of 1000. These calculations make the point that the evolution has to be very drastic, which it is for radio galaxies and quasars, to make a significant impact upon the intensity of the background emission due to discrete sources.

4.4 The Background Radiation and the Source Counts

Let us look at the relation between the observed source counts and the background radiation. The background radiation from a population of sources with differential source count $\mathrm{d}N \propto S^{-\beta}\mathrm{d}S$ is

$$I \propto \int_{S_{\min}}^{S_{\max}} S\mathrm{d}N \propto \int_{S_{\min}}^{S_{\max}} S^{-(\beta-1)}\mathrm{d}S = \frac{1}{2-\beta}\left[S^{(2-\beta)}\right]_{S_{\min}}^{S_{\max}} \tag{4.24}$$

Thus, there is a critical value $\beta = 2$ for the slope of the differential source counts. If the slope of the counts is steeper than $\beta = 2$, the background intensity $I_\nu \propto S_{\min}^{(2-\beta)}$. On the other hand, if the slope of the differential source counts is less than $\beta = 2$, the background intensity is proportional to $S_{\max}^{2-\beta}$. Thus, most of the background radiation originates from that region of the counts with slope $\beta = 2$.

Now, for a Euclidean population of sources $\beta = 2.5$. In real world models, the slope is 2.5 at small redshifts but decreases at larger redshifts as discussed in Section 4.1. From the considerations which led to the expression (4.8), we showed that the slope of the differential counts is about 2 by a redshift of 0.5, showing again that the bulk of the background emission originates from redshifts $z < 1$.

4.5 Fluctuations in the Background Radiation due to Discrete Sources

Another topic of interest is the amplitude of *fluctuations* in the background radiation due to discrete sources. This is a well-known problem, first solved for the more difficult case of observations made with a radio interferometer

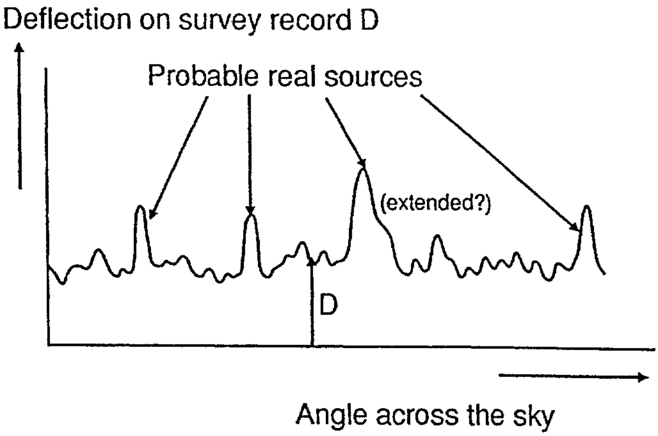
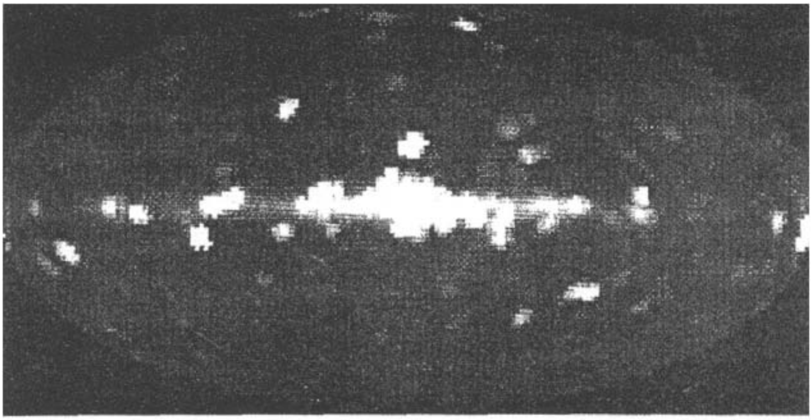


Fig. 4.4 Illustrating the fluctuations in the intensity of the background radiation due to the superposition of faint sources.

by Scheuer (1957). The problem may be stated as follows. Suppose the sky is observed with a telescope of beamwidth θ and the integral counts of sources are given by $N(\geq S) \propto S^{-\beta}$. Then, if the survey extends faint enough, eventually a flux density is reached at which there is one source per beam area and fainter, more numerous sources cannot be detected individually. In this circumstance, the noise level of the survey is due to the random superposition of faint sources within the beam of the telescope. The situation is illustrated schematically in Fig. 4.4. The problem of making observations of radio sources when the 'noise' is due to the presence of faint unresolved sources in the beam is often referred to as *confusion*. This problem afflicted the early radio surveys and is the source of fluctuations in the X-ray background emission when observed at low angular resolution (Fig. 4.5).

Scheuer (1957) provided the complete solution to the problem of determining the source counts in radio surveys which are confusion limited. In his analysis, he found the correct slope for the counts of radio sources from the early radio surveys, $N(\geq S) \propto S^{-1.8}$. This result was only in apparent contradiction with the counts of sources themselves which suggested a much steeper slope but which were very badly affected by confusion. Subsequent surveys carried out with radio telescopes with narrower beam patterns and consequently much less subject to the problems of confusion confirmed his result. I consider this to be a very important and original paper but one which is scarcely known. It was the first paper to find the correct result for the slope of the source counts at high flux densities. The steep slope showed that the counts were inconsistent with the Euclidean world model and with



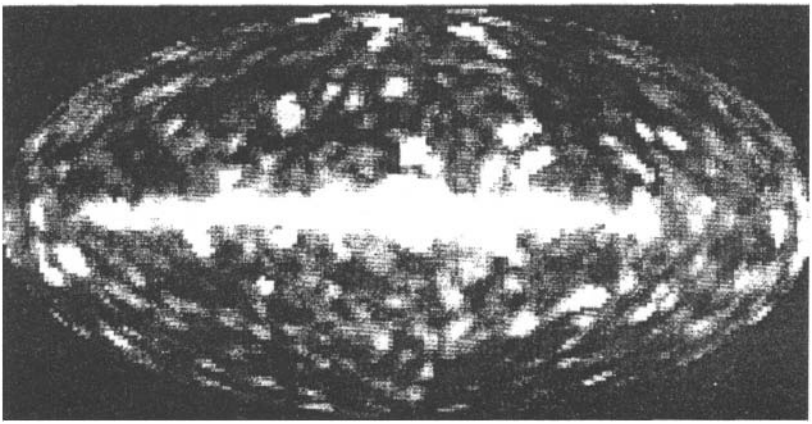


Fig. 4.5 (a) The X-ray map of the sky observed by the HEAO-1 A2 telescope at 3° resolution in Galactic coordinates. (b) A contrast enhanced image of the X-ray sky as observed by the HEAO-1 A2 experiment in the $2-10$ keV energy band, showing flucutations in the X-ray background intensity at high Galactic latitudes (Fabian and Barcons 1992).

uniform Friedman models. I should declare an interest in that Peter Scheuer was my Ph.D. supervisor.

The simplest presentation I know of the problem for a single beam telescope is also by Scheuer (1974) who went on to solve the easier problem for surveys of the sky made by early X-ray telescopes. His approach was to forget all about the detection of individual sources but to deal directly with the amplitude of the intensity fluctuations on the map. If the map is sampled at the information rate for observations with a telescope of beam-width θ, that is, at twice per beam-width, a probability distribution is found of the amplitude of the deflection D of the record from some zero level. The term 'deflection'

D was used since the original radio astronomy surveys were recorded on strip charts and the deflections really were the deflections of the recording pen. Typical probability distributions, known as $P(D)$, are illustrated in Fig. 4.6. It can be seen that the distributions are non-Gaussian but, according to the central limit theorem, the noise level is given by the standard deviation of the probability distribution $P(D)$. The very large deflections are identified as discrete sources and the $P(D)$ distribution tends asymptotically to the differential source count $P(D) \propto D^{-\beta}$. A criterion for their identification as real sources has to be established. Normally some criterion such as 5 times the standard deviation of the confusion noise or one source per twenty or thirty beam-areas is selected. The problem is that, in a confusion limited survey, the flux densities of sources are systematically overestimated because of the random presence of faint sources in each beam. It was this effect which led to the overestimation of the flux densities of faint radio sources in the early radio source catalogues and hence to an excessively steep source count (see Scheuer 1990 for the history of these problems).

By carrying out a statistical analysis of the expected function $P(D)$ for sources selected randomly from a differential source count of the form $\mathrm{d}N(S) \propto S^{-\beta}$, Scheuer showed how the slope of the source counts could be found. Fig. 4.6 shows the normalised $P(D)$ distributions for different value of β. It can be seen that the shape of the $P(D)$ distribution provides a means of determining the form of the source counts. To order of magnitude, the most probable value of $P(D)$ corresponds to the flux density of those sources which have surface density of roughly one source per beam-area — Scheuer (1974) gives a simple statistical argument to show why this should be so. At higher flux densities, the sources are too rare to make a large contribution to the beam-to-beam variation in background signal. At lower flux densities, many faint sources add up statistically and so contribute to the background intensity but the fluctuations are dominated by the brightest sources present in each beam. At roughly one source per beam area, the fluctuations can be thought of as arising from whether or not the source is by chance within the beam. Thus, whereas the reliable detection of individual sources can only be made to about 5 or 6 times the confusion noise level, statistical information concerning the source counts can be obtained to about one source per beam area.

Scheuer's analysis was entirely analytic but it is nowadays much simpler to use Monte Carlo methods to work out the functions $P(D)$ for the assumed form of source count. The first of these statistical studies using Monte Carlo modelling procedures was carried out by Hewish (1961) in his analysis of the original records of the 4C survey. He found the first evidence for the convergence of the radio source counts at low flux densities at a frequency of 178 MHz. These procedures have been used to determine the source counts to the very faintest flux densities in the radio waveband (see, for example,

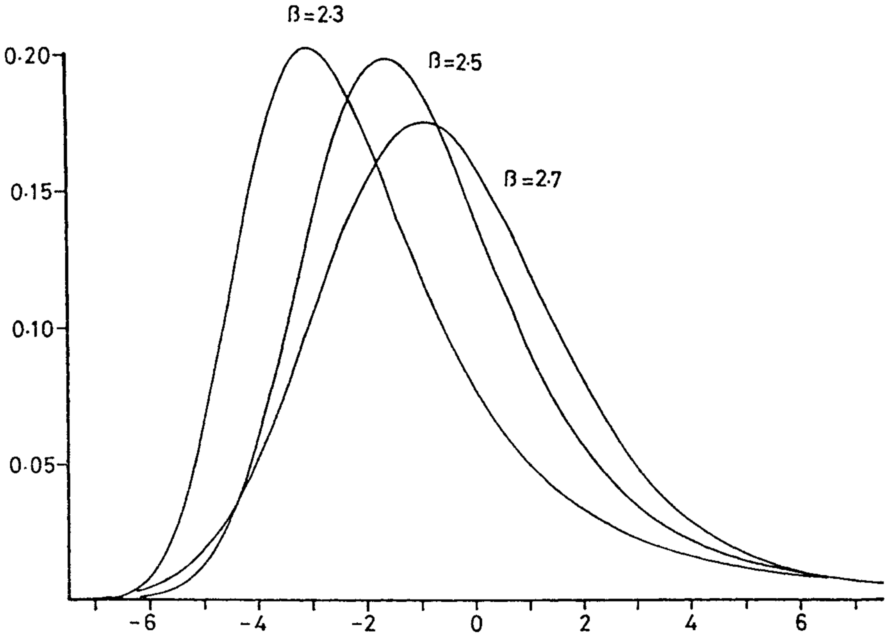


Fig. 4.6 Examples of the theoretical $P(D)$ distributions for observations made with a single-beam telescope for different assumed slopes of the differential source counts $dN(S) \propto S^{-\beta}dS$. The zero point of the abscissa is the mean amplitude $\bar{D}$ and the areas under the probability distributions have been normalised to unity. The distributions tend asymptotically to $dN \propto D^{-\beta}\, dD$ at large deflections D. (From Scheuer 1974).

Fomalont et al 1988) and in the deep ROSAT surveys by Hasinger et al (1993) (see Section 5.1).

Another approach to the interpretation of fluctuations in the background radiation is to look for a signal in the correlation function of the fluctuations. This has been carried out succesfully in the optical waveband by Shectman (1974) who found a clear signature corresponding to the two-point correlation function for galaxies. The observed fluctuation spectrum is in quite remarkable agreement with the standard correlation function found in studies of large samples of galaxies. The most recent application of this approach is the heroic work of Martin and Bowyer (1989). In a short rocket flight, they were able to make a survey of a small region of sky and found a significant correlated signal among the spatial distribution of the counts. With a number of reasonable assumptions, they were able to show that they had detected the ultraviolet emission from galaxies (see Section 1.8).

Similar analyses have been carried out for the fluctuations in the X-ray background as observed by the HEAO 1 A-2 experiment (Persic et al 1989). The binning of the background counts was in pixels 3° in size. No significant signal was found in the two-dimensional autocorrelation function on all angular scales greater than 3°. This constrains the clustering of the sources which might make up the background radiation to scales less than about 50 Mpc. They showed that the observed upper limits to the clustering would be consistent with the observed cross-correlation function for galaxies

and clusters of galaxies. A similar analysis has been carried out by Barcons and Fabian (1989) who have studied fluctations in five deep *Einstein* IPC fields. A signal is observed on the scale of 5 arcmin but it is not certain that this is of astrophysical origin. A maximum comoving clustering scale of $10h^{-1}$ Mpc is found. This rules out models in which the background is associated with clusters of galaxies at low redshifts, $z \leq 1$.

It is probably true to say that the epoch of studies of fluctuations in the background radiation due to discrete sources comes to an end as soon as source counts extend to such faint flux densities that the bulk of the background emission can be accounted for. This is now the case for the radio waveband and for the X-ray background at 1 keV.

References – Chapter 4

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