THE IDENTIFICATION AND ADAPTIVE PREDICTION OF URBAN SEWER FLOWS

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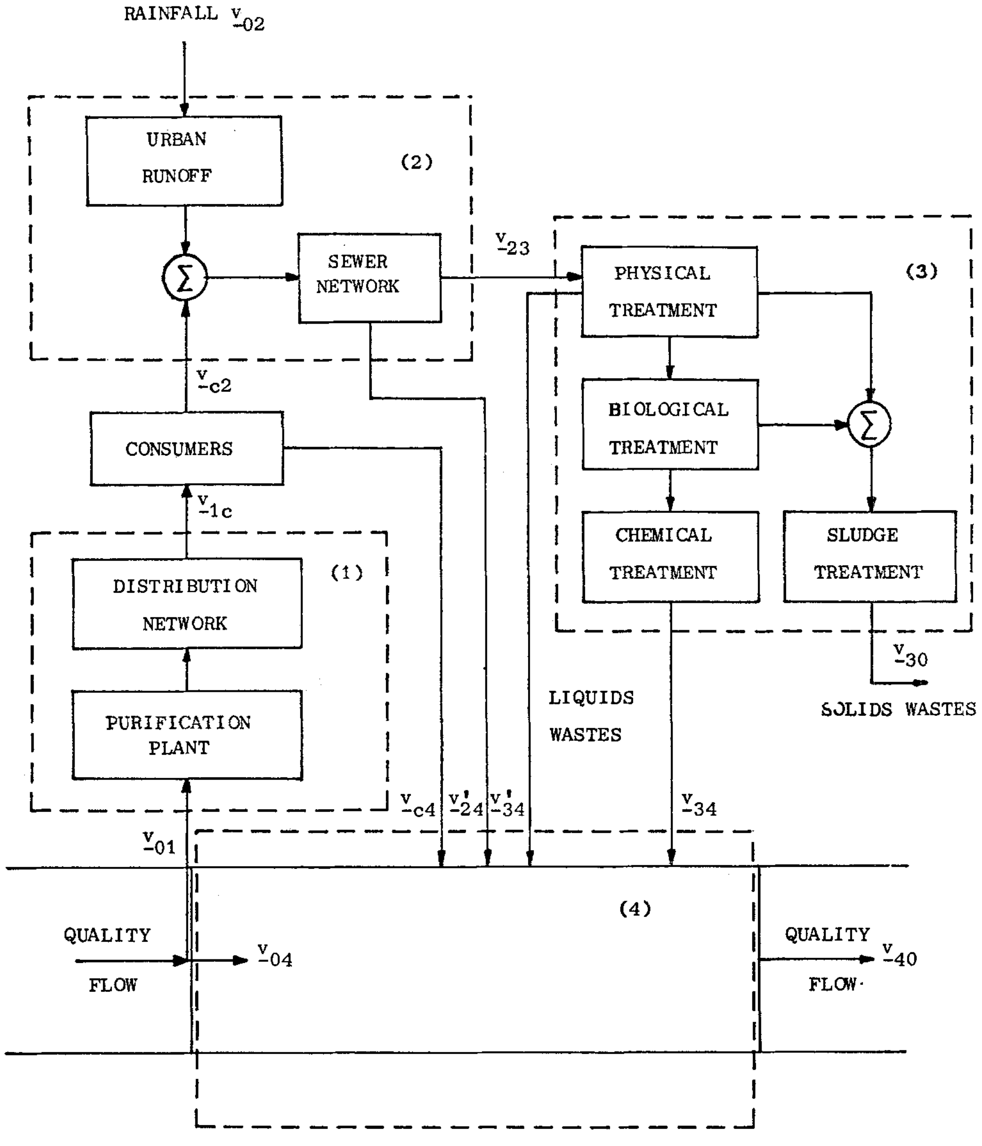
1. Introduction

Unlike most process industries a wastewater treatment plant receives a raw input material whose variations with time are large and imprecisely defined. Some of these disturbances, which result from rainfall-runoff into the urban sewer network, are quite disruptive for the operation of the treatment plant and may also cause a subsequent overloading of the receiving river's self-purification capacity. The effluent from a sewer network, i.e. the influent to a wastewater treatment plant, is as it were, the fulcrum about which the control of the sewer network and the treatment plant is balanced. Therefore, an advance knowledge of the dynamic variations of the influent flow would play an important role in the more efficient operation of the treatment plant and the minimisation of storm-water overflows from the sewers.

Most previous models for urban rainfall-runoff/sewer effluent flow relationships tend to be of the large, deterministic, internally descriptive type. For certain control objectives it may, in practice, be sufficient to use a much simpler black box conception of the system. In this paper results are presented for the identification of a stochastic input/output, time-series model using the method of maximum likelihood: the data are taken from the Käppala treatment plant and meteorological stations in the district surrounding Stockholm. The identification phase of the analysis is an introductory stage in the examination of the potential applicability of an on-line, adaptive predictor for the influent flow to the treatment plant. The prediction problem is separated into two steps: in the first step the parameters of the black box model are estimated recursively with a least squares technique; the second step makes a prediction of the plant influent flow on the basis of the newly updated model and parameter estimates.

The predictor is adaptive in the sense that it automatically adjusts the model parameters to any unknown changes in the process dynamics. It is practical in the sense that it assumes very little on-line instrumentation of the system: in general, the innovation of an automatic control for water quality is severely hampered by a lack of the relevant, reliable, and robust measuring equipment. From a comparison of the prediction with the observed Käppala data it turns out that a good advance knowledge of the influent flow variations can be obtained in the absence of any measurements of the rainfall incident on the urban land surface.



SUBSYSTEMS:

(1) POTABLE WATER ABSTRACTION, PURIFICATION, AND SUPPLY NETWORK
(2) URBAN LAND RUNOFF AND THE SEWER NETWORK
(3) WASTEWATER TREATMENT PLANT
(4) A STRETCH OF RIVER

Figure 1 The water quality system.

2. PROBLEM FORMULATION

Consider the water quality system defined by figure 1. Consider further the competing demands made on the quality "resources" of a reach of river by the assimilation of waste material from one urban community and the supply of potable water to a second, adjacent, downstream urban community. A proper understanding and control of the dynamic variations in river water quality would seem to be of vital importance to the organisation of the river's resource and amenity potential. Yet, although we have seen this stated many times before, automatic control, a common feature of most process industries, is notable by its absence from the water quality system. Much effort is still required to obtain suitable dynamic models for subsequent control system synthesis; a review of this field, with particular reference to the application of system identification and parameter estimation techniques, is given in BECK (1975)[4].

A knowledge and control of river water quality dynamics implies a knowledge of the dynamic characteristics of the sewer network and the wastewater treatment plant. In particular, figure 1 shows that the periodic oscillations of the consumer effluent, $\underline{v}_{c2}$, and the sudden, impulsive nature of urban runoff from any rainfall event, $\underline{v}_{02}$, makes the input raw material to the treatment plant, $\underline{v}_{23}$, a highly variable, and generally imprecisely known, quantity. Control objectives for the network and plant become, then, very much a matter of acquiring advance knowledge of the flow component, y , of the vector $\underline{v}_{23}$. Such a knowledge permits, in theory, the prior organisation of the network/plant operation for the minimisation of the polluting overflows $\underline{v}'_{24}$ and $\underline{v}'_{34}$ to the river, and ultimately, it provides a basis for establishing a truly controllable input to the treatment plant, which implies a greater flexibility in the regulation of the final treated effluent to the stream, $\underline{v}_{34}$.

Thus, given measurements of the rainfall u_i , say, at several spatial locations i (i=1,2, ..., m) on the urban land surface, we wish to determine a dynamic model which relates u_i to y , the effluent from the sewer network; this is the identification problem. Most previous investigations of this problem have divided it into two sub-problems: (i) given u_i , determine the inlet discharge to the sewer network from runoff (e.g. CHEN and SHUBINSKI (1971)[7] , PAPADAKIS and PREUL (1973)[19]) ; (ii) given all flows entering the network, determine the output flow-rate y (e.g. HARRIS (1970)[13]) . One striking feature of the currently available models is their purely deterministic and highly complex structure. Clearly these large, internally descriptive models reflect the complexity of the laws which describe the underlying physical phenomena governing the system's behaviour. Yet a theoretically complete analysis produces an unwieldy and possibly intractable model, with a multitude of parameters to be evaluated, and it is admitted that "to a varying degree most of these methods rely upon empirical relationships and experience" (PAPADAKIS and PREUL (1973)[19]) . It seems, therefore, that a stochastic model derived from time-series analysis, herein the maximum likelihood method (ÅSTRÖM and BOHLIN (1965)[2]), might yield equally usable results. Such an input/output, black box model for the relationships between u_i and y assumes little or no a priori knowledge of the physical laws of the system and takes a relatively macroscopic view of the cause/effect relationships involved. A similar approach to sewage flow modelling, after BOX and JENKINS (1970)[6], has been adopted by GOEL and LaGREGA (1972)[11]. For adaptive prediction of the effluent sewer flow the identification part of the analysis is required primarily for the determination of a suitable order and structure for the predictor model. The prediction problem can be stated as follows: at time t, given the noisy observations y(t), y(t-1), ..., of the present and past values of the influent to the plant and a model for the dynamic variations of y, we wish to make a k-step ahead prediction of y(t+k). In addition, the predictor should be simple, adaptable to changes in the process dynamics, and require little on-line instrumentation.

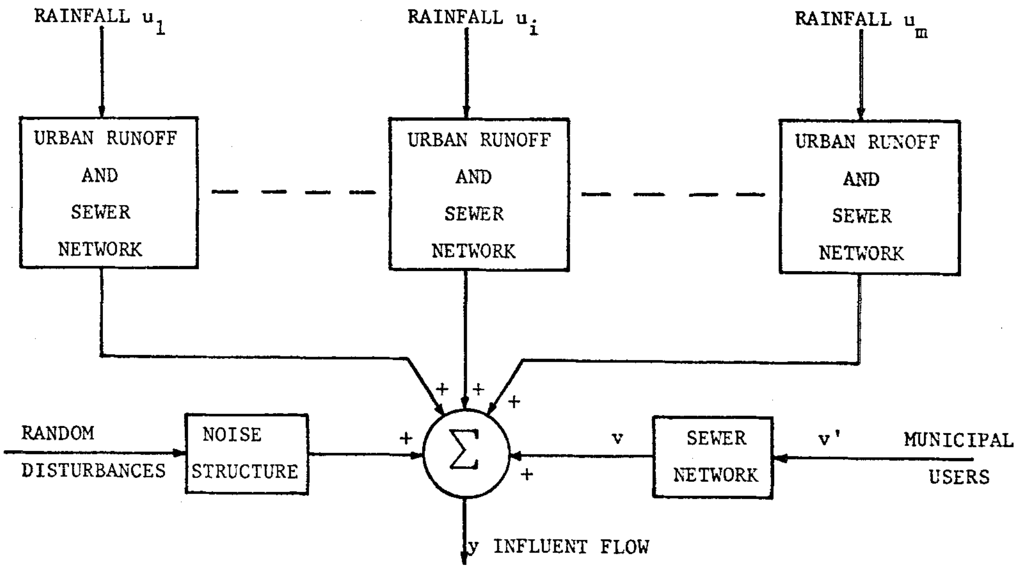


Figure 2 A schematic representation of the system for the identification of input/output flow models

3. IDENTIFICATION and ADAPTIVE PREDICTION

The class of models to be examined is one of parametric, linear, time-invariant models of a canonical form. They are black box models in the sense that they assume no knowledge of physical relationships between the system's inputs and output other than that the inputs should produce observable responses in the output.

3.1. Maximum Likelihood Identification.

In the general case, given the set of input/output data samples ($u_i(t)$, $i = 1, 2, \ldots, m$; $y(t)$; $t = 1, 2, \ldots, N$), where $u_i(t)$, $i = 1, 2, \ldots, m$ are the m input signals, $y(t)$ is the output signal and t is the time of the t^{th} sampling instant, the identification problem is to find an estimate of the parameters of the system model (ÅSTRÖM and BOHLIN (1965)[2], GUSTAVSSON (1969)[12]),

$$A(q^{-1})y(t) = \sum_{i=1}^{m} B_i(q^{-1})u_i(t) + \lambda C(q^{-1})e(t) \qquad (1)$$

in which $e(t)$ is a sequence of independent, normal (0,1) random variables and q denotes the shift operator

$$q\{y(t)\} = y(t+1) \quad \text{etc.} \qquad (2)$$

$A(q^{-1})$, $B_i(q^{-1})$, $i = 1, 2, \ldots, m$, and $C(q^{-1})$ are the polynomials

$$\left.\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \ldots + a_n q^{-n} \\ B_i(q^{-1}) &= b_{i0} + b_{i1} q^{-1} + \ldots + b_{in} q^{-n} \quad i = 1,2,\ldots,m \\ C(q^{-1}) &= 1 + c_1 q^{-1} + \ldots + c_n q^{-n} \end{aligned}\right\} \qquad (3)$$

The residual errors of eqn. (1), $\{\varepsilon(t), t = 1, 2, \ldots, N\}$, defined by

$$C(q^{-1})\varepsilon(t) = A(q^{-1})y(t) - \sum_{i=1}^{m} B_i(q^{-1})u_i(t) \qquad (4)$$

are thus an independent and normal $(0,\lambda)$ sequence. Notice that in this application the inputs of the system correspond to the rainfall u_i measured at the locations $i = 1, 2, \ldots, m$ and the output y is the influent flow to the treatment plant (see figure 2).

3.2. Adaptive Prediction

For the derivation of an adaptive predictor (WITTENMARK (1974)[22]) let us consider once again the discrete-time process, eqn.(1), which we rewrite as,

$$A(q^{-1})y(t) = \sum_{j=1}^{m'} B_j(q^{-1})v_j(t) + \lambda C(q^{-1})\, e(t) \qquad (5)$$

Here $v_j(t)$, $j = 1,2,\ldots, m'$, are auxiliary variables which aid in the characterisation of the time-series y (see eg. WITTENMARK (1974)[22], HOLST (1974)[14]); more specifically, for reasons which become apparent later, v_j may be thought of either as a suitable deterministic, synthetic signal, e.g. a periodic function, or as measurements of the process input variable u_i (eqn.(1)). Now denote the k-step ahead prediction of the output signal y based on the sampled observations y(t),y(t-1), ..., and the signals $v_j(t+k)$, $v_j(t+k-1)$, ..., ($j=1,2,\ldots, m'$), by $\hat{y}(t+k|t)$. Introducing the loss function

$$V_k(t) = E\{\varepsilon(t+k)^2\} \tag{6}$$

where E{...} is the expectation operation and $\varepsilon(t+k)$ is the prediction error,

$$\varepsilon(t+k) = y(t+k) - \hat{y}(t+k|t) \tag{7}$$

the predictor which minimises eqn.(6) is derived in ÅSTRÖM (1970)[1] for the case where the process, eqn. (5), has known A, B_j, C polynomials. According to Åström, using the identity,

$$C(q^{-1}) = A(q^{-1})\, F(q^{-1}) + q^{-k}\, G(q^{-1}) \tag{8}$$

where

$$\left.\begin{aligned} F(q^{-1}) &= 1 + f_1 q^{-1} + \ldots + f_{k-1} q^{-k+1} \\ G(q^{-1}) &= g_o + g_1 q^{-1} + \ldots + g_{n-1} q^{-n+1} \end{aligned}\right\} \tag{9}$$

The k-step ahead predictor for eqn. (5) is given by

$$\hat{y}(t+k|t) = \frac{G(q^{-1})y(t)}{C(q^{-1})} + \frac{F(q^{-1})}{C(q^{-1})} \sum_{j=1}^{m'} B_j(q^{-1})\, v_j(t+k) \tag{10}$$

Alternatively, if the polynomials A, B_j, C are unknown, they can be estimated off-line by the method outlined in section 3.1 above and then substituted into eqns. (8) and (10) to obtain the predictor.

However, for an on-line predictor of an unknown process we should prefer to identify the process and make predictions "simultaneously". In other words, we have a learning (or adaptive, self-tuning) procedure in which the parameters of the predictor, eqn. (10), are recursively estimated at each time t, rather than estimating a priori the parameters of the process, eqn. (5). WITTENMARK (1974)[22] solves this problem by transforming it into the already solved problem of an adaptive regulator (WITTENMARK (1973)[21]).

In the current application a slightly modified version of Wittenmark's algorithms are employed (HOLST (1974)[14]). The derivation is briefly as follows. Rearranging eqn. (10), and introducing $\varepsilon(t)$ from eqn. (7), we have at time t,

$$y(t) = G(q^{-1})y(t-k) - (C(q^{-1}) - 1)\hat{y}(t|t-k) + F(q^{-1}) \sum_{j=1}^{m'} B_j(q^{-1}) v_j(t) + \varepsilon(t) \quad (11)$$

Eqn. (11) is now re-written as,

$$y(t) = A^*(q^{-1})y(t-k) - B^*(q^{-1})\hat{y}(t|t-k) + \sum_{j=1}^{m'} \Gamma_j^*(q^{-1})v_j(t) + \varepsilon(t) \quad (12)$$

such that we have the identities,

$$\left.\begin{aligned} A^*(q^{-1}) &\equiv G(q^{-1}) \quad ; \quad B^*(q^{-1}) \equiv C(q^{-1}) - 1 \\ \Gamma_j^*(q^{-1}) &\equiv F(q^{-1})\, B_j(q^{-1}) \quad j=1,2,\ldots,m' \end{aligned}\right\} \quad (13)$$

with the polynomial definitions,

$$\left.\begin{aligned} A^*(q^{-1}) &= \alpha_o + \alpha_1 q^{-1} + \ldots + \alpha_{n-1} q^{-n+1} \\ B^*(q^{-1}) &= \beta_1 q^{-1} + \ldots + \beta_n q^{-n} \\ \Gamma_j^*(q^{-1}) &= \gamma_{jo} + \gamma_{j1} q^{-1} + \ldots + \gamma_{j,n+k-1}\, q^{-n-k+1} \qquad j=1,2,\ldots,m' \end{aligned}\right\} \quad (14)$$

Eqn. (12) forms the basis of the adaptive predictor algorithms, whereby (HOLST (1974) [14]):

Step 1 : Estimation

At time t, upon receipt of a new observation y(t), the parameters, $\alpha_o,\ldots,\alpha_{n-1}$, $\beta_1,\ldots,\gamma_{1o},\ldots,\gamma_{1,n+k-1},\ldots,\gamma_{m'o},\ldots,\gamma_{m',n+k-1}$ are estimated in the prediction model,

$$y(t) = A^*(q^{-1})\, y(t-k) - B^*(q^{-1})\hat{y}(t|t-k) + \sum_{j=1}^{m'} \Gamma_j^*(q^{-1})v_j(t) + \varepsilon(t) \quad (15a)$$

by the method of least squares;

Step 2 : Prediction

Using the estimates $\hat{A}^*(q^{-1})$, $\hat{B}^*(q^{-1})$, and $\hat{\Gamma}_j^*(q^{-1})$ obtained in eqn. (15a), make a k-step ahead prediction,

$$\hat{y}(t+k|t) = \hat{A}^*(q^{-1})y(t) - \hat{B}^*(q^{-1})\, \hat{y}(t+k|t) + \sum_{j=1}^{m'} \hat{\Gamma}_j^*(q^{-1})\, v_j(t+k) \quad (15b)$$

Since a least squares estimation is readily implemented in recursive form, the

the predictor is well suited to real-time applications with each step of the procedure being repeated at each sampled instant of time. Thus, if the following vectors are defined as

$$\underline{z}^T(t) = \Big[y(t-k),\ldots,y(t-k-n+1),\ -\hat{y}(t-1|t-k-1),\ldots,-\hat{y}(t-n|t-k-n),\ v_1(t),\ldots,$$

$$v_1(t-k-n+1),\ \ldots,\ v_{m'}(t),\ \ldots,\ v_{m'}(t-k-n+1)\Big]$$

$$\underline{a}(t) = \left[\alpha_0\ ,\ldots,\ \alpha_{n-1},\ \beta_1,\ \ldots,\ \beta_n,\ \gamma_{1o},\ \ldots,\gamma_{1,n+k-1},\ \ldots,\ \gamma_{m'o},\ \ldots,\ \gamma_{m',n+k-1}\right]^T$$

eqn. (15a) becomes $\qquad y(t) = \underline{z}^T(t)\ \underline{a} + \varepsilon(t) \qquad (16)$

and the well known recursive least squares algorithms for the estimates $\hat{\underline{a}}$ of $\underline{a}$ are given by (e.g. YOUNG (1969)[23], YOUNG (1974)[24]),

$$\left.\begin{aligned} \hat{\underline{a}}(t) &= \hat{\underline{a}}(t-1) - P(t-1)\underline{z}(t)\Big(1 + \underline{z}^T(t)\ P(t-1)\underline{z}(t)\Big)^{-1}\Big[\underline{z}^T(t)\ \hat{\underline{a}}\ (t-1) - y(t)\Big] && (a)\\ P(t) &= P(t-1) - P(t-1)\underline{z}(t)\Big(1+\underline{z}^T(t)\ P(t-1)\underline{z}(t)\Big)^{-1}\underline{z}^T(t)\ P(t-1) && (b)\end{aligned}\right\}(17)$$

in which $\qquad P(t) \triangleq \left[\sum_{j=1}^{t} \underline{z}(j)\underline{z}^T(j)\right]^{-1} \qquad (18)$

Remarks

(i) Estimation bias: If $C(q^{-1}) = 1$ in eqn. (5) it can be shown that $\varepsilon(t) = \lambda e(t)$ and, providing that $e(t)$ is not correlated in time and is independent of $y(t)$, $v_j(t)$ $(j=1,2,\ldots,m')$, the estimates $\hat{\underline{a}}$ are unbiased. In practice however, where it is more probable that $C(q^{-1}) \neq 1$, the predictor still appears to behave nicely; notice that if the estimates $\hat{\underline{a}}$ converge to $\underline{a}$, it implies that certain covariances and cross-covariances equal zero (HOLST (1974)[14], WITTENMARK (1974)[22]).

(ii) Time-varying parameters: In view of the nature of this particular application (see section 4.2) it is useful to allow for the estimation of time-varying parameters. One method of achieving this is with exponential weighting of past data (e.g. YOUNG (1969)[23], EYKHOFF (1974)[8]); introducing a weighting factor μ, where $0 \ll \mu < 1$, the recursive least squares algorithms of eqn. (17) are modified to give (EYKHOFF (1974)[8]),

$$\left.\begin{aligned} \hat{\underline{a}}(t) &= \hat{\underline{a}}(t-1) - P(t-1)\underline{z}(t)\Big(\mu + \underline{z}^T(t)\ P(t-1)\underline{z}(t)\Big)^{-1}\Big[\underline{z}^T(t)\hat{\underline{a}}(t-1) - y(t)\Big] && (a)\\ P(t) &= \frac{1}{\mu}\Big\{P(t-1) - P(t-1)\underline{z}(t)\Big(\mu + \underline{z}^T(t)\ P(t-1)\underline{z}(t)\Big)^{-1}\underline{z}^T(t)\ P(t-1)\Big\} && (b)\end{aligned}\right\}(19)$$

where now $\qquad P(t) \triangleq \left[\sum_{j=1}^{t} \mu^{t-j}\underline{z}(j)\underline{z}^T(j)\right]^{-1}$

(iii) Auxiliary variables: When measurements of any process inputs, e.g. rainfall in this instance, are to be used as auxiliary variables, note that the measurements $v_j(t+k)$, $v_j(t+k-1),\ldots$, are required at time t for prediction according to eqn. (15b).

Clearly, additional information of this kind is only of real benefit providing $\gamma_{jo} = \gamma_{j1} = \ldots = \gamma_{jk} = 0$, i.e. there exists a pure time delay τ in the input/output process dynamics where $\tau \geq k$. Other forms of auxiliary variables are typically (here) average weekly and daily periodic dry-weather flow profiles.

A **full** analysis of the adaptive predictor of eqn. (15) will appear in a forthcoming report (HOLST (1975)[15]).

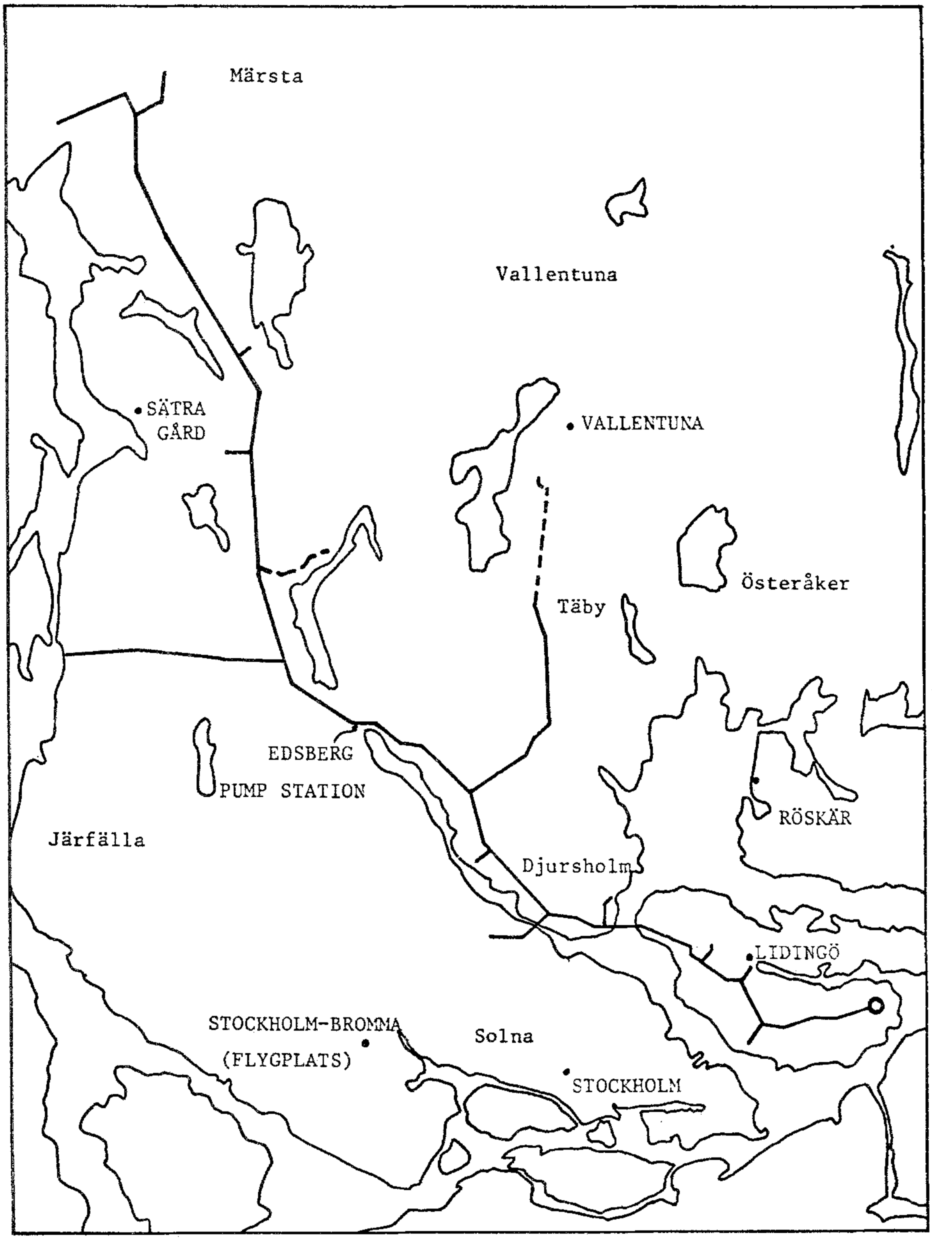


Figure 3 The Käppala sewer tunnel system

• = Meteorological station
o = Käppala treatment plant

4. A CASE STUDY - THE KÄPPALA PLANT, STOCKHOLM.

Figure 3 shows the sewer tunnel system which collects waste water from an area of some 1191 km^2 covering part of the City of Stockholm and it's surrounding districts and serving a population of 290,000. The observed influent flow-rate to the Käppala plant is analysed for the period between October 1st (08.00 hrs) and October 31st (07.00 hrs), a total of 720 hourly samples. For the same interval data are available (from the Swedish Institute of Meteorology and Hydrology, Stockholm) for the rainfall measured at four stations, Röskär, Stockholm-Bromma, Stockholm, and Lidingö (see figure 3). Because of several imperfections in the data, notably the presence of large pumping disturbances in the flow-rate measurements and the low frequency of the sampling in the rainfall measurements, e.g. once, twice, per day, it is particularly difficult to obtain suitably identified input/output model for the urban runoff/sewer effluent dynamics.

4.1. Identification.

The problems of the data and of maximum likelihood identification are discussed in greater detail elsewhere (BECK (1974)[3]). Briefly, it is necessary to low-pass filter the data before any useful analysis can be attempted. A rainfall-runoff flow (RRF) model, which incorporates a deterministic component describing the dry-weather weekly flow periodicity and a stochastic model for rainfall-runoff flows, is identified:

$$y_r(t) = -a_1 y_r(t-1) + b_3\ u(t-3) + b_7(t-7) + \lambda(e(t) + c_1 e(t-1)) \qquad (20)$$

where

$$y_r(t) = y(t) - \bar{y}_w\ (t) \qquad (21)$$

$y_r(t)$ may be considered as the excess flow resulting from runoff sources ($m^3\ s^{-1}$), $y(t)$ is the (low pass filtered) observation of the treatment plant influent ($m^3 s^{-1}$), and $\bar{y}_w(t)$ is a mean weekly dry-weather flow pattern ($m^3\ s^{-1}$) computed a priori from the (low-pass filtered) data on $y(t)$; the input $u(t)$ is a signal representing a single (low-pass filtered), spatially-averaged rainfall time-series (mm). The estimates of the parameters are given by,

$$a_1 = -0.739 \pm 0.028\ ,\ b_3 = 0.063 \pm 0.032\ ;\ b_7 = 0.086 \pm 0.031\ ;$$
$$c_1 = 0.984 \pm 0.001\ ;\ \lambda = 0.027 \pm 0.001.$$

Figure 4 shows the data, deterministic output response, and model error for the deterministic component of eqn. (20) substituted in eqn. (21) to obtain $y(t)$. The effects of the pumping disturbances are visible as regularly placed (approx. once daily) spikes in the observed output and model error sequences; there are also recognisable daily and weekly fluctuations in $y(t)$ with additional peak responses from the

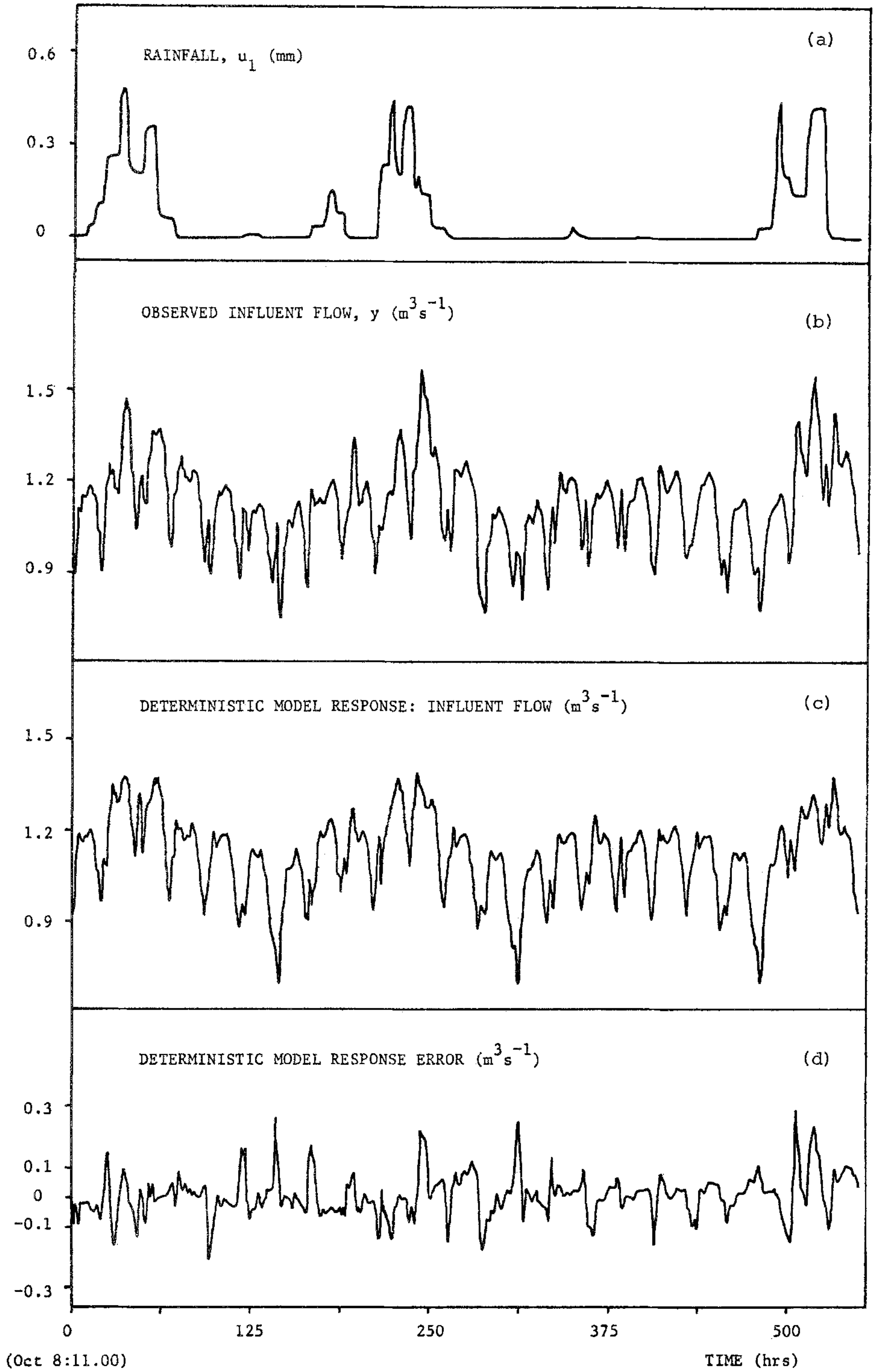


Figure 4 Comparison between the deterministic response of an RRF model (superimposed upon a mean weekly profile) and the observed influent flow

rainfall-runoff. The residuals $\varepsilon(t)$ of the stochastic model, eqn. (20), have a standard deviation (λ) of $\pm$ 0.027 ($m^3 s^{-1}$), a large part of which is contributed by the pumping disturbances. Notice that these residuals are also the one-step ahead prediction errors of the model, eqn. (20), c.f. section 4.2. In short, there are considerable difficulties in the estimation of the $B(q^{-1})$ and $C(q^{-1})$ polynomials and the model is only as good as the quality of the data, which, as we have indicated, leaves much to be desired.

4.2. Adaptive prediction.

In an initial study of the feasibility of an on-line adaptive predictor it would have been advantageous to identify the process, eqn. (1) by maximum likelihood methods, compute the optimal (minimum variance) predictor through eqns. (8) and (10), and then compare the adaptive predictor, eqn. (15), for the unknown process with the optimal predictor for the known process. The preceding remarks do not really support the use of such a procedure, although, at the very least, it is possible to conclude that a suitable predictor would have low-order $A^*(q^{-1})$ and $B^*(q^{-1})$ polynomials.

We consider the case where we have only (low-pass filtered) measurements of the influent flow-rate to the treatment plant, $y(t)$.

One-step ahead prediction

It is found that an appropriate structure for the one-step ahead predictor ($k=1$) is defined by,

$$\hat{y}(t+1|t) = \alpha_o y(t) - \beta_1 \hat{y}(t|t-1) + \gamma_{1o} v_1(t+1) + \gamma_{11} v_1(t) + \gamma_{2o} v_2(t+1) + \gamma_{21} v_2(t) \qquad (22)$$

where $v_1(t) = \bar{y}_w(t)$ and $v_2(t) = \bar{y}_d(t)$ are respectively synthetic, deterministic, weekly and daily dry-weather flow profiles computed a priori from the data. Given the a priori estimates $\underline{\hat{a}}(0)$,

$\hat{\alpha}_o(0) = 1.20$ $\quad$ $\hat{\gamma}_{1o}(0) = 0.47$ $\quad$ $\hat{\gamma}_{2o}(0) = 0.37$

$\hat{\beta}_1(0) = 0.42$ $\quad$ $\hat{\gamma}_{11}(0) = -0.27$ $\quad$ $\hat{\gamma}_{21}(0) = -0.30$

and a priori diagonal matrix $P(0) = (0.1)\ I$, where I is the identity matrix, for the algorithms of eqn. (10) (exponential weighting factor $\mu = 0.995$), the results of figures 5 and 6 are obtained for the adaptive predictor of eqn (22). Despite the inevitable errors from the pumping disturbances, the one-step ahead prediction is, perhaps surprisingly, remarkably close to the observed data; in particular, the runoff from rainfall is well described, even though the predictor is operating in the absence of any information on these events. Notice that the recursive parameter estimates are relatively insensitive to the intermittent effects of runoff, which are

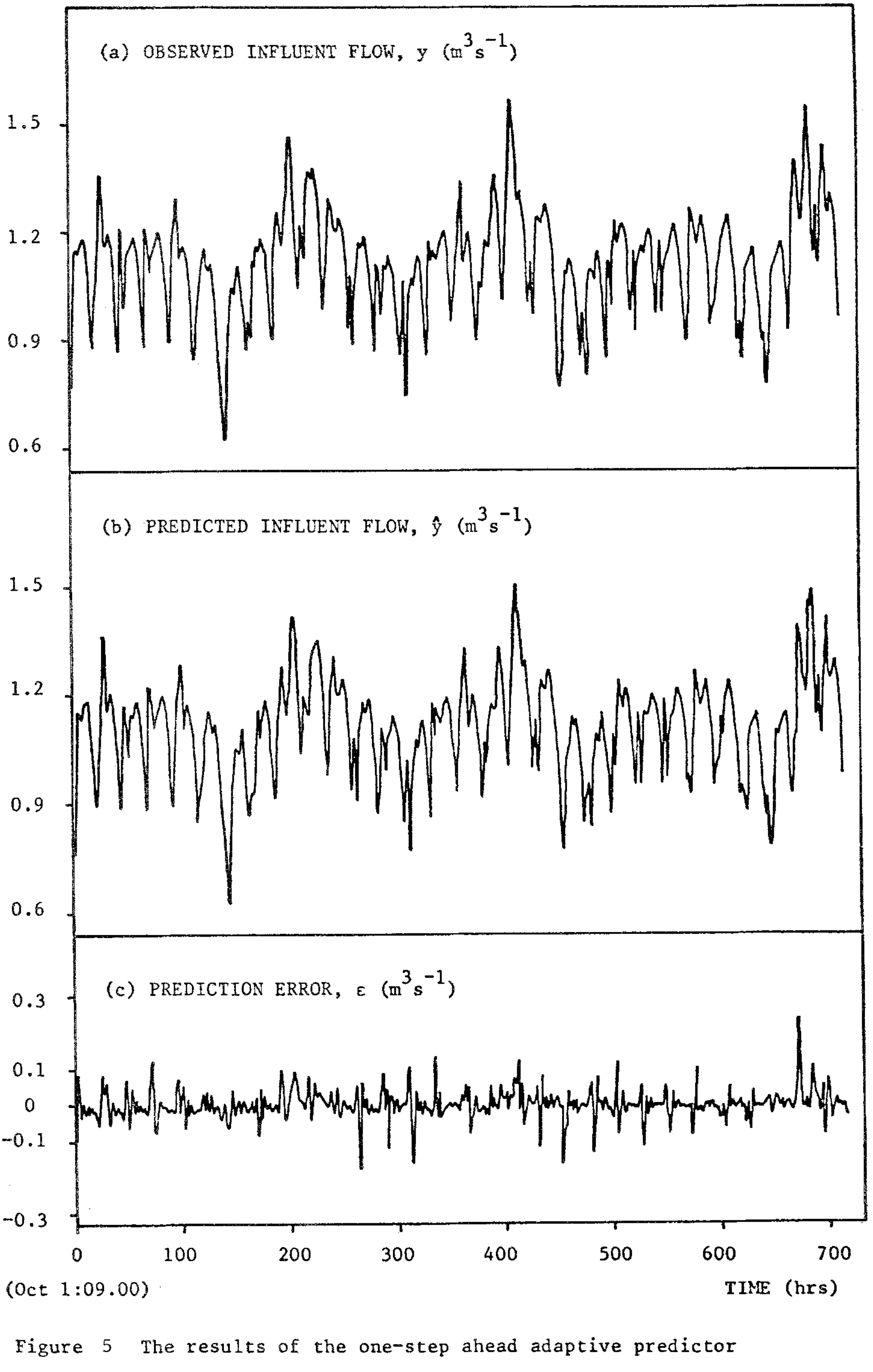


Figure 5 The results of the one-step ahead adaptive predictor

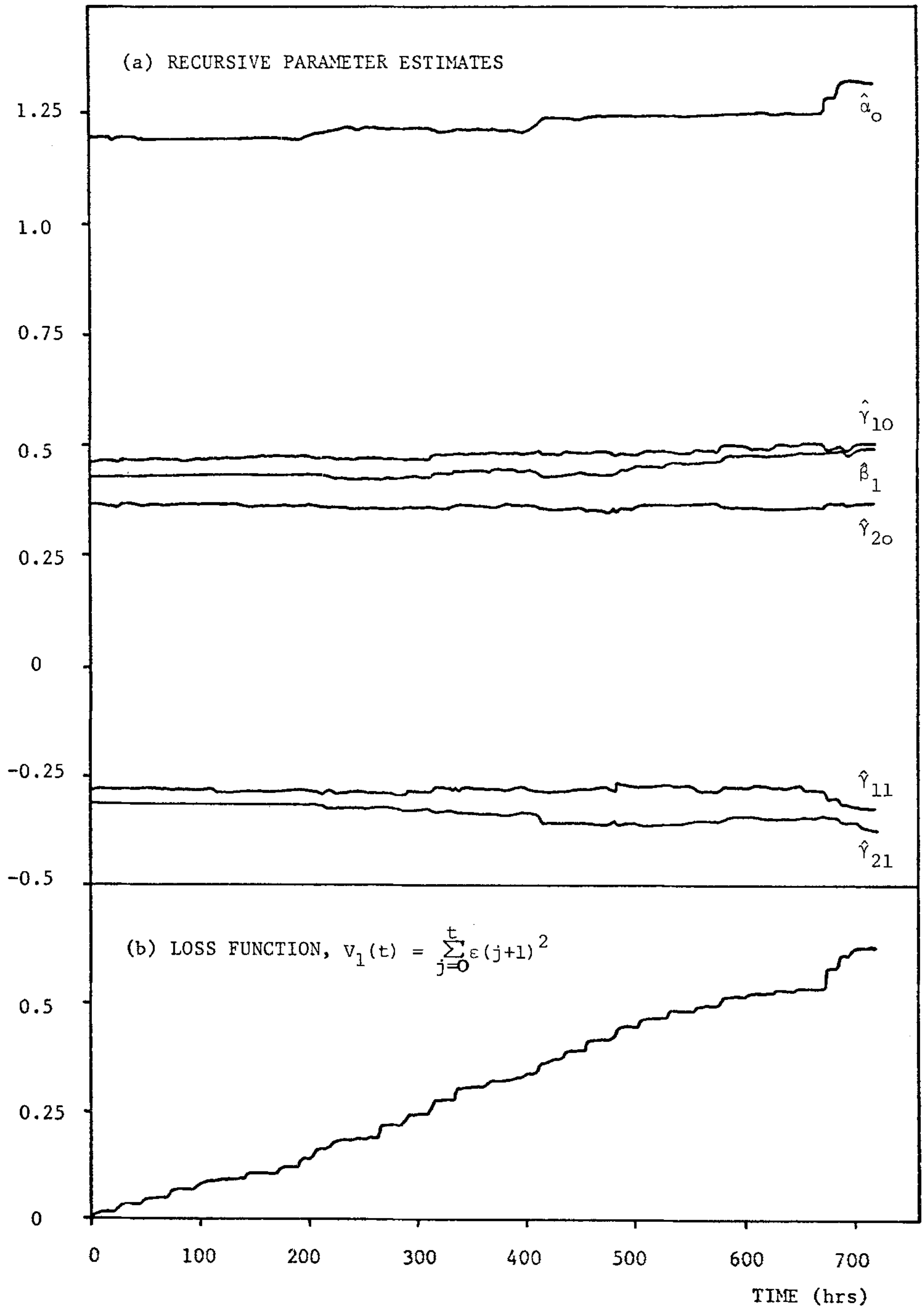


Figure 6 The results of the one-step ahead adaptive predictor

tantamount to an apparent change in the process dynamics. It is argued that a spatially-distributed rainfall event produces a temporally distributed response in the plant influent flow, $y(t)$. Hence the predictor is capable of quickly recognising such a dynamic disturbance through $y(t)$ and $\hat{y}(t|t-1)$ in eqn. (22) and significant adaptation of the parameters, e.g. α_o and β_1, becomes redundant.

The standard deviation of the predictor errors ε is $\pm$ 0.029 ($m^3 s^{-1}$), which compares well with that for the residuals of the RRF model.

Multiple-step ahead prediction.

Both a two-step and a four-step ahead predictor have been analysed; some results for the latter are given in BECK (1974)[3]. The salient features of the four-step ahead prediction results are as follows. The prediction $\hat{y}(t+4|t)$ is found to be independent of the previous prediction $\hat{y}(t+3|t-1)$ and there is altogether a stronger dependence upon the auxiliary variables, especially the weekly component $v_1(t)$. The runoff process is not well predicted and the resultant peak flows are substantially attenuated. Simultaneously, the estimate of α_o is considerably adapted in order to track the changing properties of the system's dynamics, of which the structure of the four-step ahead predictor is relatively "ignorant". However after such temporary disturbances $\hat{\alpha}_o$ returns slowly to its steady-state value for dry-weather conditions thus giving a good illustration of the adaptability of the predictor (see figure 7).

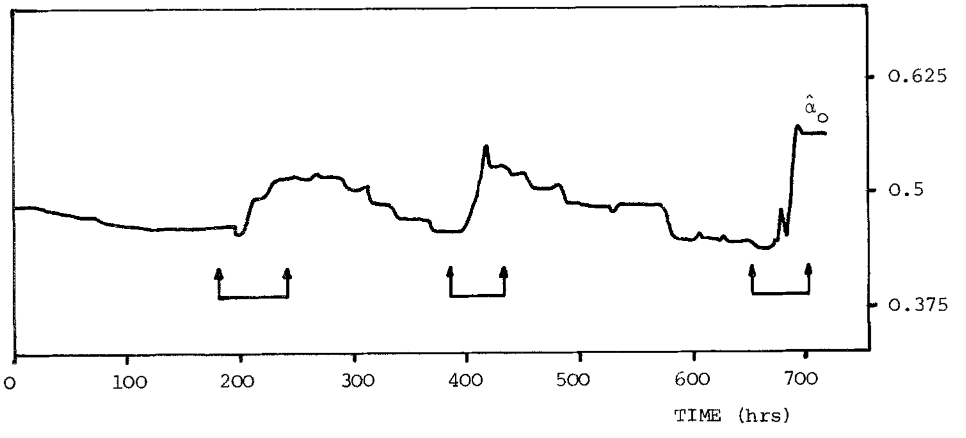


Figure 7 Recursive estimates of α_o for a four-step ahead adaptive predictor

Major rainfall events are denoted ⌊⌋

5. SOME COMMENTS ON ADAPTIVE PREDICTION AND CONTROL APPLICATIONS.

Time-varying parameters: Exponential weighting of past data (EWP) is one method of allowing for the recursive estimation of time-varying parameters. But it is a method which does not allow any prior selection between those parameters which may be expected to be time-varying and those which are not. For instance, we might expect the $\Gamma^*_j(q^{-1})$ polynomials for the auxiliary variables to be constant, while the $A^*(q^{-1})$ and $B^*(q^{-1})$ polynomials could be expected to vary much more. Furthermore, the EWP method is only appropriate for the case of slowly-varying parameters, yet the rainfall-runoff is a relatively fast, almost impulsive, disturbance of the system. It seems likely, therefore, that a more sophisticated, but easily programmed, technique of time-variable parameter estimation could improve the operation of an adaptive predictor. For example, if the parameters $\underline{a}(t)$ are assumed to vary in a simple random-walk manner, a modified version of the least squares algorithms, eqn. (17), are given by (YOUNG (1969)[23], YOUNG (1974)[24]),

$$\left.\begin{aligned} \hat{\underline{a}}(t) &= \hat{\underline{a}}(t-1) - \left[P(t-1)+D\right]\underline{z}(t)\left\{1+\underline{z}^T(t)\left[P(t-1)+D\right]\underline{z}(t)\right\}^{-1}\cdot \\ &\qquad \cdot\left[\underline{z}^T(t)\hat{\underline{a}}(t-1)-y(t)\right] & \text{(a)} \\ P(t) &= P(t-1) - \left[P(t-1)+D\right]\underline{z}(t)\left\{1+\underline{z}^T(t)\left[P(t-1)+D\right]\underline{z}(t)\right\}^{-1}\cdot \\ &\qquad \cdot\,\underline{z}^T(t)\left[P(t-1)+D\right] & \text{(b)} \end{aligned}\right\} \quad (24)$$

where D is a positive, definite, (usually) diagonal matrix which reflects the expected rates of change in the parameters $\underline{a}$ (c.f. eqn. (19)).

Auxiliary variables and additional measurements: For higher values of k the use of rainfall measurements would be of benefit to the predictor. Note that from eqn. (20) there is good reason to believe that the Käppala sewer system has a pure time delay $\tau \approx 3$ (hrs). In a practical situation, therefore, an on-line predictor could cope with a delay of up to 3 hours in the receipt of rainfall measurements, although the time to the peak runoff response $\tau_p (> \tau)$ is perhaps a more critical measure for determining the benefits of using these data. It is, after all, the peak flows which cause the greatest upset to the operational control of the network and treatment plant.

Sewer network flow control: Sewer flow control is exercised largely by the installation of storage tanks in the network, although a small amount of storage is available in the sewers themselves (see e.g. PEW et al (1973)[20]). This large-scale control problem seems to be amenable to the hierarchical approach (e.g. LABADIE et al (1975)[16]). (Similar approaches have been applied to the analogous problem of potable water supply network control, FALLSIDE and PERRY (1975)[9], FALLSIDE et al (1975)[10]).

Wastewater treatment plant control: Currently there are many more problems than solutions in wastewater treatment plant control (e.g. OLSSON et al (1973)[18]). However, the prediction and control of the influent flow has wide-ranging implications not only for the treatment plant but also for the whole water quality system (YOUNG and BECK (1974)[25], BECK (1975)[4]): the essential point is that flow control alleviates gross overloading of the water quality system and, at the same time, it regulates the dynamic behaviour of many of the unit processes of wastewater treatment. A first study of flow equalisation, i.e. the modulation of diurnal variations, shows that significant benefits might accrue, for example, in the operation of sedimentation processes (LaGREGA and KEENAN (1974)[17]). Of course, flow prediction is only a part of the problem of characterising the plant input raw material; in addition, it is necessary to describe the quality of the sewage flow. Recently, BERTHOUEX et al (1975)[5] have used similar time-series analysis techniques (BOX and JENKINS (1970)[6]) for the modelling of plant input biochemical oxygen demand variations.

6. CONCLUSIONS

The major limitation in this study of the adaptive prediction of urban sewer flows has been the poor quality of the data. In any future study it can be expected that, while pumping disturbances may not be eliminated completely, better data would be available for analysis. With a view to on-line implementation of the predictor it would,therefore, be important to site the flow-measuring equipment at a carefully chosen location.

One-step ahead forecasts of the plant influent flow are obtained from an adaptive predictor which closely approaches the satisfaction of the practical constraints on the system: namely, as little automated instrumentation as possible should be assumed. The salient feature of the black box model for the predictor is its simplicity and compactness when compared with other, largely deterministic, models based on the physical laws of the system behaviour. For it should be remembered that the currently existing technology of wastewater treatment favours the simple rather than the sophisticated.

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REFERENCES

1. ÅSTRÖM K.J., "Introduction to stochastic control theory", Academic Press, New York, 1970.
2. ÅSTRÖM K.J., and BOHLIN T., "Numerical identification of linear dynamic systems from normal operating records", Proc. IFAC symp. Theory of self-adaptive control systems, Teddington, England, 1965.
3. BECK M.B., "The identification and prediction of urban sewer flows - a preliminary study", Report 7432(C), Lund Inst. Techn., Div. Aut. Contr., 1974.

4. BECK, M.B., "Dynamic modelling and control applications in water quality maintenance", Report CUED/F - Contr/TR92, Control Group, Univ. Eng. Dept., Cambridge, 1975.
5. BERTHOEUX P.M., HUNTER W.G., PALLESEN L.C., and SHIH C-Y., "Modelling sewage treatment plant influent data", Proc. A.S.C.E., J. Env. Eng. Div., Vol.101, No. EE1, pp 127-138, Feb. 1975.
6. BOX G.E.P., and JENKINS G.M., "Time-series analysis, forecasting and control", Holden-Day, San Francisco, (1970).
7. CHEN C.W. and SCHUBINSKI R.P., "Computer simulation of urban storm water runoff", Proc. A.S.C.E., J. Hydr. Div., Vol.97, No.HY2, pp 289-301, Feb. 1971.
8. EYKHOFF P., "System identification-parameter and state examination", John Wiley and Sons, London, 1974.
9. FALLSIDE F., and PERRY P.F., "Hierarchical optimisation of a water supply network", Proc. IEE, Vol. 122, No.2., pp 202-208, Feb. 1975.
10. FALLSIDE F., PERRY P.F., and RICE P.D., "On-line prediction of consumption for water supply network control", Reprints 6th IFAC Congress, Boston, Massachusetts, Aug. 1975.
11. GOEL A.L., and LaGREGA M.D., "Stochastic models for forecasting sewage flows", Paper presented at 41st Natl. Meeting, Operations Res. Soc. Amer., New Orleans, 1972.
12. GUSTAVSSON I.,"Parametric identification of multiple input, single output linear dynamic systems", Report 6907, Lund Inst. Techn., Div. Aut. Contr., 1969.
13. HARRIS G.S., "Real time routing of flood hydrographs in storm sewers", Proc. A.S.C.E., J. Hydr. Div., Vol.96, No.HY6, pp 1247-1260, June, 1970.
14. HOLST J., "The use of self-tuning predictors for forecasting the loading of a power system" (in Swedish), Report 7433C,Lund Inst. Techn., Div. Aut. Contr., 1974.
15. LABADIE J.W., GRIGG N.S., and BRADFORD B.H., "Automatic control of large-scale combined sewer systems", Proc. A.S.C.E., J. Env. Eng. Div., Vol.101, No. EE1, pp. 27-39, Feb. 1975.
17. LaGREGA M.D., and KEENAN J.D., "Effects of equalising wastewater flows", J.W.P.C.F., Vol.46, No.1, p.p. 123-132, Jan. 1974.
18. OLSSON, G., DAHLQVIST K-I., EKLUND K., and ULMGREN L., "Control problems in wastewater treatment", Report, Axel Johnson Inst. for industr. Res., Nynäshamn, Sweden, 1973.
19. PAPADAKIS C.N., and PREUL H.C., "Testing of methods for determination of urban runoff", Proc. A.S.C.E., J. Hydr. Div., Vol.99 No. HY9, pp. 1319-1335, Sept.1973.
20. PEW K.A., CALLERY R.L., BRANDSETTER A., and ANDERSON J.J., "Data acquisition and combined sewer controls in Cleveland", J.W.P.C.F., Vol.45, No.11, pp.2276-2289, Nov.1973.
21. WITTENMARK B., "A self-tuning regulator", Report 7311, Lund. Inst. Techn.,Div. Aut. Contr., 1973.
22. WITTENMARK B., "A self-tuning predictor", I.E.E.E., Trans. Aut. Contr., Vol.AC-19 No.6, pp. 848-851, Dec. 1974.
23. YOUNG P.C., "Applying parameter estimation to dynamic systems - Part I", Control Engng., Vol.16, No.10, pp. 119-125, Oct.1969.
24. YOUNG P.C., "A recursive approach to time-series analysis", Bull Inst. Maths. Appl. (IMA), Vol.10, Nos.5/6, pp. 209-224, May/June, 1974.
25. YOUNG P.C., and BECK M.B., "The modelling and control of water quality in a river system", Automatica, Vol.10, No.5, pp. 455-468, Sept. 1974.