

STRUCTURAL EQUIVALENCE OF CONTEXT-FREE GRAMMAR FORMS IS DECIDABLE

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ABSTRACT:

Two grammars G_1 and G_2 are structural equivalent if the corresponding parenthesized grammars generate the same language. This definition transfers to grammar forms in a natural way. It is shown that structural equivalence of context-free grammar forms is decidable.

KEY WORDS AND PHRASES:

Contextfree grammars, parenthesized grammars, grammar forms, grammar homomorphisms, reduction, grammatical language families.

CR Categories: 4.12, 5.23, 5.25

0. INTRODUCTION.

In [2] Cremers and Ginsburg define grammar families by the concept of grammar forms. Intuitively a grammar form F defines a grammar family such that any member of this family is "similar" to a fixed prototype grammar G_F . Two natural equivalence problems arise by these definitions, first the problem whether two grammars G_{F_1} and G_{F_2} generate the same grammar family (strong equivalence) and second the problem whether two grammars G_{F_1} and G_{F_2} generate the same language family (weak equivalence).

In [2] it is shown that the first problem is decidable. The second one is still open. We want to show that another equivalence problem which is more general than strong equivalence and less general than weak equivalence is decidable.

Following [5] we introduce the structural equivalence of two grammars G_1 and G_2 . G_1 and G_2 are structural equivalent if the associated parenthesized grammars generate the same language. This definition transfers in a natural way to grammar forms. It is known ([5], [6]) that structural equivalence of context-free grammars is decidable. We generalize this result to structural equivalence of context-free grammar forms.

1. BASIC NOTATIONS.

We adopt the usual notations of phrase structure grammars $G=(V, \Sigma, P, \sigma)$, context-free grammars, etc. ([3], [6]) with the (trivial) change $\sigma \subseteq V - \Sigma$. To avoid trivialities we assume (without loss of generality), that all grammars in consideration are reduced ([3], [6]) and contain no unnecessary symbols.

By $\xrightarrow{*}$ (\vdash) we denote the notion of a (direct) derivation.

The set $\mathcal{L}(G) = \{w \in \Sigma^* \mid \exists \sigma_1 \in \sigma : \sigma \xrightarrow{*} w\}$

is the generated language and

$$D(G) = \{\sigma_1 \xrightarrow{*} w \mid w \in \Sigma^* \text{ \& } \sigma_1 \in \sigma\}$$

is the set of derivations.

A grammar morphism $\phi : G_1 \rightarrow G_2$ is a monoid homomorphism

$\phi : V_1^* \rightarrow V_2^*$ with

- (i) $\phi(\sigma_1) \in \sigma_2$
- (ii) $\phi(V_1 - \Sigma_1) \in (V_2 - \Sigma_2)$
- (iii) $\phi(\Sigma_1) \in \Sigma_2^*$
- (iv) $\phi(P_1) = \{(\phi(p), \phi(q)) \mid (p, q) \in P_1\} \in P_2$.

We single out various classes of grammar morphisms.

If ϕ is lengthpreserving, we call ϕ a fine morphism, if $\phi(\xi) = \xi$ ($\xi \in V_1 - \Sigma_1$) ϕ is external and if $\phi(t) = t$ ($t \in \Sigma_1$) ϕ is internal.

In a natural way ϕ induces $\hat{\phi} : D(G_1) \rightarrow D(G_2)$. ([4])

We call ϕ closed if $\hat{\phi}$ is surjective. For closed morphisms ϕ we get: $\phi(\mathcal{L}(G_1)) = \mathcal{L}(G_2)$. A closed, internal morphism ϕ is called a reduction. ϕ is an isomorphism if ϕ and ϕ^{-1} are morphisms.

To any grammar G we associate a grammar family Γ_G to be the collection of all grammars G' such that there exists a diagram

$$G \xleftarrow{\phi_1} G'' \xrightarrow{\phi_2} G'$$

where ϕ_1 is fine and ϕ_2 is external and closed. By $\lambda(G)$ we denote the collection of all languages $\mathcal{L}(G')$ with $G' \in \Gamma_G$.

The existence of such a diagram is by [10] equivalent to the notion of grammar forms [2] and therefore $\lambda(G)$ is exactly the grammatical language family ([2]) associated to this grammar form. We call G_1 and G_2 strong equivalent ($G_1 \sim G_2$) if $\Gamma_{G_1} = \Gamma_{G_2}$, and weak equivalent ($G_1 \sim G_2$) if $\lambda(G_1) = \lambda(G_2)$.

Now, consider a universal bracket pair $\{(\cdot), \cdot\}$.

If G is a grammar, then the associated parenthesized grammar $G^{(1)}$ is obtained by

$$G^{(1)} = (V \cup \{(\cdot), \cdot\}, \Sigma \cup \{(\cdot), \cdot\}, \{(p, (q)) \mid (p, q) \in P\}, \sigma) \quad ([5], [6]).$$

Denote by $\lambda^{(1)}(G)$ the collection of all languages $\mathcal{L}(G'^{(1)})$ with $G' \in \Gamma_G$.

G_1 and G_2 are called structural equivalent ($G_1 \equiv G_2$) if $\lambda^{(1)}(G_1) = \lambda^{(1)}(G_2)$.

It is easy to check:

$$G_1 \sim G_2 \implies G_1 \equiv G_2 \implies G_1 \sim G_2.$$

Example: Consider G_1 and G_2 specified by

$$P_1 : \sigma \rightarrow \sigma_1 + a\sigma + \varepsilon$$

$$\sigma_1 \rightarrow \sigma_1\sigma_1 + \sigma_1 + \varepsilon$$

$$P_2 : \sigma \rightarrow \sigma_1 + a\sigma + \varepsilon$$

It is easy to check that $\lambda(G_1) = \mathcal{L}_{\text{reg}} = \lambda(G_2)$, $\lambda^{(1)}(G_2) \subseteq \mathcal{L}_{\text{lin}}$ and $D_1 \in \lambda^{(1)}(G_1)$, where D_1 is the Dyck language over $\{(\cdot)\}$, hence $G_1 \sim G_2$ but $G_1 \not\equiv G_2$.

From now on, we assume without further mentioning all grammars to be context-free.

2. PRELIMINARY RESULTS.

In this section we derive some results on grammar morphisms and parenthesized grammars, which are necessary to prove our main result.

We state without proof.

Lemma 1: Let $\phi : G_1 \rightarrow G_2$ be a morphism

There exists a factorization $\phi = \phi_1 \circ \phi_2$ resp. $\phi_2 \circ \phi_1$ with ϕ_2 internal and ϕ_1 external. If ϕ is closed then ϕ_2 is closed.

Lemma 2: If $\phi_1 : G_0 \rightarrow G_1$ is closed and external and $\phi_2 : G_0 \rightarrow G_2$ is internal, then there exist $\psi_1 : G_2 \rightarrow G_3$ closed and external and $\psi_2 : G_1 \rightarrow G_3$ internal with $\psi_2\phi_1 = \psi_1\phi_2$. Moreover, if ϕ_1 is fine then ψ_1 is fine and if ϕ_2 is a reduction then ψ_2 is a reduction.

We now study the effect of parenthesizing. Obviously there is a canonical external and closed morphism $e : G^{(1)} \rightarrow G$ defined via bracket erasing.

Consider a morphism $\phi : G_1 \rightarrow G_2$, then ϕ induces a unique morphism $\phi^{(1)} : G_1^{(1)} \rightarrow G_2^{(1)}$ such that

$$\begin{array}{ccc} G_1^{(1)} & \xrightarrow{\phi^{(1)}} & G_2^{(1)} \\ e \downarrow & & \downarrow e \\ G_1 & \xrightarrow{\phi} & G_2 \end{array}$$

is commutative. Observe that in a certain sense parenthesizing is a functor. Obviously properties of ϕ transfer to $\phi^{()}$, especially if ϕ is a reduction then $\phi^{()}$ is a reduction.

This yields

Fact: If $\phi : G_1 \rightarrow G_2$ is a reduction, then $\mathcal{L}(G_1^{()}) = \mathcal{L}(G_2^{()})$.

Conversely if $\phi : G_1^{()} \rightarrow G_2^{()}$ is a "bracket" morphism ($\phi((\)) = (\&\phi(\)) =)$), then ϕ induces $\hat{\phi} : G_1 \rightarrow G_2$ with $\phi = \hat{\phi}^{()}$. Again properties of ϕ transfer to $\hat{\phi}$.

Call a grammar G invertible if $(p, q) \ \& \ (p, q') \in P \implies q = q'$.

Lemma 3: To any grammar G there exists a diagram

$$G \xleftarrow{\phi_1} G_0 \xrightarrow{\phi_2} G^{\text{inv}}$$

where ϕ_1 and ϕ_2 are reductions and G^{inv} is invertible.

Proof: We analyze the construction of G^{inv} given in [5], [6].

Step 1: " G^{inv} ":

Consider $Z' = 2^{V-\Sigma} - \{\emptyset\}$. Define a substitution $\mu : (Z' \cup \Sigma)^* \rightarrow 2^{V^*}$ by

$$\mu(z_1) = \begin{cases} z_1 & , z_1 \in Z' \\ \{z_1\} & , z_1 \in \Sigma. \end{cases}$$

to any $q \in (Z' \cup \Sigma)^*$ consider

$$S_1 = \{\xi \mid q' \text{ with } \xi \rightarrow q' \in P \ \& \ q' \in \mu(q)\}.$$

Now define G^{inv} as follows

$$G^{\text{inv}} = (V', \Sigma, P', \sigma')$$

- (i) $V' - \Sigma = \{S_q \mid S_q \neq \emptyset, q = t_0 s_{q_1} t_1, \dots, s_{q_i} t_i$
with $S_{q_\lambda} \neq \emptyset, t_\lambda \in \Sigma^*, q, q_\lambda \in (Z' \cup \Sigma)^*, 0 \leq \lambda \leq i\}$
- (ii) $P' = \{(S_q, q) \mid q \in (Z' \cup \Sigma)^* \ \& \ S_q \in V' - \Sigma\}$
- (iii) $\sigma' = \{S_q \mid S_q \wedge \sigma \neq \emptyset\}$

It is easy to check, that G^{inv} is invertible.

Step 2: "G₀, φ₁, φ₂"

Define $G_0 = (V_0, \Sigma, P_0, \sigma_0)$ by

- (i) $V_0 - \Sigma = \{(S_q, \xi) \mid \xi \in S_q, q \in V' - \Sigma\}$
(ii) $\sigma_0 = \{(S_q, \sigma_1) \mid \sigma_1 \in \sigma \cap S_q\}$
(iii) $P_0 = \{(S_q, \xi) \rightarrow t_0(S_{q_1}, \eta_1), \dots, (S_{q_i}, \eta_i)t_i \mid$
 $\alpha) i \geq 0, t_\lambda \in \Sigma^* (0 \leq \lambda \leq i)$
 $\beta) \xi \in S_q, \eta_\lambda \in S_{q_\lambda} (1 \leq \lambda \leq i)$
 $\gamma) q = t_0 S_{q_1}, \dots, S_{q_i} t_i$
 $\delta) \xi \rightarrow t_0 \eta_1, \dots, \eta_i t_i \in P\}$.

Induce ϕ_1 and ϕ_2 by $\phi_1((S_q, \xi)) = \xi$ and $\phi_2((S_q, \xi)) = S_q$.

Step 3: "φ₁ is a reduction"

We observe first for all $\sigma_1 \in \sigma$

$$\phi_1^{-1}(\sigma_1) = \{(S_q, \sigma_1) \mid \sigma_1 \in S_q\} \subseteq \sigma_0.$$

Now consider $(S_q, \xi) \rightarrow t_0(S_{q_1}, \eta_1) \dots (S_{q_i}, \eta_i)t_i \in P_0$ and

$q' = t'_0(S_{q'_1}, \eta'_1) \dots (S_{q'_j}, \eta'_j)t_j$ such that

$$t_0 \eta_1 \dots \eta_i t_i = \phi_1(q') = t'_0 \eta'_1 \dots \eta'_j t'_j.$$

We conclude $j = i, t_0 = t'_0, \dots, t_i = t'_i, \eta_1 = \eta'_1, \dots, \eta_i = \eta'_i$.

On the other hand, by $\xi \rightarrow t_0 \eta_1 \dots \eta_i t_i \in P$ we get $\xi \in S_q$, hence

$$(S_q, \xi) \rightarrow t_0(S_{q_1}, \eta_1) \dots (S_{q_i}, \eta_i)t_i \in P_0.$$

This proves, that ϕ_1 is coperfect in the sense of [11]. Hence ϕ_1 is a reduction ([11]).

Step 4: "φ₂ is a reduction"

Again, we observe the following fact.

If $S_q \in \sigma'$, then there exists $\sigma_1 \in \sigma \cap S_q$, hence $(S_q, \sigma_1) \in \sigma_0$.

This proves $\phi_2(\sigma_0) = \sigma'$.

Consider

$$(S_q, \xi) \rightarrow t_0(S_{q_1}, \eta_1) \dots (S_{q_i}, \eta_i) t_i \in P_0$$

and

$$(S_q, \xi') \text{ with } \phi_2(S_q, \xi') = \phi_2((S_q, \xi)).$$

We get $S_q = S_{q_1}$. Since $\xi' \in S_q$ and $q = t_0 S_{q_1} \dots S_{q_i} t_i$ there exist $\eta'_1 \dots \eta'_i$ with $\xi' \rightarrow t_0 \eta'_1 \dots \eta'_i t_i \in P$ and $\eta'_\lambda \in S_{q_\lambda}$ ($1 \leq \lambda \leq i$).

We conclude

$$(S_q, \xi') \rightarrow t_0(S_{q_1}, \eta'_1) \dots (S_{q_i}, \eta'_i) t_i \in P_0$$

and

$$\phi_2(t_0(S_{q_1}, \eta'_1) \dots (S_{q_i}, \eta'_i) t_i) = \phi_2(t_0(S_{q_1}, \eta_1) \dots (S_{q_i}, \eta_i) t_i).$$

This proves that ϕ_2 is perfect in the sense of [11]. Hence ϕ_2 is a reduction ([11]).

We are now able to prove one of the key theorems.

Theorem 1: The following statements are equivalent for two grammars G_1 and G_2

(1) $\mathcal{L}(G_1^{(1)}) = \mathcal{L}(G_2^{(1)})$

(2) There exists a diagram

$$G_1 \xleftarrow{\phi_1} G_0 \xrightarrow{\phi_2} G_2$$

where ϕ_1 and ϕ_2 are reductions.

Proof: (2) \implies (1) with the above fact.

(1) \implies (2). By lemma 3 there exist two diagrams

$$G_i \xleftarrow{\psi_1^i} G_i' \xrightarrow{\psi_2^i} G_i^{\text{inv}} \quad (i = 1, 2)$$

with ψ_1^i and ψ_2^i reductions ($i=1,2$).

Parenthezising leads to diagrams

$$G_i^{(1)} \xleftarrow{\hat{\psi}_1^i} G_i'^{(1)} \xrightarrow{\hat{\psi}_2^i} G_i^{\text{inv}(1)} \quad (i = 1, 2)$$

where $\hat{\psi}_1^i$ and $\hat{\psi}_2^i$ are reductions and $G_i^{\text{inv}(1)}$ is invertible ($i = 1, 2$). Since

$$\mathcal{L}(G_1^{\text{inv}(1)}) = \mathcal{L}(G_1^{(1)}) = \mathcal{L}(G_2^{(1)}) = \mathcal{L}(G_2^{\text{inv}(1)})$$

we obtain from [5], that there exists an internal isomorphism between

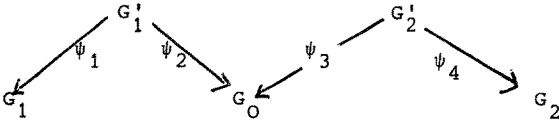
$G_1^{n()}$ and $G_2^{n()}$.

Hence we get a diagram

$$G_1^{()} \xleftarrow[\psi_1]{\hat{\psi}_1} G_1^{()} \xrightarrow[\psi_2]{\hat{\psi}_1} G_1^{n()} \xleftarrow[\psi_2]{\hat{\psi}_2} G_2^{()} \xrightarrow[\psi_1]{\hat{\psi}_2} G_2^{()},$$

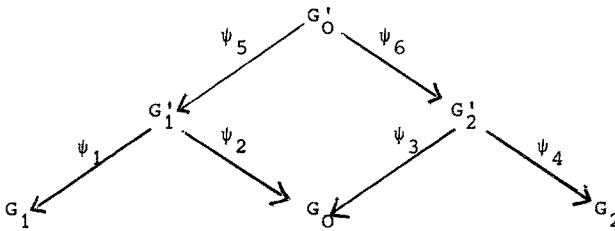
where all arrows are reductions.

Removing the brackets, we get a diagram



where all $\psi_i (i = 1, 2, 3, 4)$ are reductions.

By a theorem of [8], [9] we can fill in this diagram with reductions ψ_5 and ψ_6 .



Now, letting $\phi_1 = \psi_1\psi_5$ and $\phi_2 = \psi_4\psi_6$, we get the theorem.

3. THE MAIN THEOREM

We are now in the position to prove a characterization of the structural equivalence.

Theorem 2: The following statements are equivalent for two grammars G_1 and G_2 :

(1) $\lambda^{()} G_2 \sqsubseteq \lambda^{()} (G_1)$

(2) There exists a diagram

$$G_1 \xleftarrow{\phi_1} G'_1 \xrightarrow{\phi_2} G_0 \xleftarrow{\phi_3} G'_2 \xrightarrow{\phi_4} G_2$$

with

- (α) ϕ_1 is internal
- (β) ϕ_2 is a reduction
- (γ) ϕ_3 is fine, external and closed
- (δ) ϕ_4 is external and closed.

Proof: (1) \implies (2)

Consider G_2 . Since $\lambda(G_2^{(1)}) \in \lambda^{(1)}(G_1)$, there is a $\hat{G}_2 \in \Gamma_{G_1}$ with $\lambda(\hat{G}_2^{(1)}) = \lambda(G_2^{(1)})$.

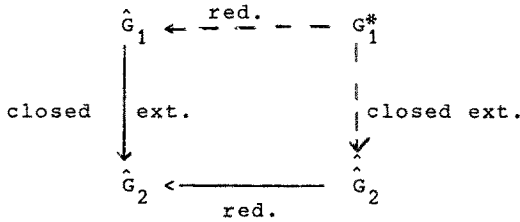
By theorem 1 and the definition of Γ_{G_1} we get a diagram

$$G_1 \xleftarrow{\text{fine}} \hat{G}_1 \xrightarrow[\text{external}]{\text{closed}} \hat{G}_2 \xleftarrow{\text{reduction}} \hat{\hat{G}}_2 \xrightarrow{\text{reduction}} G_2$$

consider the diagram

$$\hat{G}_1 \xrightarrow[\text{external}]{\text{closed}} \hat{G}_2 \xrightarrow{\text{reduction}} \hat{\hat{G}}_2.$$

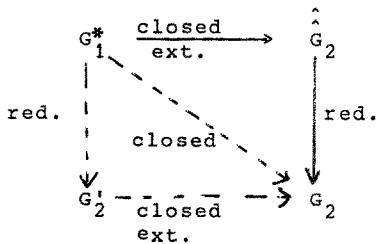
By a theorem of [10] we can fill in this diagram in the following way



Consider the diagram

$$G_1^* \xrightarrow[\text{ext.}]{\text{closed}} \hat{\hat{G}}_2 \xrightarrow{\text{red.}} G_2.$$

By lemma 1 we can fill in this diagram in the following way



Now, we reached the following situation:

$$G_1 \xleftarrow{\text{fine}} G_1^* \xrightarrow{\text{red.}} G_2' \xrightarrow[\text{closed}]{\text{ext.}} G_2$$

By lemma 1 again we find a diagram

$$G_1 \xleftarrow[\text{closed, fine}]{\text{ext.}} G_1^* \xleftarrow[\text{fine}]{\text{internal}} G_1$$

Consider the diagram

$$G_1' \xleftarrow[\text{fine}]{\text{ext.}} G_1^* \xrightarrow{\text{red.}} G_2'$$

By lemma 2 we can fill in this diagram:

$$\begin{array}{ccc} G_1' & \xleftarrow[\text{closed, fine}]{\text{ext.}} & G_1^* \\ \text{red.} \downarrow & & \downarrow \text{red.} \\ G_0 & \xleftarrow[\text{closed, fine}]{\text{ext.}} & G_2' \end{array}$$

But this proves the if-part of theorem 2.

(2) \implies (1) Consider an arbitrary diagram

$$G_2 \xleftarrow{\text{fine}} G_3' \xrightarrow[\text{closed}]{\text{ext.}} G_3$$

By a theorem of [10] we get a diagram

$$G_0 \xleftarrow{\text{fine}} G_2' \xleftarrow{\text{fine}} G_3^* \xrightarrow[\text{closed}]{\text{ext.}} G_3$$

On the other hand by a theorem of [8], [9] we get a diagram

$$G_1' \xleftarrow{\text{fine}} G_3^{**} \xrightarrow{\text{red.}} G_3^*$$

In summary we obtain

$$G_1 \xleftarrow{\text{fine}} G_3^{**} \xrightarrow{\text{red.}} G_3^* \xrightarrow[\text{closed}]{\text{ext.}} G_3$$

By definition: $G_3^{**} \in \Gamma_{G_1}$. By section 2 we get

$$\mathcal{L}(G_3^{**}) = \mathcal{L}(G_3^*) \text{ and } \mathcal{L}(G_3) = h(\mathcal{L}(G_3^*))$$

where h is a certain, bracket preserving homomorphism. But this proves

$$\mathcal{L}(G_3) \in \lambda(G_1)$$

It remains to show the decidability of statement (2) of theorem 2.

Denote by $||G|| = \text{Max } |q|$. To any G and Z with $Z \cap \Sigma = \emptyset$

$$(p, q) \in P$$

associate $\mathcal{G}(G, Z) = \{G' \mid G' = (Z \cup \Sigma, \Sigma, P', \sigma')\}$, $||G'|| \leq ||G||$

& there exists a diagram
 $G \xrightarrow[\phi_1]{\text{int.}} G'' \xrightarrow[\phi_2]{\text{red.}} G'$

Lemma 4: $|\mathcal{G}(G, Z)| < \infty$ and there is an algorithm which computes $\mathcal{G}(G, Z)$.

Proof: We show first, that to any diagram

$$G \xrightarrow[\phi_1]{\text{int.}} G'' \xrightarrow[\phi_2]{\text{red.}} G'$$

exists a diagram

$$G \xrightarrow[\phi_1]{\text{int.}} G''' \xrightarrow[\phi_2]{\text{red.}} G'$$

with $V''' - \Sigma \subseteq (V - \Sigma) \times (V' - \Sigma)$.

To prove this we consider the following equivalence relation on $V'' - \Sigma$:

$$\xi \equiv \xi' \iff \phi_1(\xi) = \phi_1(\xi') \text{ \& \ } \phi_2(\xi) = \phi_2(\xi').$$

Then define $G''' = \{[\xi]_{\equiv} \mid \xi \in V'' - \Sigma\} \cup \Sigma, \Sigma, P''', \{[\sigma_1]_{\equiv} \mid \sigma_1 \in \sigma''\}$ where

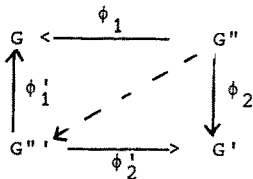
$$[\xi]_{\equiv} \rightarrow t_0[\eta_1]_{\equiv} \dots [\eta_i]_{\equiv} t_i \in P'''$$

if and only if

$$\xi \rightarrow t_0 \eta_1 \dots \eta_i t_i \in P''$$

for all $\xi, \eta_1, \dots, \eta_i \in V'' - \Sigma, t_0, \dots, t_i \in \Sigma^*$ and $i \geq 0$.

By definition of $\xi \equiv \xi'$ we get in a natural way internal ϕ_1' and ϕ_2' such that



is commutative. This shows: ϕ_2' is a reduction. By construction $[\xi]_{\equiv}$ is uniquely determined by $\phi_1'([\xi]_{\equiv})$ and $\phi_2'([\xi]_{\equiv})$. This proves the above assertion.

$$t \equiv t' \text{ iff } \phi_1(t) = \phi_1(t') \ \& \ \phi_2(t) = \phi_2(t').$$

Define $G''' = (\{[t]_{\equiv} \mid t \in \Sigma''\} \cup (V'' \setminus \Sigma''), \{[t]_{\equiv} \mid t \in \Sigma''\}, P''', \sigma'')$

where

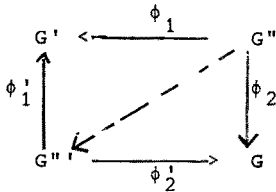
$$\xi \mapsto \eta_0 [t_1]_{\equiv} \dots [t_i]_{\equiv} \eta_i \in P'''$$

iff

$$\xi \mapsto \eta_0 t_1 \dots t_i \eta_i \in P''$$

for all $\eta_0, \dots, \eta_i \in (V'' \setminus \Sigma'')^*, t_1, \dots, t_i \in \Sigma''$ and $i \geq 0$.

Again we obtain by construction a commutative diagram



where ϕ_1' is closed, external and fine and ϕ_2' is closed and external. Moreover $[t]_{\equiv}$ is now uniquely determined by $\phi_1'([t]_{\equiv})$ and $\phi_2'([t]_{\equiv})$.

To compute $\mathcal{X}(G, \Sigma', k)$ we have to consider all pairs (G'', G') with

(i) $||G'|| \leq k, \quad ||G''|| \leq k$

(ii) $\Sigma'' \subseteq \Sigma' \times \bigcup_{j=0}^{\infty} \Sigma^j$

(iii) $V'' \setminus \Sigma'' = V' \setminus \Sigma' = V \setminus \Sigma$.

Obviously, there are only a finite number of such pairs and all these pairs can be constructed effectively. Now, associate to any such pair (G'', G') two mappings ϕ_1 and ϕ_2 by $\phi_1(t, w) = t$ and $\phi_2(t, w) = w$ for all $(t, w) \in \Sigma''$. Decide whether both mappings define external morphisms. If they do, check: $P' = \phi_1(P'')$ and $P = \phi_2(P'')$ which is a necessary and sufficient condition for ϕ_1 resp. ϕ_2 to be closed.

Consider now a diagram from theorem 2(2)

$$G_1 \xleftarrow{\text{int.}} G_1' \xrightarrow{\text{red.}} G_0 \xleftarrow[\text{ext., fine}]{\text{closed}} G_2' \xrightarrow[\text{ext.}]{\text{closed}} G_2$$

By definition of $\mathcal{G}(G, Z)$ and $\mathcal{X}(G, \Sigma', k)$ we get

$$G_0 \in \mathcal{G}(G_1, V_2 \setminus \Sigma_2) \ \& \ G_0 \in \mathcal{X}(G_2, \Sigma_1, ||G_1||).$$

This shows, that the existence of such a diagram is equivalent to

$$\mathcal{G}(G_1, V_2 - \Sigma_2) \sim \mathcal{L}(G_2, \Sigma_1, ||G_1||) \neq \emptyset.$$

Using lemma 4 and lemma 5 we can decide this last relation. This completes the proof of our

Theorem 3: For any two contextfree grammars G_1 and G_2 it is decidable whether or not $G_1 \equiv G_2$ holds.

Remark: We have shown, that the relation

$$\lambda^{(1)}(G_2) \subseteq \lambda^{(1)}(G_1) \text{ is decidable, too.}$$

REFERENCES:

- [1] E. Bertsch, An Observation on Relative Parsing Time, JACM(2) 4, 1975, 493-498
- [2] A. Cremers - S. Ginsburg, Contextfree Grammar Forms JCSS 11, 1975, 86-117
- [3] S. Ginsburg, The Mathematical Theory of Contextfree Languages, McGraw-Hill, Book Co., New York, 1966
- [4] G. Hotz, Eindeutigkeit und Mehrdeutigkeit formaler Sprachen, EIK(2) 4, 1966, 235-246
- [5] R. McNaughton, Parenthesis Grammars, JACM 14, 1967, 490-500
- [6] A. Salomaa, Formal Languages, Academic Press, New York, 1971
- [7] C.P. Schnorr, Vier Entscheidbarkeitsprobleme für kontext-sensitive Sprachen, Computing 93, 1968, 311-317
- [8] C.P. Schnorr, On Transformational Classes of Grammars, Inf. and Control 14, 1969, 252-277
- [9] H. Walter, Verallgemeinerte Pullbackkonstruktionen bei Semi-Thue-Systemen, EIK 6, (4/5), 1970, 239-254
- [10] H. Walter, Grammar Forms and Grammar Morphisms, to appear in Acta Informatica, 1976
- [11] H. Walter, Grammatik-und Sprachfamilien, Teil IV TB AFS 75-22, TH Darmstadt, 1975