# A FOUR-VARIABLE WORLD SYSTEM*

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## ABSTRACT

This is a model to study aspects of the short and long-range growth and balance between four world quantities: food, energy, fertilizer, and population. The model may be used as a tool to study implications of various policies for coordinated world planning. The model operates as follows: The world is subdivided into a number of regions. Consider time period t. In each region two factors, investments and population, are used to determine supplies of fertilizer, energy, arable acreage, and workforce availability. The regional investment stream is an exogenous input to the system. In each region, demand functions are specified for foods, fertilizer, energy for agriculture, energy for other uses, acreage, and labor. These demands are functions of all prices, population, and income in period t - 1. A spatial equilibrium model links all regions and determines equilibrium imports, exports, and prices for each region. This gives, for each region, the income in period t, and specific consumption of fertilizer, energy for agriculture, acreage, and workforce. Based on this consumption, and taking account of weather, regional agricultural outputs are determined. This provides an exogenous food supply for the spatial equilibrium model in year t + 1. The supplies in t + 1 of fertilizer, energy, acreage and labor are determined, as functions of population and the investment stream, and the procedure is repeated.

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*Since the writing of this material preliminary reports have been released on the MOIRA model of H. Linnemann and Associates. Because of this timing of circumstances it should be explicitly stated that this document was written prior to any knowledge of the Linnemann efforts and hence the latter must be excepted from any comments herein which refer to other works in global modeling.

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## I. Introduction

The purpose of this paper is to sketch a framework for investigating some aspects of the short and long-range growth and balance between four world quantities: food, energy, fertilizer, and population. The model may be used as a tool to study the implications of various policies for coordinated world planning.

We shall describe the overall flow of the system with its important links and couplings. At the outset it should be emphasized that many of the details are only briefly indicated and remain to be filled in by individuals with expertise in the areas treated.

The model operates as follows. The world is subdivided into a number of regions. Consider time period $t$. In each region two factors, investments and population, are used to determine supplies of fertilizer, energy, arable acreage, and workforce availability. The regional investment stream is an exogenous input to the system. The population growth is an endogenous model which remains to be filled in. In each region, demand functions are specified for foods, fertilizer, energy for agriculture, energy for other uses, acreage, and labor. These demands are functions of all prices, population, and income in period $t - 1$. A spatial equilibrium model links all regions and determines equilibrium imports, exports, and prices for each region. This gives, for each region, the income in period $t$, and specific consumption of fertilizer, energy for agriculture, acreage, and workforce. Based on this consumption, and taking account of weather, regional agricultural outputs are determined. This provides an exogenous food supply for the spatial equilibrium model in year $t + 1$. The supplies in $t + 1$ of fertilizer, energy, acreage and labor are determined, as functions of population and the investment stream, and the procedure is repeated (see Figures 1, 2, and 3).

Previous efforts in global modeling are reported and critiqued in [1], [2], [3], [4], and [5]. The approach described herein differs from these previous works in several respects. By way of brief comparison:

i. Speaking generally, previous studies tend to describe the world with unrealistic minimally structured functions and restrictive logic. This logic is usually in the form of difference equation/simulation models which are built at a modest level of methodology, though this is claimed to have been appropriate, given the level of

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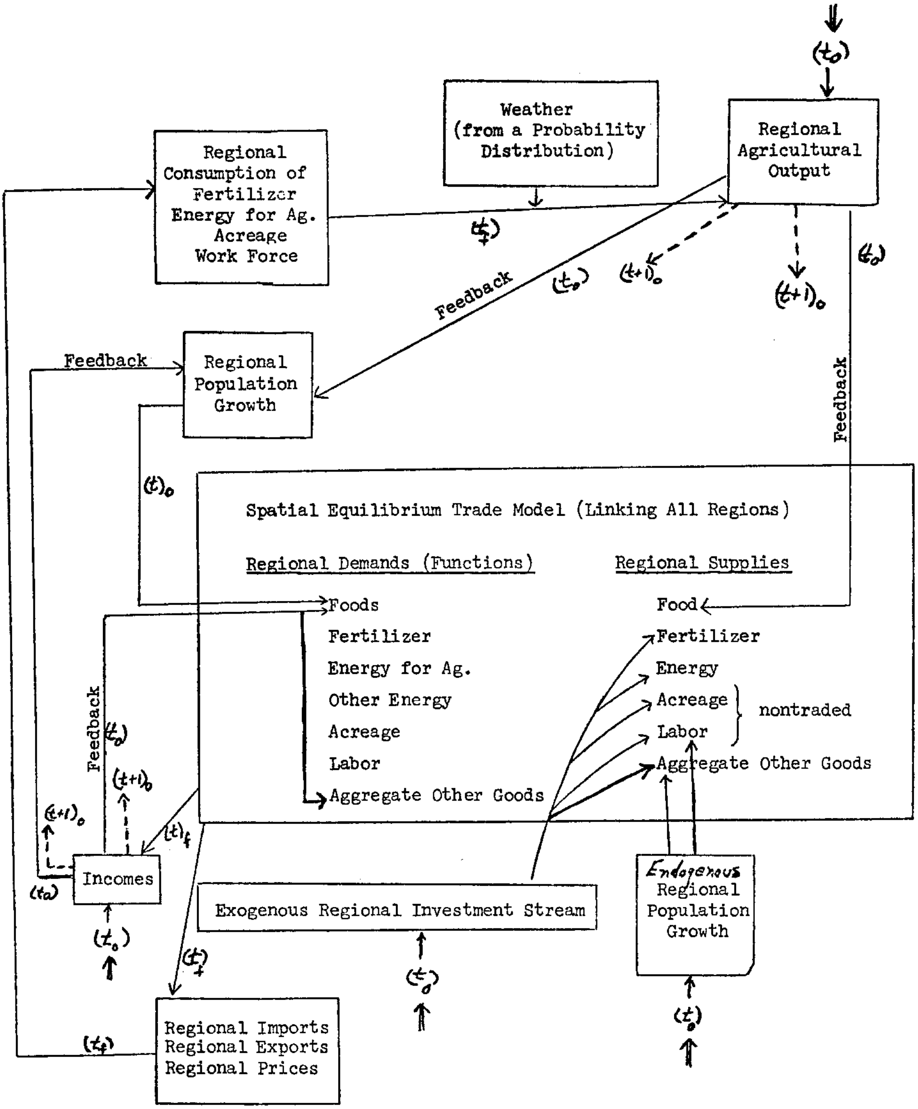

FIGURE 1: OVERALL SYSTEM

0 subscript = beginning of period
f subscript = end of period
⇒ initiating conditions

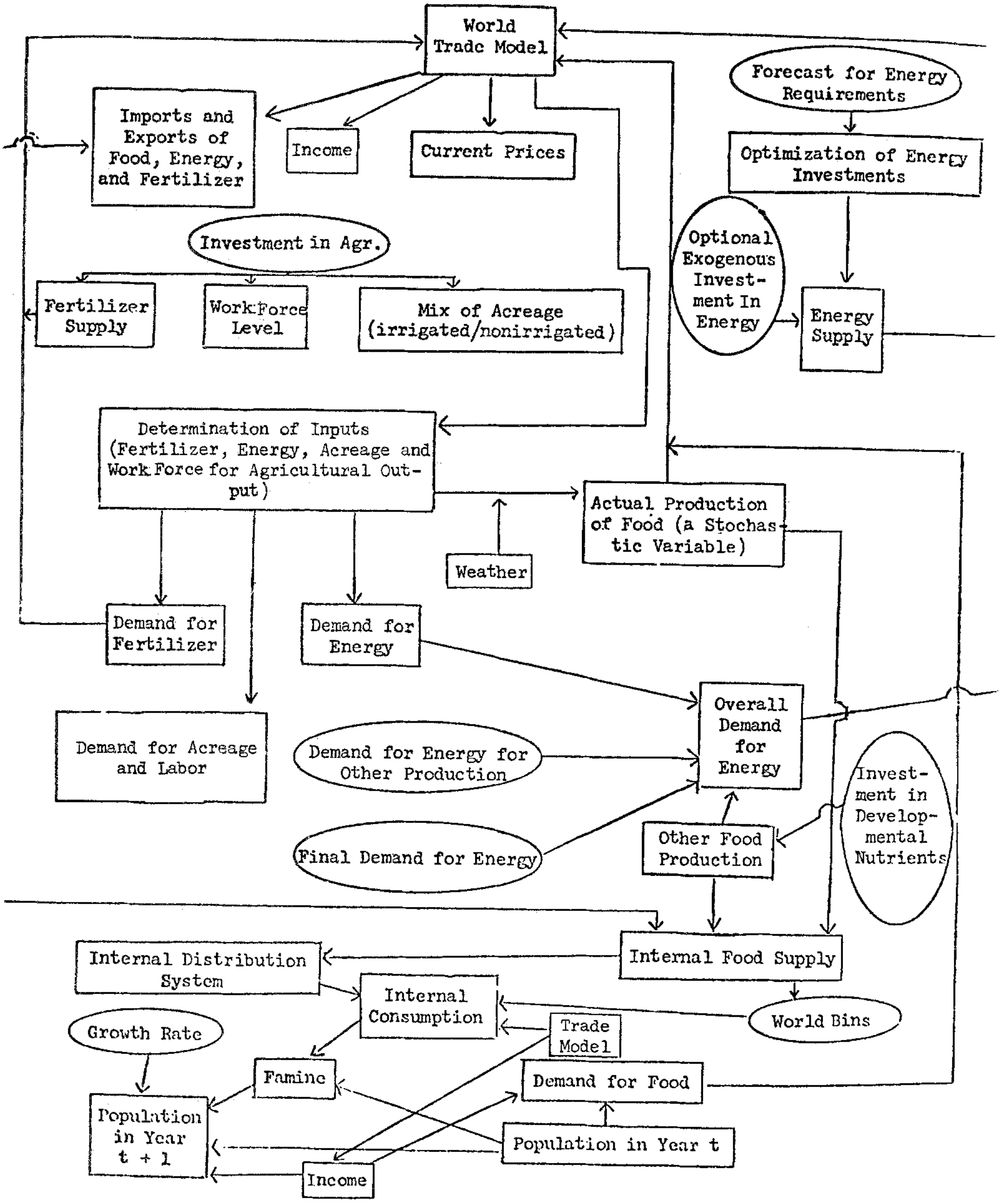

FIGURE 2: PROTOTYPE REGIONAL MODEL, DETAILED

⬭= Exogenous Inputs

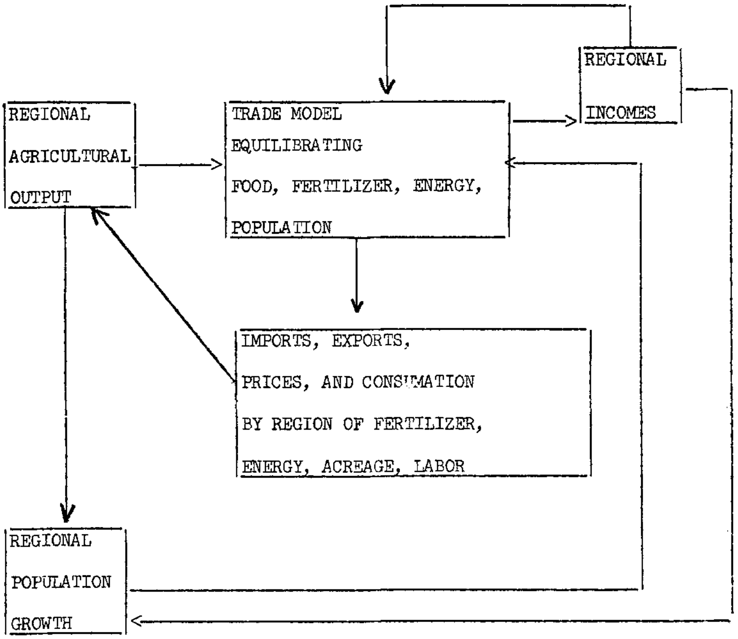

FIGURE 3: DYNAMIC LOOPS

available information and the levels of approximation inherent in the technical structure of the model. The approach herein is along more basic methodological lines, justified by the belief that, ceteris paribus, a stronger technical base will produce more interesting results. Moreover, the accomplishments of mathematical programming and operations research in large-scale cost-effectiveness studies are now abundantly evident in defense, transportation, and other areas of planning in both government and industry. This would seem to provide further support for the prospects of such an approach.

ii. Speaking more specifically, previous models for estimation of grain output tend to be log linear extrapolations over time. This provides estimates which are at best only grossly sensitive in an aggregate sense to changes in inputs and climatological probabilities, and entirely insensitive to potential new trends that can develop via the motivations of humans, organizations, and government. At worst, such estimates can be very misleading, and indeed are thought to be misleading by some prominent workers in the field. By contrast, the regional models in the present discussion are designed to produce agricultural output functions as climate-induced probability distributions. More specifically, for several commodities, such as corn, wheat, rice, and soybeans, regional output functions are estimated in terms of the following input variables: acreage planted in each of several categories (such as irrigated or non-irrigated), labor, fertilizer, a single aggregated measure of all other energy inputs, and, finally, weather patterns. Several weather patterns are defined, and based on the probabilities of these various patterns a probability distribution for regional output can be obtained as a function of the input variables. The mix of inputs for labor, acreage, energy, and fertilizer is derived from the spatial equilibrium trade model. The weather can either be exogenously input or can be drawn from a probability distribution for the region. The selection of factor inputs and weather then leads to a simulated actual regional output. This food supply model is more structured than anything existing in previous models and it should serve to provide the most specific information available on global grain outputs in the near future.

iii. None of the known global analysis models tend to deal in any depth with international trade. Much of the discussion of the food situation confuses need with market demand. During all of the years of the so-called food "surpluses" in North America there was much undernourishment and starvation in the world and even in the U.S. The disappearance of the grain reserves in 1972-1973 was caused not only by a coincidence of crop failures but by a spurt in demand brought about by heightened affluence in Europe and Japan and a policy decision in the USSR. The world food market is outstandingly a price market--concessional sales notwithstanding. Even trade restrictions and subsidies tend to work to a large extent through the price mechanism. Consequently price determination is seen as a key component of the present work. The spatial equilibrium model determines, for each region, exports and imports of foods, energy, and fertilizer along with the appropriate market clearing prices. Though a number of International Trade Models are currently being developed [6], [7], none are linked in a dynamic way to the other systems modeled in this project (food, energy, fertilizer, and population).

It is contemplated that the system we have outlined can serve several functions. Certainly it can provide a useful tool for the study and analysis of alternatives. It can offer assistance in decision making at many levels. Another objective could be to provide a tool for studying the implications of various policies for coordinated world planning. Related to this is another important function, more pedagogic in nature: the model may be used as a powerful learning tool. The possibility for making alternative exogenous inputs is consistent with the idea of a parallel "gaming version" of the model. This would be a man-machine interactive mode of operation along the following lines. Experts in such areas as public policy, financial investment, agronomy, energy, international trade, and nutrition would assemble with executives and policy analysts for periods of several days during which the model would be executed, interactively, and a future of events would unfold in accord with a variety of exogenously input constraints and decisions. Such an environment of mock decision making, guided by expert advice, with continous updating of information, has proved in other contexts to be an effective way of expanding horizons and maturing judgment. In terms of the massive dynamics of the problems herein confronted a device for "hands-on" experience, even in a mock scenario, can help to bring the situation home.

One of the most recent and distinguished efforts on world modeling is the work of Mesarovic and Pestel reported in [5]. Their results seem not only to generally support the type of earlier results reported by Meadows et al., but even to outline potentials for more dire consequences for several regions of the world. The Mesarovic-Pestel group has disaggregated the world into about ten regions, and regional submodels have been constructed with numerous interacting components. Though there is a repeated emphasis in the Mesarovic-Pestel discussion on the need for coordinated world planning, satisfaction of overall objectives, etc., there is little if anything along the lines of optimization or suboptimization in their model. The quantity of detail in their logical structure is enormous, but at least qualitatively the functional specifications seem not unlike the systems of proportionalities of the earlier Meadows effect. It is our contention that optimization results can be useful at a minimum in guiding the search for acceptable policies and that the state of the art has reached the point that optimization options can be built-in and successfully handled. Moreover, it is felt that more complex mathematical representations will better approximate the non-linear interactions in world dynamics, and, again, the state of the art is able to handle the added complexity.

We wish not to detract from the fact that other projects on world models have made important and initiative steps in shedding light on policy issues in areas where global activities and interactions are influential. Our basic assumption is merely that by making systematic use of more information, more data, more structure, and more methodology, we at least allow for the possibility of improved, more sensitive forecasting, and this in turn will produce more feasible, perhaps more convincing, world plans for further study and consideration.

The spirit of this effort can be illustrated by reference to the work of Forrester, Meadows, and associates [1], [2], [3] who have produced well-publicized scenarios of doom in perhaps as little as a hundred years based on projections of current technologies and trends. By comparison, the economists' view, at least, as expressed by T. W. Schultz, tends to be generally calm and unsympathetic, for, as Schultz argues, regaring food [8]:

> "There are two wholly inconsistent views of the future availability of food. The natural earth view is one of space, depletion of energy and a virtually fixed land area suitable for growing good crops that make it impossible to feed the increasing world population. The social-economic view is based on the ability and intelligence of man to lessen his dependency on cropland and on traditional agriculture, and thereby to reduce the real costs of producing food even in spite of the current population growth. Is is possible to resolve this extraordinary inconsistency? I shall try, but it will not be easy because of the strong prevailing commitment to the natural earth view. I find it ironic that economics, which has long been labelled the dismal science, must bear the cross of showing that the bleak earth outlook for food is not compatible with economic behavior."

The framework herein described should assist in reconciling these two positions, the "limited-earth" view and the more optimistic "social-economic" view. The proponents of the latter position argue that as new needs and conditions are perceived, modifications of behavior, investments in research and technology, etc., will ward off disaster. The limited-earth/exponential growth theorist basically claims that economic adjustments are not instantaneous, lead times are required, and unwittingly we may not allow for enough time.

From one point of view it might be said that the general model to be discussed is an effort toward allowing for "enough time." We seek to recognize explicitly the investment process in various new technologies, both in food and in energy, and to tune accurately enough to changing interactions so that there is sufficient lead time to modify policy and redirect resources without paying catastrophic costs. The remaining sections of this paper describe in more detail the overall framework.

## II. Overall Logic of the System

This discussion is a nontechnical summary. It should be mentioned that optional capabilities can be developed for suboptimizing (or otherwise computing) more of the components which are presently described as purely exogenous. It may be helpful to refer back to Figures 1 and 3 during the following discussion.

### 1. Investment

In each region an investment stream is input for agriculture and energy. In agriculture this includes fertilizer technology, acreage development, irrigation, and work force training. Moreover, there is allowance for investment in selected developmental nutrients. In energy, the investment stream includes development of new technologies. All investments are exogenous.

### 2. Agricultural Production and Food Supply

For each region a production methodology has been developed for agriculture.

Input levels are derived from the trade model. Given any choice of factor inputs (acreage of various types planted, work force, fertilizer, and other energy inputs) a probability distribution of output can be determined as a function of regional weather patterns. Other possible stochastic shocks may be included (attacks of insects, fungus, etc.). In each region, a simulated actual (as opposed to expected) food production can be obtained by using a Monte Carlo technique with the probability distribution obtained from weather and any other stochastic factors. However, agricultural production cannot be equated with food supply, since much of the labor, capital, energy technology, and material that goes into the food supply system is expended after the crop leaves the farm. Such factors as internal storage facilities and the logistics of land transport are considered in an internal distribution component. There is also provision in the model for a subsystem of other nutrient production to augment agricultural output. These "other foods" supply functions remain to be developed.

### 3. Supply of Energy and Fertilizer

Supply functions are to be developed for fertilizer and for a variety of energy technologies. These will be dependent on the input investment stream. For fertilizer and energy, for each technology, time dependent cost functions will be estimated and the derived marginal cost curves will represent the supply functions. In the short run supplies are fairly inelastic with respect to price.

### 4. Demands for Food, Energy, Fertilizer, Acreage, and Labor

Regional demands for food as a function of all prices, population, and income must be estimated. Demands for fertilizer, acreage, workforce and energy for agriculture are derived from solving the agricultural output optimization model with alternative prices. Demands for energy for all other products (including energy production) and for final usage are exogenous.

## III. The Agricultural Production System

### A. The Inputs

Weather. Along with technology weather is a major determinant of regional and global output. The influence of weather is mainly stochastic, and with a given technology and specified inputs the probability of various levels of yield of a given crop can be related to the probabilities of various weather conditions at specified critical periods of the crop cycle. Examples of methods for describing and analyzing these stochastic relations are found in references [9], [10], [11]. In general, it is important that the analysis be disaggregated with respect to region, crop, and time (intraseasonal variations must be recognized).

Though the influence of weather on yields is qualitatively obvious, the implication of the quantitative importance of this factor on U.S. grain yields may be less appreciated, at least to the extent suggested by the following excerpt from a U.S. Department of Agriculture study performed in 1973 [12]:

> The conclusions [of this study] indicate very strongly that the production of grain in the United States has been favored by extremely good weather in recent years. Any national policy that does not take into consideration the fact that less favorable weather is far more likely than recent nearly optimum conditions, is likely to place us in most unfortunate circumstances. . . . The weather in recent years has been extremely favorable for high grain yields. ... the recent string of consistent high yields, especially for corn, is a weather phenomenon. It is without any basis to suppose that technology has removed the susceptibility of yields to weather fluctuations.

Technology. This is input in the forms of acreage, labor, fertilizer, and energy. Acreage will be classified as irrigated or not, and also according to the variety of seed in the sense of high or low yield. All inputs other than labor and fertilizer (i. e., machines, fuel, pesticides, herbicides, etc.) will be measured in units of energy and aggregated into a single energy input. The feasibility of aggregating in terms of energy has been demonstrated in [13] and [14].

## B. Definition of Terms in the Agricultural Production Model

1. Region: A subset of the world assumed self-ruling and independent in terms of policy, trade, and production.

2. Zone: A subset of a region which is homogeneous with regard to weather.

3. Acreage Type: In each zone there are three possible types of acreage: irrigated with a high yield variety; nonirrigated with a high yield variety; and nonirrigated with a low yield variety.

4. Crops and Planting: In each zone on each acreage type there are four possible crops, some of which may be planted more than once a year on the same acreage: wheat, corn, soybeans, and rice.

5. Weather Pattern: Three key time periods are specified for each growing season (each such period being an interval of a specified number of days at a specified time of the year). For each time period weather is characterized as being in one of the conditions good, normal, or poor. A weather pattern is one of the 27 possible triples of such conditions.

6. Total output of crop $i$ in planting $j$ on acreage of type $k$ in zone $z$, given weather pattern $w$, is given by

$$Q^T_{ijkzw}(L^T_{ijkz}, F^T_{ijkz}, E^T_{ijkz}, A^T_{ijkz})$$

where

$L^T_{ijkz}$ = total labor input on $ijkz$

$F^T_{ijkz}$ = total fertilizer input on $ijkz$

$E^T_{ijkz}$ = total energy input other than fertilizer on $ijkz$

$A^T_{ijkz}$ = total acreage devoted to $ijkz$

Assuming $Q^T$ is homogeneous of degree 1 we write

$$Q_{ijkzw}(L_{ijkz}, F_{ijkz}, E_{ijkz})$$

for output per acre as a function of inputs per acre.

7. Expected output of crop $i$ in planting $j$ in region $R$

$$ERO_{ij} = \sum_z \sum_w P_{wz} \sum_k A^T_{ijkz} \cdot Q_{ijkzw}(L_{ijkz}, F_{ijkz}, E_{ijkz})$$

where

$P_{wz}$ = probability of weather pattern $w$ in zone $z$ .

## C. A Maximization Model for Determining Factor Demands

$$\max \sum_{i,j} [\pi_i ERO_{ij} - \sum_{k,z} (\pi_{A_i} A^T_{ijkz} + \pi_F F^T_{ijkz}) - \sum_z (\pi_{E_{ik}} E^T_{ijkz} + \pi_{L_{ik}} L^T_{ijkz})] \quad ,$$

where

$\pi_i$ = current regional price of food type $i$

$\pi_{A_i}$ = regional rental price of acreage of type $i$

$\pi_F$ = current regional price of fertilizer

$\pi_{E_{ik}}$ = current regional price of energy input to acreage of type k for crop i

$\pi_{L_{ik}}$ = current regional price of labor employed for production of crop i on acreage of type k .

As prices are varied, and the model re-solved, approximations are obtained for the demand functions for fertilizer, energy, acreage, and labor. Each of these is a function of $(\pi_i, \pi_{A_i}, \pi_F, \pi_{E_{ik}}, \pi_{L_{ik}})$.

### D. Methodology for the Agricultural Production System

Econometric methods must be used to estimate the production function $Q_{ijkzw}$ given in (6) above. Initially a log linear form independent of Region might be investigated.

The problem in C must be analyzed after the complication of the Q function is discovered. It may be desirable to add constraints to the problem, in which case potential algorithms include piecewise linearization, decomposition, and generalized Lagrangian techniques developed and previously reported in published literature [15], [16], [17], [18], [19], [20], [21].

## IV. The Energy and Fertilizer Components

Regional cost curves must be obtained for producing given amounts of energy with various technologies. Let the cost of producing $x_{it}$ kilocalories of energy in period t with technology i, given past investments $k_{i1}, k_{i2}, \ldots, k_{i,t-1}$, be given by functions

$$C_{it} = H_{it}(k_{i1}, k_{i2}, \ldots, k_{i,t-1}, x_{it}) \ .$$

Given these relations, for each technology (in each region) the marginal cost relations and supply functions can be derived.

There is also an opportunity to optimize investment and operating expenditures over the various energy technologies so as to satisfy estimated or forecast requirements at minimum cost over a given time horizon. For example

$$x_{it} = O_{it}(k_{i1}, k_{i2}, \ldots, k_{i,t-1}, C_{it})$$

gives output as a function of expenditures. One can then formulate the nonlinear program

$$\min \sum_{t=1}^{T} \sum_{i=1}^{K} k_{it} + C_{it}, \quad \text{s.t.}$$

$$O_{it}(k_{i1}, \; k_{i2}, \; \ldots, \; k_{i,t-1}, \; C_{it}) \geq R_t, \quad t = 1, \; \ldots, \; T$$

which will allocate funds to technologies so as to satisfy estimated requirements $R_1, \ldots, R_T$ at minimum overall cost. It may be of interest to disaggregate this problem over certain subsets of technologies.

Methods similar to the above will be used for the fertilizer sector.

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