THE FORECAST AND PLANNING OF MANPOWER WITH IMPLICATIONS TO HIGHER EDUCATIONAL INSTITUTIONS-MATHEMATICAL MODELS

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ABSTRACT: The article investigates the possibilities of planning a national higher educational system, namely the ways and the means wherein a desired output of the system is to be obtained from a given input. This examination generates guidelines whereupon a detailed mathematical model can be erected to effectuate the planning objectives. However, since the actual construction of the model involves a bulk of mathematical machinery and notations too burdensome to be comprised in the manuscript, the model itself is not developed here. Rather, a detailed exemplification of the planning operations is illustrated on a lilliputian system which serves as a demonstrative vehicle.

1. INTRODUCTION

This article considers the managerial aspects of a national higher educational system.

It is customary to think, on one hand, that its "raison d'etre" is to enable any knowledge pursuer the accomplishment of his wishes, thereby preserving a free educational process. On the other hand, the phenomenon of surplus and shortage in graduates of either this or that profession in the economy is widely recognized; a phenomenon that raises the question: Isn't it possible to regulate the process that transforms candidates to graduates so that these gaps will be diminished?

The latter viewpoint regards the higher educational system as an input/output system absorbing students as inputs and producing professionals as outputs.

The movement of students in the system is partly affected by factors that do not depend on the students themselves or their capabilities but on matters that may be controlled externally. These are composed of factors that directly influence the students, as: scholarships, help in securing accommodation and employment etc., and means affecting the absorption capacity of the various departments, like: numbers of teachers, classrooms, laboratories, etc.

Maneuvers in the allocation of these resources may yield distinct outputs to a same given input by routing the flow in the system with their aid. If there exists a desired output for the economy, it is possible to direct the flow towards it. The location of these means-and their appropriate utilization-which will enable the regulation of the movement in such a way that a desired output will be obtained from a given input, amounts to manpower planning in the higher educational institutions.

A survey of the possibilities to accomplish this goal with the aid of a mathematical model demonstrated on a simulative lilliputian educational system is the core of the present manuscript. The general approach supporting this review and employed to solve the manpower planning problem is the mathematical programming approach.

A voluminous literature exists on the subject of planning educational systems via mathematical models; see the detailed surveys of Correa [2], McNamara [3], Charnes et al. [1] and the series of papers published at the University of California at Berkeley [4], to mention only a few. The principal contribution of the present work is in developing a mathematical model of a higher educational system as an input-output system for professional manpower, and in utilizing this model for operating the possible planning means to regulate the students' flow in the system. This model constitutes an extension, a unification and an improvement compared to existing models in the literature which do not consider all the characteristics of this complicated manpower planning problem.

2. A LILLIPUTIAN PROBLEM.

We proceed to introduce a lilliputian problem that illuminates the ideas of the model. The presentation will be made without using explicit mathematical machinery. The lilliputian problem is computerized in the APL language and is available for free usage.

2.1 A Lilliputian System.

Consider a higher educational system consisting of two institutions. The first institution comprises two departments, an engineering department and an economics department. The engineering department confers one degree towards which one studies two studying units, whereas the economics department confers one degree towards which one studies one studying unit. The second institution contains one department, of engineering, which confers two consecutive degrees for each one of them one studies one studying unit.

The system includes, then, three departments and five studying units that, for the sake of convenience, will be enumerated from 1 to 3 and from 1 to 5, respectively, by their order of appearance. Thus, studying units 1, 2 are in department 1; studying unit 3 is in department 2; and studying units 4, 5 are in department 3.

A planning policy is operated on the system during 2 periods and keeps its impact for 3 periods, i.e., one period beyond the planning period itself. In real life, both a studying unit and a period may be interpreted as a year.

The description of the input-output activities of the system consists of two main elements, namely, the flow of students, and the changes in the inventory levels of the teaching facilities as a consequence of budgets allocations. A transformation which translates the quantities of teaching facilities to absorption capacity of students links the two parts. Hence, the process under study is: allocations of budgets which affect the quantities of teaching facilities. Those are translated to absorption capacity of students that with the numbers of enrolled students and the flow rates yield the output of the system.

2.2 The Flow of Students in the Lilliputian System.

Table 22.1 summarizes the flow of students to, within, and from the higher educational system. The table consists of three parts - from top to bottom - each giving a snapshot of the numbers of students in one period. Every part contains five rows that represent the studying units. The lines between the rows group the studying units in the appropriate departments. For instance, studying units 4, 5 are in department 3 (see column (1)). Column (2) shows the numbers of students in the system at the beginning of each period before absorbing the new students. For example, in the beginning of the second period, the number of students in the third studying unit before absorbing the new students is 2.55. Evidently, for period 1 this column discloses the numbers of students that are inherited from the pre-planning period, and serves as a known initial condition. Column (3) gives the numbers of the newly enrolled, qualified students. These numbers are known via statistical forecasts. Column (4) comprises the numbers of students that can be

Table 22.1

The Flow of Students in the Lilliputian System

Period t=1

(1) i	(2) $n^1 \equiv k^0$	(3) e^1	(4) g^1	(5) k^1	(6) m^1	(7) n^2	(8) ℓ^1	(9) d^1	(10) ℓ^1-d^1	(11) α_1^1	(12) α_2^1	(13) α^1	(14) $\alpha^1\lvert\ell^1-d^1\rvert$
1	2	10	3	1	3	0.96	0	0	0	0	0	0	0
2	6	0	7	0	6	3.16	4.2	3	1.2	3	1	1	1.2
3	3	7	7.5	4.5	7.5	2.55	3.6	2	1.6	4	2	2	3.2
4	1	8	3.6	2.6	3.6	1.01	2.06	3	-0.94	3	2	3	2.82
5	4	11	6	2	6	3.76	2.6	4	-1.4	2	3	2	2.8
													10.02

Period t=2

(1) i	(2) n^2	(3) e^2	(4) g^2	(5) k^2	(6) m^2	(7) n^3	(8) ℓ^2	(9) d^2	(10) ℓ^2-d^2	(11) α_1^2	(12) α_2^2	(13) α^2	(14) $\alpha^2\lvert\ell^2-d^2\rvert$
1	0.96	8	6	5.04	6	0.50	0	0	0	0	0	0	0
2	3.16	1	14	1	4.16	4.88	3.7	5	-1.3	4	1	4	5.2
3	2.55	9	11.1	8.55	11.1	4.63	7.48	6	1.48	5	1	1	1.48
4	1.01	5	8	5	6.01	0.50	2.76	4	-1.24	2	1	2	2.48
5	3.76	8	8	4.24	8	3.73	5.36	4	1.36	3	4	4	5.44
													14.60

Period t=3

(1) i	(2) n^3	(3) e^3	(4) g^3	(5) k^3	(6) m^3	(7) n^4	(8) ℓ^3	(9) d^3	(10) ℓ^3-d^3	(11) α_1^3	(12) α_2^3	(13) α^3	(14) $\alpha^3\lvert\ell^3-d^3\rvert$
1	0.50	12	6.8	6.3	6.8		0	0	0	0	0	0	0
2	4.88	3	25.2	3	7.88		7.18	7	0.18	2	3	3	0.54
3	4.63	14	12.3	7.67	12.3		10.80	11	-0.20	4	5	4	0.80
4	0.50	15	9	8.5	9		4.25	5	-0.75	5	1	5	3.75
5	3.73	6	9	5.27	9		6.13	8	-1.87	6	2	6	11.22
													16.31
													40.93

absorbed by the system according to the quantities of its teaching facilities and according to the intensity of their utilization. This column, namely the "absorption capacity" of the system, is controllable by the planning policy as will be explicated in the sequel.

Column (5) contains the numbers of the new actually absorbed students in every period. They depend on the system's absorption capacity, on the number of students found in the studying units at the beginning of the period, and on the number of enrolled students. The number of vacancies in each studying unit is the difference between the corresponding numbers in Columns (4), (2), whereas the numbers of the qualified, newly enrolled students is shown in Column (3). Hence, the entry in Column (5) is the smaller of these two values. For instance, the absorption capacity of studying unit 4 in period 1 is 3.6. Since there is already 1 student in the studying unit inherited from the past (Column (2)), it is possible to absorb in it at most 2.6 students out of 8 enrolled (Column (5)). On the other hand, the absorption capacity of the same studying unit in period t=2 is 8 students (Column (4)), and the number of students from previous periods in it is 1.01 (Column (2)). Thus, the number of absorbable students is 6.99. However, there are only 5 newly enrolled students (Column (3)). Consequently, these 5 are accepted (Column (5)). Obviously, this 5 is an upper bound and it is possible to accept only part of the enrolled students. In the first example there is a surplus of enrolled students, and 5.4 of them will be left out. In the second example there is a deficiency in qualified, enrolled students and 1.99 studying places are left vacant after the absorption of the new students. Column (6) introduces the total numbers of students in the system in each period; i.e., the sum of Columns (2) and (5).

Column (7) includes the numbers of students to be in the system at the beginning of the next period before the absorption of the newly enrolled students and, actually, it is identical, by definition, to Column (2) of the next part. The entries of this column are obtained by transitions of students from all the previous time periods. Column (7) in period t=2 was figured out, for instance, from transitions of students from Columns (5) of periods t=1 and t=2 in addition to transitions from Column (2) of period t=1. Column (7) in the bottom was obtained from transitions of students from Columns (5) of all parts of Table 22.1 in addition to transitions from Column (2) of the top. For example, the number 4.88 in studying unit 2 of Column (7) in period t=2 is computed in the following manner:

$$0.4\times1+0.1\times4.5+0.7\times5.04+0.1\times5 = 4.88.$$

The transition rates (the decimal fractions) of students in the system are obtained through statistical estimates which are based upon the movement of students in it in periods preceding the planning horizon and which are not found in Table 22.1. Tracing the calculation it is apparent that, from Column (2) of period t=1 no student was transferred, 0.4 of the students of studying unit 1 of Column (5) of period t=1 and 0.1 of the students of studying unit 3 of this column moved to studying unit 2 in period t=3, 0.7 of the students of studying unit 1 and 0.1 of the students of studying unit 4 of Column (5) of period t=2 moved also to studying unit 2 in period t=3. It is conspicuous that the possible transitions are the most diversified ones; progressing in studies, tarrying in one studying unit more than one period, altering studying orbits, dropping out, etc.

Column (8) gives the numbers of graduates at the end of every period, i.e., the system's output. It is also obtained, similar to Column (7), through graduating rates of students from all previous Columns (5) of the Table, including the section in which the particular Column (8) is located, in addition to graduating rates of students from Column (2) of the top section. These are based upon graduating rates in periods preceding the planning time and are not found in the Table.

The entry 3.6, for instance, in studying unit 3 of Column (8) of period t=1 was obtained as follows:

$$0.6\times3+0.4\times4.5 = 3.6 \quad .$$

Namely, 0.6 of the students of studying unit 3 of Column (2) of period t=1 and 0.4 of the students of studying unit 3 of Column (5) of the same period graduated at the end of period t=1 in studying unit 3. The first component of Column (8) is identically 0 since studying unit 1 does not confer any degrees.

Column (9) includes the desired numbers of graduates that ought to be produced by the system. It is estimated according to the predicted demand of the economy for academic manpower. Column (10) gives the differences between the actual output and the desired one. A negative sign stands for a deficiency in graduates of the profession whereas a positive sign stands for a surplus.

Columns (11), (12) introduce the penalties incurred for a deficiency, surplus of one graduate, respectively. These penalties represent the losses of the economy that are implied by dissatisfying its demand to academic manpower. Note that they vary with the studying unit, with surplus and shortage, and with the period as well. For instance, the penalty for surplus of one graduate in studying unit 3 in period t=1 is 2, whereas the same penalty in period t=2 is 1. In period t=1, the penalty for shortage in the graduates of studying unit 3 is 4.

Column (13) gives the relevant penalties of the current case according to the signs of the numbers in Column (10).

Column (14) contains the total penalty for either shortage or surplus in each studying unit, i.e., the products of the corresponding numbers in Columns (10) and (13). Its sum is the total penalty incurred for either surplus or shortage in graduates against the desired numbers in every period. The sum of these penalties, for all the periods, is found in Column (14) at the bottom of Table 22.1.

The total incurred penalty is 40.93. This sum is implied by Column (4) of Table 22.1, which stems from certain allocations of the budgets. Distinct allocations will yield different entries in Column (4) and thereby, a different penalty amount.

The objective of the planning model is to detect that allocation of the budgets that will minimize the total penalty in the period during which the system is subject to the planning policy.

2.3 The Growth of the Absorption Capacity of the Lilliputian System as a Result of the Allocations of the Development Budgets.

We shall commence with the exemplification of the impact of the development budgets on the growth of the inventory levels of the teaching facilities of the system. The effect of this budget is a fundamental one and is complemented by that of the current budget.

Table 23.1 summarizes the information about the growth of the inventory levels of the teaching facilities of the system. The table is partitioned into 3 parts, each of them devoted to one period. Every part consists of 6 rows which correspond to the teaching facilities. The lines between the rows group them in the appropriate departments. For instance, facilities 4, 5 and 6 serve in deparment 3, which in Table 22.1 contains studying units 4 and 5 (see Column (1)).

Column (2) shows the initial inventory levels of the teaching facilities in each time period. These inventory levels stem from previous periods and hence in t=1, this column gives the quantities inherited from the preplanning time. The initial inventory level of teaching facility 4 in the beginning of period t=2 is, for example, 3.

Column (3) comprises the numbers of money units that were allotted for investments in teaching facilities in every time period. For instance, in period t=1, out of a total sum of 41 money units the planner has allotted 7 units to teaching facility 5. This allocation is the controlled factor that, via a sequence of stages, establishes the total incurred penalty, namely 40.93 of Table 22.1. The goal of the optimization problem is to find the allocation that will yield the minimal penalty.

Columns (4), (5) and (6) introduce the incremented quantities that are added to the inventory levels of the teaching facilities in every period as a consequence of an investment of 1 money unit in periods t=1, t=2, t=3, respectively. Therefore, an investment of 1 money unit in teaching facility 2 in period t=1 will increase the inventory level in 0.2 units of it in period t=2 (row 2 in Column (4) of the middle part of Table 23.1). Obviously, materializations of investments from later time periods are identically zero.

Columns (7), (8), (9) give the increments to the inventory levels of the teaching facilities in the appropriate period as resulted from the development budget's money units allotted to them in periods t=1, t=2, t=3, respectively. The number 1.4 in row 5 of Column (7) of period t=2 is the increment to the inventory level of teaching facility 5 in the beginning of period t=2 as a consequence of an

Table 23.1

The Growth of the Inventory Levels of Teaching Facilities of the Lilliputian System as a Result of the Allocation of the Development Budgets

Period t=1

(1) j	(2) c^0	(3) y^1	(4) h^{11}	(5) h^{12}	(6) h^{13}	(7) Δc^{11}	(8) Δc^{12}	(9) Δc^{13}	(10) c^1	(11) f^1	(12) x^1
1	1	5	0	0	0	0	0	0	1	0.5	2
2	1	5	0.2	0	0	1	0	0	2	0.4	5
3	4	10	0.1	0	0	1	0	0	5	0.5	10
4	3	10	0	0	0	0	0	0	3	0.2	15
5	2	7	0.3	0	0	2.1	0	0	4.1	0.4	10.25
6	1	4	0.2	0	0	0.8	0	0	1.8	0.3	6
		41									48.25

Period t=2

(1) j	(2) c^1	(3) y^2	(4) h^{21}	(5) h^{22}	(6) h^{23}	(7) Δc^{21}	(8) Δc^{22}	(9) Δc^{23}	(10) c^2	(11) f^2	(12) x^2
1	1	8	0.2	0	0	1	0	0	2	0.5	4
2	2	3	0.1	0.2	0	0.5	0.6	0	3.1	0.4	7.75
3	5	4	0.2	0.1	0	2	0.4	0	7.4	0.5	14.8
4	3	5	0.1	0	0	1	0	0	4	0.2	20
5	4.1	3	0.2	0.3	0	1.4	0.9	0	6.4	0.4	16
6	1.8	10	0.1	0.2	0	0.4	2	0	4.2	0.3	14
		33									76.55

Period t=3

(1) j	(2) c^2	(3) y^3	(4) h^{31}	(5) h^{32}	(6) h^{33}	(7) Δc^{31}	(8) Δc^{32}	(9) Δc^{33}	(10) c^3	(11) f^3	(12) x^3
1	2	0	0	0.2	0	0	1.6	0	3.6	0.5	7.2
2	3.1	0	0	0.1	0.2	0	0.3	0	3.4	0.4	8.5
3	7.4	0	0	0.2	0.1	0	0.8	0	8.2	0.5	16.4
4	4	0	0	0.1	0	0	0.5	0	4.5	0.2	22.4
5	6.4	0	0	0.2	0.3	0	0.6	0	7	0.4	17.5
6	4.2	0	0	0.1	0.2	0	1	0	5.2	0.3	17.3
		0									89.30
		74									214.10

investment of 7 money units in period t=1 (row 5 of column (3) of the top part of Table 23.1). These 7 money units multiplied by 0.2 (row 5 of Column (4) in the middle part) will bring about a growth of 1.4.

Column (10), which is the sum of Columns (2), (7), (8) and (9), discloses the current inventory levels of teaching facilities in each period. Column (10) of every part is, in fact, Column (2) of the next one.

Column (11) presents the quantities of teaching facilities that are maintained during the relevant period by 1 money unit. Consequently, Column (12), which is Column (10) divided by Column (11), gives the necessary amounts of money units needed to keep up the current inventory levels of the teaching facilities.

It is left now to translate the inventory levels of the teaching facilities of Table 23.1 to absorption capacity of students (Column (4) of Table 22.1).

We shall demonstrate how the entry in row 1 of Column (4) of period t=1 of Table 22.1 was calculated. Department 1 uses teaching facilities 1 and 2. It was decided that on the account of one unit of teaching facility 1, 3 students can be absorbed, and each unit of teaching facility 2 can absorb 2 students. These figures establish the extent of the intensity of the utilization of the teaching facilities and cannot be found in the Tables. It follows that according to the inventory level of teaching facility 1 owned by department 1 in period t=1 (row 1 of Column (10) in the top of Table 23.1) it can absorb 3 students whereas according to the inventory level of teaching facility 2 owned by it in this period (row 2 in the same column), it can absorb 4 students. The final absorption capacity is the smallest over all the teaching facilities; i.e., the smaller of the numbers 3 and 4, namely 3. Teaching facility 1 is then the bottleneck in period t=1 to students' absorption by department 1. In the same manner the transformation maps the inventory levels of teaching facilities to absorption capacity of students for all the studying units.

We have obtained that a certain allocation of 74 money units of the development budget during the planning period that caused a certain allocation of 214.10 money units of the current budget - which is not yet taken as controllable - incurred a total penalty of 40.93 money units for deviations of graduates' numbers from the demand of the economy. Our task is to find the allocation that will minimize the penalty.

2.4 The Impact of the Current Budget on the Inventory Levels of Teaching Facilities of the Lilliputian System.

The values of Table 22.1 were calculated based on the assumption that the required sums of the current budget needed to maintain the growing inventory levels of the teaching facilities are automatically allotted. We shall now drop this supposition and incorporate the current budget into the control means.

Table 24.1 summarizes the modifications that occur in the quantities of the facilities due to the controlled allocations of the current budgets. The missing columns are identical to those of Table 23.1.

The pattern of Table 24.1 is essentially the same as that of Table 23.1. The meanings of its first five columns, from the left, are identical to the corresponding ones of Table 23.1 according to their title number. The entries themselves in the first four columns from the left are equal to those of Table 23.1. The distinction lies in the fifth column (with title number (12)), namely the current budget column. It shows the amounts of money units that were allotted for maintaining the inventory levels of the teaching facilities. These allow only the maintenance of the inventory levels that are given in the column with title number (13). The values of the last column are obtained as products of the corresponding numbers in its two preceding columns. For instance, the allocation of 14 money units to a current maintenance of teaching facility 5 in period t=2, instead of 16 as before, yielded a decrease in its active inventory level from 6.4 to 5.6 units.

The inventory levels of teaching facilities of Column (13) are translated to absorption capacity of students in the same manner that was outlined above.

Due to the lessening in these inventory levels, some modifications will occur in Columns (2), (5) and (8) of Table 22.1. The reason is that the students' absorption

Table 24.1

The Impact of the Current Budget on the Inventory Levels of Teaching Facilities of the Lilliputian System.

Period t=1

(1) j	(2) $\underline{c}^0$	(10) $\underline{c}^1$	(11) $\underline{f}^1$	(12) $\underline{x}^1$	(13) $\underline{c}_1^1$
1	1	1	0.5	2	1
2	1	2	0.4	5	2
3	4	5	0.5	10	5
4	3	3	0.2	15	3
5	2	4.1	0.4	10.25	4.1
6	1	1.8	0.3	6	1.8
				48.25	

Period t=2

(1) j	(2) $\underline{c}^1$	(10) $\underline{c}^2$	(11) $\underline{f}^2$	(12) $\underline{x}^2$	(13) $\underline{c}_1^2$
1	1	2	0.5	4	2
2	2	3.1	0.4	6	2.4
3	5	7.4	0.5	12	6
4	3	4	0.2	20	4
5	4.1	6.4	0.4	14	5.6
6	1.8	4.2	0.3	13	3.9
				69	

Period t=3

(1) j	(2) $\underline{c}^2$	(10) $\underline{c}^3$	(11) $\underline{f}^3$	(12) $\underline{x}^3$	(13) $\underline{c}_1^3$
1	2	3.6	0.5	7	3.5
2	3.1	3.4	0.4	6	2.4
3	7.4	8.2	0.5	13	6.5
4	4	4.5	0.2	21	4.2
5	6.4	7	0.4	15	6
6	4.2	5.2	0.3	14	4.2
				76	
				193.25	

capacity of the studying units is changed and consequently the numbers of actually absorbed students and the numbers of graduates have changed as well. These modifications are summarized in Table 24.2 and obviously will affect also the total incurred penalty which is now 55.28.

We have obtained that the same allocation of the 74 money units of the development budget during the planning period and a certain allocation of a trimmed current budget of 193.25 money units instead of 214.10, caused an increase from 40.93 to 55.28 in the total penalty. The purpose of the optimization procedure in this case is to point out the allocation of the development and current budgets that will minimize the total penalty.

2.5 Feedbacks in the Lilliputian System.

The illustration of the feedback phenomena will be done only for period t=1. The data is taken from Tables 22.1, 23.1 and is based on the assumption that the current budget is not subject to control.

In addition to the teaching facilities of Column (10) of Table 23.1, a certain portion of the students themselves, Columns (2) and (5) of Table 22.1, serve in academic positions as teaching assistants, thus being considered as part of the teaching facilities contributing to the absorption capacity of the studying units. This phenomenon will be named "the internal feedback". The "external feedback" will be termed after graduates that join the academic staff. These phenomena may phenomena may constitute a considerable share of the absorption capacity and cannot be ignored in delineating the input-output activities of the system. Since now Columns (4) and (5) of Table 22.1 mutually establish each other, the only way to determine them is rather by a solution of a system of equations and not by the direct method that was applied above. The formulation of the set of equations is quite cumbersome and is beyond the scope of this article.

Table 24.2

The Modifications in Table 22.1 Due to Cuts in the Current Budgets of the Lilliputian System.

Period t=1

(1) i	(2) $\underline{n}^1=\underline{k}^0$	(5) $\underline{k}^1$	(8) $\underline{\ell}^1$
1	2	1	0
2	6	0	4.2
3	3	4.5	3.6
4	1	2.6	2.06
5	4	2	2.6

Period t=2

(1) i	(2) $\underline{n}^2$	(5) $\underline{k}^2$	(8) $\underline{\ell}^2$
1	0.96	3.84	0
2	3.16	1	3.7
3	2.55	6.45	6.22
4	1.01	5	2.76
5	3.76	4.24	5.356

Period t=3

(1) i	(2) $\underline{n}^3$	(5) $\underline{k}^3$	(8) $\underline{\ell}^3$
1	0.384	4.416	0
2	4.038	3	6.338
3	3.674	6.076	7.927
4	0.384	8.016	4.008
5	3.732	4.668	5.773

Table 25.1

The Flow of Students in the Lilliputian System With Feedbacks

Period t=1

(1) i	(2) $\underline{n}^1\equiv\underline{k}^0$	(3) $\underline{k}^1$	(4) $Q^{20}\underline{k}^0$	(5) $Q^{21}\underline{k}^1$	(6) $\underline{n}^2$	(7) $P^{10}\underline{k}^1$	(8) $P^{11}\underline{k}^1$	(9) $\underline{\ell}^1$
1	2	2	0.50	0.66	1.16	0	0	0
2	6	0	2.40	0.76	0.76	2.94	0	2.94
3	3	7	1.10	2.30	2.30	1.44	1.96	3.40
4	1	2.6	0.20	0.96	0.96	0.45	1.40	1.85
5	4	5	2.70	2.26	2.26	4.80	0.75	5.55

Table 25.1 presents the modifications in Table 22.1 which happen due to the feedbacks. Columns (1) and (2) are identical to those of Table 22.1. Column (3) contains the numbers of absorbed students (the column which corresponds to Column (5) of Table 22.1) that will be obtained after the solution of the system of equations. Columns (4), (5) show the numbers of students to be in the system at the beginning of period t=2 out of Columns (2), (3), respectively, while Column (6) is their sum; i.e., the corresponding column to Column (7) of Table 22.1. The decomposition of Column (6) as the sum of Columns (4) and (5) is now necessary since the internal feedback rates, namely students' rates serving in teaching positions, for period t=2 are derived from the components of the numbers of students in the system sorted by their absorption period, Columns (4) and (5) of Table 25.1, in addition to the currently absorbed students.

Columns (7), (8) give the numbers of graduates in the end of period t=1 which are implied by Columns (2), (3), respectively, after discarding the external feedback, namely graduates that join the academic staff.

Column (9) introduces the numbers of graduates of the system which are directed to the economy to satisfy its demand for academic manpower; i.e., the corresponding column to Column (8) of Table 22.1. Trivially, it is the sum of Columns (7) and (8). The external feedback rates in our case show, for instance, that only 0.8, 0.7 of the graduates in the end of period t=1 that come from the absorbed students of Columns (2) and (3), respectively, are directed to meet the economy's needs for academic manpower. The rest returned and joined the system as part of the teaching facilities. The rates 0.7 and 0.8 are not found in Table 25.1 and are estimated by statistical methods based upon past observations.

The total incurred penalty in this period for surplus and shortage in academic manpower is 11.08. Again, the optimization procedure must reveal the allocation of budgets that will minimize this penalty.

In case we assume that the current budget is also controllable, the feedback's rates are not constants but depend on its allocation. This extension highly complicates the problem and will not be demonstrated here.

2.6 A Summary of the Budgetary Activities of the Lilliputian System

Table 26.1 summarizes the budgetary activities of the Lilliputian System during the three periods, see Column (1). Column (2) exhibits the total amounts of the development budgets in each period. Columns (3), (4) give the total amounts of the current budget needed to keep up the growing inventory levels of teaching facilities, the penalties incurred for either surplus or shortage in graduates, respectively.

Table 26.1

Summary of the Budgetary Activities of the Lilliputian System

(1) t	(2) $\bar{y}^t$	(3) $\bar{x}^t$	(4) u^t	(5) $\bar{x}^t$	(6) u^t	(7) u^t
1	41	48.25	10.02	48.25	10.02	11.08
2	33	76.55	14.60	69	13.34	12.43
3	0	89.30	16.31	76	31.92	
	74	214.10	40.93	193.25	55.28	

Columns (5), (6) follow the same pattern as Columns (3), (4), respectively, except that the current budget is subject to control. Column (7) shows the penalties in case there are feedbacks and the current budget is provided to maintain the full growing inventory levels of teaching facilities.

When the current budget is not controllable, the optimization's task is to find the allocation of the development budget that will yield the minimal value of the penalty. In case there is a possibility to cut the current budget as a control means, the optimization procedure ought to detect the allocation of the development as well as the current budgets for the sake of obtaining the minimal penalty. Similar cases exist when the feedback phenomena of the system are considered. A detailed technical formulation of the mathematical model and the optimization procedure - which may be either linear or not, depending on the assumptions concerning the functions of the model - is rather involved. It is strongly hoped that the lilliputian system has succeeded in conveying the principal ideas, and has acutely pinpointed the vertices on which intelligent, controlled operations may be implemented thereby enabling the planning of a national higher educational system.

2.7 Overview

The starting point of the planning model is the controlled allocations of the budgets. This allocation establishes the inventory levels of the teaching facilities. These are translated to absorption capacity of students. The absorption capacity, the numbers of students that are in the system from previous years, and the flow from outside determine the numbers of actually absorbed students. The flow rates of students in the system will yield the numbers of graduates. A comparison between the graduates' numbers and the economy's demand appropriately weighted gives the total penalty incurred for shortage or surplus in academic manpower. The goal of the optimization procedure of the planning model is to single out the allocation of budgets that will minimize the total penalty.

3. REFERENCES

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