Section 4

EXECUTION TIMES FOR EISPACK

In this section we display approximate execution times of the individual subroutines and various paths in EISPACK measured with sample matrices on many of the computer systems for which the subroutines have been certified. The elements of the sample matrices are random numbers sampled from uniform distributions. Tables of execution times appear in the first subsection, and considerations regarding the reliability of the timing measurements, their dependence upon the elements of the matrices, and the extrapolation of these results to other machines are discussed in later subsections. The section concludes with listings of the program segments that generate the sample matrices.

Section 4.1

TABLES OF EXECUTION TIMES

For each of 15 computer systems there follow three tables displaying sample execution times respectively for complex general matrices, real general matrices, and those matrices that can be reduced to real symmetric tridiagonal form. A further table follows comparing the performances of the computer systems with each other. Finally, one additional table is included reporting path timings on the IBM 370/195 when the control program EISPAC is used.

The tables report execution times both for the individual subroutines and for many of the recommended paths. Each column of a table reports times for a matrix of different order. The entries in any column are expressed as multiples of the time unit that appears at the head of the column. The time unit chosen is the absolute time for subroutine COMHES, ELMHES, or TRED1 as appropriate. A dash appears in place of an entry representing an execution time too small to be measured within the resolution of the clock.

For a subroutine that computes some eigenvalues or eigenvectors, the tabulated time represents that required for the subroutine to compute all eigenvalues or eigenvectors. The times for the eigenvector subroutines INVIT, CINVIT, and TINVIT are measured when the eigenvalues are provided by the subroutines HQR, COMLR, and BISECT respectively; these times could be significantly longer if less accurate eigenvalue subroutines were substituted.

For paths where a driver subroutine exists, the time for the actual call of the driver subroutine is recorded. For other paths, the entries are built by summing the times for the separate subroutines. The entries for those paths that compute partial eigensystems are expressed as the sum of two terms: the first term is the base time for the path and the second term is that part which depends upon the number of eigenvalues and eigenvectors computed. For complex general matrices, this latter term is the product of the estimated time to compute one eigenvector and M, the number of eigenvectors computed. For real general matrices, it is the product of the estimated time to compute one column of the eigenvector matrix and M, the number of columns required to store the requested eigenvectors. For matrices that can be reduced to real symmetric tridiagonal form, it is either the product of the estimated time to compute one eigenvalue-eigenvector pair and M, the number of pairs computed, or the product of the estimated time to compute one eigenvalue and M, the number of eigenvalues computed. Program segments that generate the sample matrices are listed in Section 4.5.

TABLE 4

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: IBM 370/195, Fortran H, OPT=2
ARGONNE NATIONAL LABORATORY

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.002	.014	.099	.33	.86
CBAL	.17	.07	.03	.02	.01
CBABK2	.11	.05	.03	.02	.01
COMHES	1	1	1	1	1
COMBAK	.46	.53	.61	.66	.91
CORTH	2.3	2.6	3.0	3.0	2.8
CORTB	1.1	1.3	1.4	1.5	1.3
COMQR2	15	14	14	14	13
COMLR2	11	10	9.3	9.1	8.0
COMQR	7.7	5.8	4.8	4.4	3.8
COMLR	6.9	4.9	3.5	3.1	2.6
CINVIT	7.4	5.5	4.3	3.7	3.4
PATHS					
CG (CBAL,CORTH,COMQR2,CBABK2)	18	17	17	17	15
CG (CBAL,CORTH,COMQR)	10	8.5	7.8	7.5	6.7
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	10+.86M	8.5+.34M	7.8+.14M	7.5+.09M	6.6+.06M

TABLE 5

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: IBM 370/195, Fortran H, OPT=2
ARGONNE NATIONAL LABORATORY

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.001	.006	.047	.15	.36
BALANC	.40	.13	.05	.03	.02
BALBAK	.17	.07	.03	.02	.02
ELMHES	1	1	1	1	1
ELTRAN	.28	.13	.06	.04	.03
ELMBAK	.63	.59	.60	.60	.64
ORTHES	2.6	2.7	2.7	2.6	2.6
ORTRAN	.72	.60	.54	.52	.51
ORTBAK	1.0	.91	.83	.79	.78
HQR2	19	15	13	12	12
HQR	12	7.9	5.4	4.7	4.2
INVIT	6.3	5.2	4.3	3.9	3.7
PATHS					
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	20	17	14	14	13
RG (BALANC,ELMHES,HQR)	13	9.0	6.6	5.8	5.2
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	13+.71M	9.0+.29M	6.5+.12M	5.8+.08M	5.3+.05M

TABLE 6

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: IBM 370/195, Fortran H, OPT=2
ARGONNE NATIONAL LABORATORY

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.001	.007	.041	.12	.27
TRED1	1	1	1	1	1
TRED2	1.6	1.8	2.0	2.1	2.2
TRED3	.98	1.1	1.1	1.2	1.2
TRBAK1	1.2	1.4	1.6	1.7	1.8
TRBAK3	.85	1.1	1.3	1.5	1.5
HTRIDI	2.1	2.6	3.0	3.2	3.4
HTRID3	2.2	2.8	3.3	3.6	3.8
HTRIBK	2.3	3.4	4.3	4.9	5.1
HTRIB3	2.6	4.0	5.1	5.7	6.0
FIGI	.13	.05	.02	.01	.00
FIGI2	.18	.10	.05	.03	.02
BAKVEC	.10	.05	.03	.02	.01
TQL2	3.4	3.5	3.3	3.2	3.2
IMTQL2	3.8	3.8	3.7	3.5	3.5
TQLRAT	1.2	.67	.36	.24	.18
TQL1	2.1	1.5	.94	.66	.52
IMTQL1	2.2	1.6	1.0	.73	.57
BISECT	12	8.1	4.9	3.5	2.7
TRIDIB	12	7.7	4.6	3.3	2.5
IMTQLV	2.4	1.7	1.1	.77	.60
RATQR	2.6	2.2	1.5	1.2	.93
TINVIT	.95	.64	.40	.30	.30
TSTURM	13	8.6	5.3	3.8	3.0
PATHS					
RS (TRED2,TQL2)	5.0	5.3	5.3	5.3	5.3
RS (TRED1,TQLRAT)	2.2	1.7	1.3	1.2	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+1.4M	1+.51M	1+.17M	1+.09M	1+.06M
TRED1,BISECT	1+1.2M	1+.40M	1+.12M	1+.06M	1+.03M
CH (HTRIDI,TQL2,HTRIBK)	8.1	9.5	11	11	12
CH (HTRIDI,TQLRAT)	3.3	3.3	3.3	3.5	3.6
HTRIDI,BISECT,TINVIT,HTRIBK	2.1+1.5M	2.6+.61M	3.0+.24M	3.2+.15M	3.4+.10M
HTRIDI,BISECT	2.1+1.2M	2.6+.40M	3.0+.12M	3.2+.06M	3.4+.03M

TABLE 7

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: IBM 360/75, Fortran H, OPT=2
UNIVERSITY OF ILLINOIS

	ORDER OF MATRIX			
SUBROUTINE	N=20	N=40	N=60	N=80
Time Unit (Sec)	.20	1.6	5.2	12
CBAL	.04	.02	.01	.01
CBABK2	.05	.01	.01	.01
COMHES	1	1	1	1
COMBAK	.48	.52	.55	.55
CORTH	2.2	2.3	2.3	2.4
CORTB	1.2	1.2	1.2	1.3
COMQR2	15	14	14	14
COMLR2	7.7	6.6	6.5	6.3
COMQR	5.7	4.7	4.5	4.4
COMLR	3.5	2.4	2.3	2.1
CINVIT	3.2	2.4	2.2	2.1
PATHS				
CG (CBAL,CORTH,COMQR2,CBABK2)	17	17	16	16
CG (CBAL,CORTH,COMQR)	7.9	7.1	6.8	6.7
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	7.9+.22M	7.1+.09M	6.8+.06M	6.7+.04M

TABLE 8

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: IBM 360/75, Fortran H, OPT=2
UNIVERSITY OF ILLINOIS

SUBROUTINE	ORDER OF MATRIX N=20	N=40	N=60	N=80
Time Unit (Sec)	.063	.49	1.6	3.9
BALANC	.11	.04	.02	.02
BALBAK	.03	.02	.02	.01
ELMHES	1	1	1	1
ELTRAN	.05	.03	.02	.01
ELMBAK	.65	.60	.61	.60
ORTHES	2.5	2.4	2.3	2.3
ORTRAN	.78	.69	.68	.67
ORTBAK	1.1	1.1	1.1	1.0
HQR2	17	14	14	13
HQR	8.0	5.8	5.2	4.6
INVIT	3.7	3.1	2.9	2.7
PATHS				
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	18	15	15	14
RG (BALANC,ELMHES,HQR)	9.1	6.8	6.2	5.7
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	9.1+.22M	6.8+.09M	6.2+.06M	5.7+.04M

TABLE 9

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: IBM 360/75, Fortran H, OPT=2
UNIVERSITY OF ILLINOIS

SUBROUTINE	ORDER OF MATRIX N=20	N=40	N=60	N=80
Time Unit (Sec)	.072	.48	1.6	3.5
TRED1	1	1	1	1
TRED2	1.8	1.9	1.9	2.0
TRED3	.96	.93	1.0	1.0
TRBAK1	1.3	1.5	1.5	1.5
TRBAK3	1.2	1.3	1.4	1.4
HTRIDI	2.9	3.2	3.4	3.5
HTRID3	3.0	3.3	3.6	3.7
HTRIBK	4.1	4.6	5.0	5.1
HTRIB3	4.4	5.0	5.5	5.6
FIGI	-	-	.01	.00
FIGI2	-	-	.01	.01
BAKVEC	-	-	.02	.01
TQL2	4.9	5.0	4.8	4.8
IMTQL2	5.0	5.0	4.9	4.9
TQLRAT	.58	.32	.21	.15
TQL1	1.3	.70	.47	.36
IMTQL1	1.2	.71	.50	.38
BISECT	4.9	2.7	1.8	1.4
TRIDIB	5.2	2.8	1.9	1.5
IMTQLV	1.3	.75	.49	.38
RATQR	2.2	1.4	.99	.79
TINVIT	.49	.34	.25	.28
TSTURM	5.5	3.1	2.2	1.7
PATHS				
RS (TRED2,TQL2)	6.7	6.9	6.8	6.8
RS (TRED1,TQLRAT)	1.4	1.3	1.2	1.1
TRED1,BISECT,TINVIT,TRBAK1	1+.33M	1+.11M	1+.06M	1+.04M
TRED1,BISECT	1+.24M	1+.07M	1+.03M	1+.02M
CH (HTRIDI,TQL2,HTRIBK)	12	12	13	13
CH (HTRIDI,TQLRAT)	3.5	3.5	3.6	3.7
HTRIDI,BISECT,TINVIT,HTRIBK	2.9+.47M	3.2+.19M	3.4+.12M	3.5+.09M
HTRIDI,BISECT	2.9+.24M	3.2+.07M	3.4+.03M	3.5+.02M

TABLE 10

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: IBM 370/168, Fortran H, OPT=2
UNIVERSITY OF MICHIGAN

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.008	.070	.57	2.0	4.7
CBAL	.08	.03	.01	.01	.01
CBABK2	.07	.03	.01	.01	.01
COMHES	1	1	1	1	1
COMBAK	.47	.52	.54	.55	.58
CORTH	2.3	2.3	2.2	2.2	2.2
CORTB	1.3	1.3	1.2	1.2	1.2
COMQR2	19	16	15	14	14
COMLR2	9.1	7.7	6.5	6.3	6.1
COMQR	8.1	5.9	4.8	4.6	4.3
COMLR	4.9	3.3	2.3	2.1	1.9
CINVIT	4.5	3.0	2.2	2.0	1.9
PATHS					
CG (CBAL,CORTH,COMQR2,CBABK2)	21	19	17	17	16
CG (CBAL,CORTH,COMQR)	11	8.1	7.0	6.7	6.4
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	11+.58M	8.2+.21M	7.0+.09M	6.7+.05M	6.4+.04M

TABLE 11

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: IBM 370/168, Fortran H, OPT=2
UNIVERSITY OF MICHIGAN

SUBROUTINE	ORDER OF MATRIX N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.003	.020	.16	.56	1.4
BALANC	.20	.07	.03	.02	.01
BALBAK	.14	.06	.03	.02	.01
ELMHES	1	1	1	1	1
ELTRAN	.12	.05	.02	.01	.01
ELMBAK	.56	.58	.59	.59	.60
ORTHES	2.7	2.5	2.4	2.3	2.3
ORTRAN	.87	.81	.78	.77	.76
ORTBAK	1.4	1.3	1.2	1.2	1.2
HQR2	23	18	15	15	14
HQR	13	8.8	6.1	5.5	4.9
INVIT	4.7	3.6	2.8	2.6	2.4
PATHS					
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	25	19	16	16	15
RG (BALANC,ELMHES,HQR)	14	9.8	7.2	6.5	5.9
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	14+.54M	9.8+.21M	7.2+.09M	6.5+.05M	5.9+.04M

TABLE 12

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: IBM 370/168, Fortran H, OPT=2
UNIVERSITY OF MICHIGAN

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.004	.023	.16	.50	1.2
TRED1	1	1	1	1	1
TRED2	1.6	1.8	1.9	2.0	2.0
TRED3	.98	.99	1.0	1.0	1.0
TRBAK1	1.1	1.3	1.5	1.5	1.5
TRBAK3	.99	1.2	1.4	1.5	1.5
HTRIDI	2.8	3.1	3.4	3.6	3.7
HTRID3	2.7	3.1	3.5	3.6	3.7
HTRIBK	3.3	4.3	5.0	5.2	5.3
HTRIB3	3.4	4.5	5.2	5.4	5.5
FIGI	.09	.03	.01	.00	.00
FIGI2	.12	.04	.02	.01	.01
BAKVEC	.10	.05	.03	.02	.02
TQL2	5.4	5.8	5.7	5.6	5.4
IMTQL2	5.4	5.5	5.6	5.3	5.3
TQLRAT	1.2	.66	.34	.22	.16
TQL1	2.3	1.4	.81	.55	.42
IMTQL1	2.3	1.5	.83	.56	.42
BISECT	8.0	4.9	2.7	1.8	1.4
TRIDIB	8.4	5.0	2.7	1.8	1.4
IMTQLV	2.4	1.5	.85	.57	.44
RATQR	2.8	2.1	1.4	1.0	.80
TINVIT	.86	.54	.31	.23	.26
TSTURM	8.8	5.3	2.9	2.0	1.6
PATHS					
RS (TRED2,TQL2)	7.0	7.6	7.6	7.6	7.4
RS (TRED1,TQLRAT)	2.2	1.7	1.3	1.2	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+1.0M	1+.34M	1+.11M	1+.06M	1+.04M
TRED1,BISECT	1+.80M	1+.24M	1+.07M	1+.03M	1+.02M
CH (HTRIDI,TQL2,HTRIBK)	11	13	14	14	14
CH (HTRIDI,TQLRAT)	4.1	3.8	3.8	3.8	3.8
HTRIDI,BISECT,TINVIT,HTRIBK	2.8+1.2M	3.1+.48M	3.4+.20M	3.6+.12M	3.7+.09M
HTRIDI,BISECT	2.8+.80M	3.1+.24M	3.4+.07M	3.6+.03M	3.7+.02M

TABLE 13

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: IBM 370/165, Fortran H Extended, OPTIMIZE(2)
THE UNIVERSITY OF TORONTO

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.006	.043	.39	1.5	4.0
CBAL	-	.06	.03	.02	.01
CBABK2	-	-	.02	.02	.01
COMHES	1	1	1	1	1
COMBAK	1	.53	.62	.71	.98
CORTH	2	2.2	2.0	1.9	1.7
CORTB	1	1.2	1.1	.95	.85
COMQR2	17	16	13	12	10
COMLR2	11	9.3	7.5	6.5	5.9
COMQR	8	5.9	4.3	3.7	3.2
COMLR	6	4.1	2.7	2.2	1.9
CINVIT	7	4.8	3.3	2.6	2.5
PATHS					
CG (CBAL,CORTH,COMQR2,CBABK2)	20	18	15	13	12
CG (CBAL,CORTH,COMQR)	11	8.2	6.4	5.6	4.9
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	10+.8M	8.2+.30M	6.4+.11M	5.6+.06M	4.9+.04M

TABLE 14

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: IBM 370/165, Fortran H Extended, OPTIMIZE(2)
THE UNIVERSITY OF TORONTO

	ORDER OF MATRIX			
SUBROUTINE	N=20	N=40	N=60	N=80
Time Unit (Sec)	.016	.12	.43	1.1
BALANC	-	.05	.03	.02
BALBAK	-	.03	.02	.02
ELMHES	1	1	1	1
ELTRAN	-	.07	.04	.03
ELMBAK	.62	.63	.66	.80
ORTHES	2.3	2.4	2.2	2.1
ORTRAN	.83	.75	.71	.64
ORTBAK	1.2	1.1	1.1	.95
HQR2	16	15	13	11
HQR	8.5	5.9	4.9	4.1
INVIT	4.2	3.7	3.1	2.8
PATHS				
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	18	16	14	12
RG (BALANC,ELMHES,HQR)	9.1	6.8	5.9	5.1
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	9.5+.24M	7.0+.11M	5.9+.06M	5.1+.05M

TABLE 15

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: IBM 370/165, Fortran H Extended, OPTIMIZE(2)
THE UNIVERSITY OF TORONTO

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.003	.017	.11	.35	.84
TRED1	1	1	1	1	1
TRED2	2	1.7	1.9	2.0	2.0
TRED3	1	.92	1.0	1.0	1.0
TRBAK1	1	1.5	1.6	1.7	1.6
TRBAK3	1	1.1	1.3	1.3	1.3
HTRIDI	2	2.7	3.1	3.6	4.1
HTRID3	3	2.7	3.2	3.4	3.5
HTRIBK	3	3.7	4.7	5.0	5.5
HTRIB3	4	4.0	5.1	5.1	5.1
FIGI	-	-	-	.01	.00
FIGI2	-	-	.04	.03	.02
BAKVEC	-	-	.03	.02	.02
TQL2	4	4.8	4.7	4.8	4.4
IMTQL2	4	4.9	4.8	4.7	4.6
TQLRAT	2	.80	.42	.27	.19
TQL1	2	1.5	.92	.61	.46
IMTQL1	2	1.7	.96	.63	.47
BISECT	10	6.7	3.9	2.6	1.9
TRIDIB	9	6.5	3.8	2.6	1.9
IMTQLV	3	1.7	.98	.64	.48
RATQR	2	2.3	1.6	1.1	.86
TINVIT	1	.76	.42	.30	.30
TSTURM	10	7.3	4.2	2.9	2.2
PATHS					
RS (TRED2,TQL2)	6	6.8	6.6	6.7	6.3
RS (TRED1,TQLRAT)	3	1.8	1.4	1.3	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+1.2M	1+.45M	1+.15M	1+.08M	1+.05M
TRED1,BISECT	1+1.0M	1+.34M	1+.10M	1+.04M	1+.02M
CH (HTRIDI,TQL2,HTRIBK)	11	11	13	13	14
CH (HTRIDI,TQLRAT)	4	3.4	3.6	3.9	4.3
HTRIDI,BISECT,TINVIT,HTRIBK	4+1.4M	2.7+.56M	3.1+.22M	3.6+.13M	4.1+.10M
HTRIDI,BISECT	4+1.0M	2.7+.34M	3.1+.10M	3.6+.04M	4.1+.02M

TABLE 16

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: Burroughs 6700, Fortran IV (2.6)
UNIVERSITY OF CALIFORNIA, SAN DIEGO

SUBROUTINE	ORDER OF MATRIX N=10	N=20	N=40	N=60
Time Unit (Sec)	.27	2.1	17	59
CBAL	.10	.04	.02	.01
CBABK2	.05	.02	.01	.01
COMHES	1	1	1	1
COMBAK	.58	.56	.59	.59
CORTH	1.7	1.6	1.6	1.6
CORTB	.92	1.0	.99	.94
COMQR2	13	11	10	9.6
COMLR2	8.6	7.9	6.8	6.8
COMQR	5.0	3.9	3.3	3.1
COMLR	4.3	3.1	2.4	2.3
CINVIT	3.3	2.6	2.3	2.2
PATHS				
CG (CBAL,CORTH,COMQR2,CBABK2)	14	13	12	11
CG (CBAL,CORTH,COMQR)	6.7	5.6	4.9	4.6
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	6.8+.43M	5.5+.18M	4.9+.08M	4.8+.05M

TABLE 17

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: Burroughs 6700, Fortran IV (2.6)
UNIVERSITY OF CALIFORNIA, SAN DIEGO

	ORDER OF MATRIX			
SUBROUTINE	N=10	N=20	N=40	N=60
Time Unit (Sec)	.095	.79	6.3	20
BALANC	.18	.06	.03	.02
BALBAK	.08	.04	.02	.01
ELMHES	1	1	1	1
ELTRAN	.17	.07	.04	.03
ELMBAK	.57	.58	.59	.60
ORTHES	1.7	1.7	1.6	1.7
ORTRAN	.83	.70	.66	.66
ORTBAK	1.1	.99	.94	1.0
HQR2	12	9.9	8.9	8.8
HQR	6.3	4.3	3.6	3.1
INVIT	3.1	2.4	2.1	2.1
PATHS				
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	14	11	10	9.9
RG (BALANC,ELMHES,HQR)	7.8	5.4	4.5	4.3
BALANC,ELMHES,HQR,INVIT ELMBAK,BALBAK	7.5+.38M	5.4+.15M	4.6+.07M	4.1+.04M

TABLE 18

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: Burroughs 6700, Fortran IV (2.6)
UNIVERSITY OF CALIFORNIA, SAN DIEGO

	ORDER OF MATRIX			
SUBROUTINE	N=10	N=20	N=40	N=60
Time Unit (Sec)	.093	.56	4.1	13
TRED1	1	1	1	1
TRED2	1.6	1.9	2.0	2.2
TRED3	.77	.82	.76	.80
TRBAK1	1.1	1.5	1.6	1.8
TRBAK3	1.1	1.4	1.5	1.6
HTRIDI	2.5	2.9	3.1	3.1
HTRID3	2.4	2.8	3.0	3.1
HTRIBK	3.5	4.6	5.1	5.3
HTRIB3	3.7	4.9	5.1	5.3
FIGI	.05	.01	.00	.00
FIGI2	.12	.05	.03	.02
BAKVEC	.09	.06	.03	.02
TQL2	4.2	4.7	4.6	4.7
IMTQL2	4.4	5.1	4.8	5.2
TQLRAT	.77	.38	.19	.13
TQL1	1.1	.68	.36	.25
IMTQL1	1.2	.69	.37	.25
BISECT	3.7	2.1	1.1	.73
TRIDIB	4.1	2.5	1.2	.83
IMTQLV	1.1	.61	.31	.22
RATQR	1.4	1.1	.62	.46
TINVIT	.68	.43	.24	.19
TSTURM	5.0	3.0	1.5	1.1
PATHS				
RS (TRED2,TQL2)	5.8	6.9	6.6	6.9
RS (TRED1,TQLRAT)	1.5	1.3	1.1	1.1
TRED1,BISECT,TINVIT,TRBAK1	1+.55M	1+.20M	1+.07M	1+.04M
TRED1,BISECT	1+.37M	1+.11M	1+.03M	1+.01M
CH (HTRIDI,TQL2,HTRIBK)	11	12	13	13
CH (HTRIDI,TQLRAT)	3.2	3.2	3.3	3.3
HTRIDI,BISECT,TINVIT,HTRIBK	2.5+.79M	2.9+.35M	3.1+.16M	3.1+.10M
HTRIDI,BISECT	2.5+.37M	2.9+.11M	3.1+.03M	3.1+.01M

TABLE 19

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: CDC 6600, FTN (4.2) Compiler
KIRTLAND AIR FORCE BASE

	ORDER OF MATRIX				
SUBROUTINE	N=40	N=50	N=60	N=70	N=80
Time Unit (Sec)	.52	1.0	1.8	2.8	4.2
CBAL	-	-	-	-	-
CBABK2	-	-	-	-	-
COMHES	1	1	1	1	1
COMBAK	.72	.57	.55	.56	.56
CORTH	2.8	2.6	2.7	2.7	2.7
CORTB	1.0	.96	.93	.95	.97
COMQR2	14	12	12	12	12
COMLR2	7.5	6.9	6.6	6.5	6.6
COMQR	4.9	4.5	4.3	4.2	4.2
COMLR	3.1	2.7	2.5	2.4	2.4
CINVIT	2.6	2.3	2.2	2.2	2.2
PATHS					
CG (CBAL,CORTH,COMQR2,CBABK2)	16	15	15	14	15
CG (CBAL,CORTH,COMQR)	7.8	7.1	6.9	6.8	7.0
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	7.7+.09M	7.1+.07M	6.9+.05M	6.8+.04M	6.9+.04M

TABLE 20

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: CDC 6600, FTN (4.2) Compiler
KIRTLAND AIR FORCE BASE

	ORDER OF MATRIX				
SUBROUTINE	N=40	N=50	N=60	N=70	N=80
Time Unit (Sec)	.25	.49	.80	1.4	2.0
BALANC	-	-	-	-	-
BALBAK	-	-	-	-	-
ELMHES	1	1	1	1	1
ELTRAN	-	-	-	-	-
ELMBAK	.46	.59	.59	.61	.58
ORTHES	2.6	2.6	2.6	2.5	2.5
ORTRAN	.92	.81	.81	.76	.75
ORTBAK	1.2	1.2	1.2	1.2	1.2
HQR2	12	12	12	11	10
HQR	5.1	4.9	4.6	4.0	3.8
INVIT	2.8	2.7	2.7	2.5	2.4
PATHS					
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	13	13	13	12	12
RG (BALANC,ELMHES,HQR)	6.1	5.7	5.7	5.0	4.7
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	6.1+.08M	5.8+.07M	5.6+.05M	5.0+.04M	4.8+.04M

TABLE 21

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: CDC 6600, FTN (4.2) Compiler
KIRTLAND AIR FORCE BASE

	ORDER OF MATRIX				
SUBROUTINE	N=40	N=50	N=60	N=70	N=80
Time Unit (Sec)	.19	.37	.60	.92	1.3
TRED1	1	1	1	1	1
TRED2	2.2	2.1	2.1	2.2	2.2
TRED3	1.8	1.8	1.7	1.8	1.7
TRBAK1	1.7	1.7	1.7	1.8	1.8
TRBAK3	1.9	1.9	2.0	2.1	2.0
HTRIDI	2.2	1.9	2.2	2.3	2.1
HTRID3	1.9	2.1	2.2	2.3	2.0
HTRIBK	3.4	3.0	3.5	3.7	3.3
HTRIB3	3.0	3.1	3.4	3.6	3.4
FIGI	-	-	-	-	-
FIGI2	-	-	-	-	-
BAKVEC	-	-	-	-	-
TQL2	4.2	4.0	4.0	4.2	4.0
IMTQL2	4.5	4.3	4.4	4.4	4.4
TQLRAT	.39	.28	.25	.21	.19
TQL1	.63	.49	.42	.37	.32
IMTQL1	.68	.56	.50	.44	.38
BISECT	6.2	4.9	4.2	3.7	3.2
TRIDIB	6.0	4.8	4.2	3.6	3.2
IMTQLV	.76	.62	.54	.47	.42
RATQR	2.0	1.6	1.4	1.3	1.1
TINVIT	.52	.40	.39	.36	.40
TSTURM	6.6	5.3	4.6	4.1	3.7
PATHS					
RS (TRED2,TQL2)	6.3	6.0	6.1	6.2	6.2
RS (TRED1,TQLRAT)	1.4	1.3	1.2	1.2	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+.21M	1+.14M	1+.11M	1+.08M	1+.07M
TRED1,BISECT	1+.16M	1+.10M	1+.07M	1+.05M	1+.04M
CH (HTRIDI,TQL2,HTRIBK)	8.8	8.9	9.7	10	9.2
CH (HTRIDI,TQLRAT)	2.1	2.3	2.5	2.6	2.3
HTRIDI,BISECT,TINVIT,HTRIBK	2.2+.25M	2.0+.15M	2.1+.11M	2.3+.09M	2.1+.07M
HTRIDI,BISECT	2.2+.16M	2.0+.10M	2.1+.07M	2.3+.05M	2.1+.04M

TABLE 22

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: CDC 6600, FTN 4.3+P393 Compiler, OPT=1
NASA LANGLEY RESEARCH CENTER

	ORDER OF MATRIX			
SUBROUTINE	N=10	N=20	N=40	N=60
Time Unit (Sec)	.009	.063	.54	1.8
CBAL	.24	.10	.04	.03
CBABK2	.12	.04	.01	.01
COMHES	1	1	1	1
COMBAK	.47	.60	.62	.64
CORTH	1.8	1.6	1.6	1.6
CORTB	.88	.90	.84	.79
COMQR2	16	15	13	13
COMLR2	9.9	8.0	6.9	7.0
COMQR	7.4	5.9	4.5	4.3
COMLR	5.5	3.9	2.8	2.8
CINVIT	3.8	2.8	2.2	2.1
PATHS				
CG (CBAL,CORTH,COMQR2,CBABK2)	18	17	14	14
CG (CBAL,CORTH,COMQR)	9.5	7.6	6.2	5.9
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	9.4+.48M	7.6+.19M	6.2+.08M	5.9+.05M

TABLE 23

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: CDC 6600, FTN 4.3+P393 Compiler, OPT=1
NASA LANGLEY RESEARCH CENTER

SUBROUTINE	ORDER OF MATRIX N=10	N=20	N=40	N=60
Time Unit (Sec)	.006	.032	.24	.77
BALANC	.27	.17	.07	.05
BALBAK	-	.05	.03	.02
ELMHES	1	1	1	1
ELTRAN	.18	.06	.04	.02
ELMBAK	.55	.58	.61	.61
ORTHES	1.8	2.0	2.2	2.2
ORTRAN	.91	.73	.85	.82
ORTBAK	1.1	1.3	1.3	1.3
HQR2	13	14	14	12
HQR	6.9	6.7	6.2	4.8
INVIT	3.6	3.3	3.1	2.9
PATHS				
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	14	15	15	13
RG (BALANC,ELMHES,HQR)	8.5	7.7	7.2	5.9
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	8.2+.41M	7.9+.20M	7.2+.09M	5.8+.06M

TABLE 24

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: CDC 6600, FTN 4.3+P393 Compiler, OPT=1
NASA LANGLEY RESEARCH CENTER

	ORDER OF MATRIX			
SUBROUTINE	N=10	N=20	N=40	N=60
Time Unit (Sec)	.006	.030	.19	.58
TRED1	1	1	1	1
TRED2	1.6	1.8	2.0	2.0
TRED3	1.2	1.5	1.7	1.8
TRBAK1	1.2	1.4	1.6	1.7
TRBAK3	1.2	1.5	1.8	1.7
HTRIDI	1.7	2.1	2.4	2.5
HTRID3	1.8	2.3	2.6	2.6
HTRIBK	1.9	2.7	3.2	3.5
HTRIB3	1.9	2.7	3.2	3.2
FIGI	-	.03	.01	.00
FIGI2	-	.07	.02	.01
BAKVEC	-	.05	.04	.02
TQL2	3.6	3.8	4.1	4.1
IMTQL2	3.8	4.0	4.4	4.7
TQLRAT	1.0	.58	.32	.25
TQL1	1.5	.95	.55	.42
IMTQL1	1.6	1.1	.68	.51
BISECT	15	9.2	5.5	3.9
TRIDIB	14	9.1	5.5	3.9
IMTQLV	1.5	1.2	.74	.53
RATQR	3.6	2.8	2.0	1.5
TINVIT	.91	.67	.44	.32
TSTURM	15	9.7	5.8	4.3
PATHS				
RS (TRED2,TQL2)	5.5	5.8	6.1	6.1
RS (TRED1,TQLRAT)	2.0	1.6	1.4	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+1.7M	1+.56M	1+.19M	1+.10M
TRED1,BISECT	1+1.5M	1+.46M	1+.14M	1+.06M
CH (HTRIDI,TQL2,HTRIBK)	7.2	8.8	9.9	10
CH (HTRIDI,TQLRAT)	2.6	2.7	2.7	2.6
HTRIDI,BISECT,TINVIT,HTRIBK	1.7+1.7M	2.1+.63M	2.4+.23M	2.5+.13M
HTRIDI,BISECT	1.7+1.5M	2.1+.46M	2.4+.14M	2.5+.06M

TABLE 25

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: CDC 7600, Local Compiler
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.002	.015	.11	.38	.89
CBAL	-	.10	.05	.03	.02
CBABK2	-	.05	.01	.01	.01
COMHES	1	1	1	1	1
COMBAK	1	.57	.60	.61	.62
CORTH	2	2.3	2.3	2.3	2.3
CORTB	1	1.1	1.1	1.1	1.1
COMQR2	15	15	13	13	13
COMLR2	10	8.1	6.9	6.7	6.4
COMQR	7	5.8	4.7	4.4	4.3
COMLR	6	3.9	2.8	2.5	2.3
CINVIT	4	3.0	2.3	2.2	2.1
PATHS					
CG (CBAL,CORTH,COMQR2,CBABK2)	18	17	16	15	15
CG (CBAL,CORTH,COMQR)	9	8.0	7.1	6.7	6.6
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	9+.5M	8.2+.21M	7.1+.09M	6.8+.05M	6.6+.04M

TABLE 26

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: CDC 7600, Local Compiler
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.001	.007	.050	.16	.38
BALANC	-	-	.07	.05	.04
BALBAK	-	-	.02	.01	.01
ELMHES	1	1	1	1	1
ELTRAN	-	-	.04	.02	.02
ELMBAK	1	.55	.60	.60	.60
ORTHES	2	2.4	2.5	2.5	2.5
ORTRAN	1	.81	.78	.78	.77
ORTBAK	1	1.3	1.2	1.2	1.2
HQR2	18	16	15	14	14
HQR	10	7.4	5.8	5.2	4.8
INVIT	4	3.4	3.0	2.8	2.7
PATHS					
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	20	17	16	15	15
RG (BALANC,ELMHES,HQR)	11	8.6	6.9	6.3	5.8
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	11+.5M	8.4+.20M	6.9+.09M	6.3+.06M	5.8+.04M

TABLE 27

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: CDC 7600, Local Compiler
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

		ORDER OF MATRIX			
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.001	.007	.040	.12	.28
TRED1	1	1	1	1	1
TRED2	2	1.7	1.9	2.0	2.0
TRED3	2	1.8	2.8	3.2	3.4
TRBAK1	1	1.3	1.5	1.5	1.6
TRBAK3	2	1.6	2.7	3.0	3.2
HTRIDI	2	2.1	2.4	2.6	2.6
HTRID3	2	2.2	2.5	2.6	2.7
HTRIBK	2	2.5	3.1	3.3	3.4
HTRIB3	2	2.6	3.2	3.4	3.5
FIGI	-	-	-	.01	.00
FIGI2	-	-	.02	.01	.01
BAKVEC	-	-	.03	.02	.01
TQL2	3	3.4	3.6	3.6	3.5
IMTQL2	4	3.5	3.8	3.8	3.8
TQLRAT	1	.69	.40	.28	.21
TQL1	2	1.0	.65	.46	.36
IMTQL1	2	1.2	.77	.55	.43
BISECT	18	11	7.0	5.0	3.9
TRIDIB	17	11	6.6	4.7	3.6
IMTQLV	2	1.3	.82	.58	.45
RATQR	3	2.7	1.9	1.5	1.2
TINVIT	1	.62	.42	.32	.33
TSTURM	18	11	7.2	5.2	4.1
PATHS					
RS (TRED2,TQL2)	5	5.1	5.5	5.6	5.5
RS (TRED1,TQLRAT)	2	1.6	1.4	1.3	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+2.0M	1+.65M	1+.22M	1+.11M	1+.07M
TRED1,BISECT	1+1.8M	1+.56M	1+.17M	1+.08M	1+.05M
CH (HTRIDI,TQL2,HTRIBK)	8	7.9	9.1	9.4	9.6
CH (HTRIDI,TQLRAT)	4	2.8	2.8	2.8	2.8
HTRIDI,BISECT,TINVIT,HTRIBK	2+2.1M	2.1+.71M	2.4+.26M	2.6+.14M	2.6+.09M
HTRIDI,BISECT	2+1.8M	2.1+.56M	2.4+.17M	2.6+.08M	2.6+.05M

TABLE 28

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: CDC 6400, FTN 3.0 Compiler, OPT=2
NORTHWESTERN UNIVERSITY

	ORDER OF MATRIX			
SUBROUTINE	N=10	N=20	N=40	N=60
Time Unit (Sec)	.036	.29	2.2	7.5
CBAL	.19	.07	.03	.02
CBABK2	.08	.03	.01	.01
COMHES	1	1	1	1
COMBAK	.51	.54	.57	.58
CORTH	2.5	2.4	2.4	2.4
CORTB	1.2	1.2	1.2	1.2
COMQR2	18	16	15	14
COMLR2	9.7	7.9	6.5	6.5
COMQR	8.3	6.2	5.2	4.9
COMLR	5.3	3.5	2.5	2.3
CINVIT	4.1	2.8	2.2	2.0
PATHS				
CG (CBAL,CORTH,COMQR2,CBABK2)	21	19	17	17
CG (CBAL,CORTH,COMQR)	11	8.7	7.6	7.3
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	11+.54M	8.7+.20M	7.6+.09M	7.3+.05M

TABLE 29

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: CDC 6400, FTN 3.0 Compiler, OPT=2
NORTHWESTERN UNIVERSITY

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.016	.11	.83	2.7	6.3
BALANC	.31	.14	.06	.04	.03
BALBAK	.15	.05	.02	.01	.01
ELMHES	1	1	1	1	1
ELTRAN	.15	.06	.03	.02	.01
ELMBAK	.53	.54	.57	.57	.58
ORTHES	2.7	2.8	2.9	3.0	3.0
ORTRAN	.83	.77	.77	.77	.77
ORTBAK	1.3	1.2	1.2	1.2	1.2
HQR2	17	15	14	13	13
HQR	10	7.2	5.6	5.1	4.7
INVIT	4.2	3.2	2.8	2.6	2.5
PATHS					
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	19	16	15	14	14
RG (BALANC,ELMHES,HQR)	11	8.3	6.7	6.1	5.7
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	11+.48M	8.3+.19M	6.7+.08M	6.1+.05M	5.7+.04M

TABLE 30

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: CDC 6400, FTN 3.0 Compiler, OPT=2
NORTHWESTERN UNIVERSITY

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.019	.11	.69	2.1	4.9
TRED1	1	1	1	1	1
TRED2	1.7	1.8	1.9	2.0	2.0
TRED3	1.2	1.3	1.5	1.6	1.6
TRBAK1	1.1	1.3	1.5	1.5	1.5
TRBAK3	1.1	1.3	1.5	1.6	1.7
HTRIDI	2.4	2.7	3.0	3.1	3.2
HTRID3	2.4	2.7	2.9	3.0	3.1
HTRIBK	2.6	3.4	4.0	4.3	4.5
HTRIB3	2.6	3.4	4.0	4.2	4.4
FIGI	.07	.03	.01	.00	.00
FIGI2	.12	.05	.02	.01	.01
BAKVEC	.11	.06	.03	.02	.02
TQL2	4.2	4.5	4.7	4.7	4.7
IMTQL2	4.4	4.7	4.9	4.9	4.9
TQLRAT	1.1	.66	.36	.24	.19
TQL1	1.7	1.2	.68	.48	.37
IMTQL1	2.0	1.3	.77	.54	.41
BISECT	9.8	6.2	3.5	2.5	1.9
TRIDIB	10	6.2	3.6	2.5	1.9
IMTQLV	2.0	1.3	.79	.56	.43
RATQR	3.1	2.4	1.6	1.2	.97
TINVIT	.94	.61	.37	.28	.31
TSTURM	11	6.7	3.9	2.8	2.2
PATHS					
RS (TRED2,TQL2)	5.9	6.3	6.6	6.7	6.7
RS (TRED1,TQLRAT)	2.1	1.7	1.4	1.2	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+1.2M	1+.40M	1+.13M	1+.07M	1+.05M
TRED1,BISECT	1+.98M	1+.31M	1+.09M	1+.04M	1+.02M
CH (HTRIDI,TQL2,HTRIBK)	9.2	10	11	12	12
CH (HTRIDI,TQLRAT)	3.5	3.4	3.4	3.4	3.4
HTRIDI,BISECT,TINVIT,HTRIBK	2.4+1.3M	2.7+.51M	3.0+.20M	3.1+.12M	3.2+.08M
HTRIDI,BISECT	2.4+.98M	2.7+.31M	3.0+.09M	3.1+.04M	3.2+.02M

TABLE 31

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: CDC 6400/6500, FUN Compiler
PURDUE UNIVERSITY

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.045	.35	2.7	9.2	22
CBAL	.13	.05	.02	.01	.01
CBABK2	.07	.03	.01	.01	.01
COMHES	1	1	1	1	1
COMBAK	.58	.62	.64	.65	.66
CORTH	2.7	2.7	2.7	2.7	2.7
CORTB	1.4	1.3	1.3	1.3	1.3
COMQR2	20	18	16	16	16
COMLR2	11	8.7	7.4	7.1	6.7
COMQR	9.5	7.4	6.4	6.2	5.8
COMLR	5.7	3.8	2.7	2.4	2.2
CINVIT	3.9	2.9	2.4	2.2	2.2
PATHS					
CG (CBAL,CORTH,COMQR2,CBABK2)	23	20	19	19	18
CG (CBAL,CORTH,COMQR)	12	10	9.1	8.9	8.5
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	12+.54M	10+.21M	9.1+.09M	8.9+.06M	8.5+.04M

TABLE 32

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: CDC 6400/6500, FUN Compiler
PURDUE UNIVERSITY

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.018	.12	.90	3.0	6.9
BALANC	.25	.10	.05	.03	.02
BALBAK	.14	.06	.03	.02	.01
ELMHES	1	1	1	1	1
ELTRAN	.17	.07	.03	.02	.02
ELMBAK	.55	.60	.61	.62	.63
ORTHES	2.6	2.7	2.7	2.8	2.8
ORTRAN	.91	.83	.81	.81	.81
ORTBAK	1.4	1.3	1.3	1.3	1.2
HQR2	33	31	30	30	29
HQR	18	14	12	11	10
INVIT	5.1	4.9	4.6	4.6	4.6
PATHS					
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	35	32	31	31	30
RG (BALANC,ELMHES,HQR)	19	15	13	12	11
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	19+.58M	15+.28M	13+.13M	12+.09M	11+.06M

TABLE 33

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: CDC 6400/6500, FUN Compiler
PURDUE UNIVERSITY

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.024	.13	.81	2.5	5.6
TRED1	1	1	1	1	1
TRED2	1.7	1.8	1.9	2.0	2.0
TRED3	.92	1.1	1.2	1.2	1.2
TRBAK1	1.1	1.3	1.5	1.6	1.6
TRBAK3	1.1	1.4	1.6	1.7	1.7
HTRIDI	2.4	2.8	3.2	3.4	3.5
HTRID3	2.6	3.1	3.5	3.7	3.9
HTRIBK	2.8	3.7	4.5	4.9	5.1
HTRIB3	3.0	4.0	4.9	5.3	5.5
FIGI	.06	.02	.01	.00	.00
FIGI2	.11	.05	.02	.01	.01
BAKVEC	.09	.05	.03	.02	.02
TQL2	3.8	4.2	4.4	4.6	4.6
IMTQL2	4.0	4.2	4.5	4.5	4.6
TQLRAT	.83	.51	.28	.19	.15
TQL1	1.4	.92	.56	.39	.30
IMTQL1	1.5	1.0	.62	.44	.34
BISECT	9.4	6.1	3.6	2.5	2.0
TRIDIB	9.1	5.9	3.5	2.5	1.9
IMTQLV	1.7	1.1	.70	.50	.39
RATQR	2.9	2.3	1.6	1.2	.98
TINVIT	.97	.68	.43	.33	.37
TSTURM	10	6.7	4.0	2.9	2.3
PATHS					
RS (TRED2,TQL2)	5.4	6.0	6.4	6.6	6.6
RS (TRED1,TQLRAT)	1.9	1.5	1.3	1.2	1.1
TRED1,BISECT,TINVIT,TRBAK1	1+1.2M	1+.40M	1+.14M	1+.07M	1+.05M
TRED1,BISECT	1+.94M	1+.30M	1+.09M	1+.04M	1+.02M
CH (HTRIDI,TQL2,HTRIBK)	9.0	10	12	13	13
CH (HTRIDI,TQLRAT)	3.3	3.3	3.5	3.6	3.6
HTRIDI,BISECT,TINVIT,HTRIBK	2.4+1.3M	2.8+.52M	3.2+.21M	3.4+.13M	3.5+.09M
HTRIDI,BISECT	2.4+.94M	2.8+.30M	3.2+.09M	3.4+.04M	3.5+.02M

TABLE 34

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: CDC 6600/6400, RUN Compiler
THE UNIVERSITY OF TEXAS

	ORDER OF MATRIX		
SUBROUTINE	N=10	N=20	N=40
Time Unit (Sec)	.015	.084	.69
CBAL	-	.17	.05
CBABK2	-	-	.02
COMHES	1	1	1
COMBAK	-	.64	.64
CORTH	2.1	3.0	2.9
CORTB	1.1	1.4	1.3
COMQR2	17	19	17
COMLR2	9.5	9.8	7.8
COMQR	8.9	8.9	7.4
COMLR	5.5	4.6	3.1
CINVIT	4.2	3.9	2.9
PATHS			
CG (CBAL,CORTH,COMQR2,CBABK2)	19	22	20
CG (CBAL,CORTH,COMQR)	12	12	10
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	19+.53M	12+.26M	10+.11M

TABLE 35

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: CDC 6600/6400, RUN Compiler
THE UNIVERSITY OF TEXAS

SUBROUTINE	ORDER OF MATRIX N=20	N=40	N=60	N=80
Time Unit (Sec)	.045	.24	.78	1.8
BALANC	-	.06	.06	.05
BALBAK	-	-	.02	.02
ELMHES	1	1	1	1
ELTRAN	-	-	.04	.03
ELMBAK	.36	.73	.70	.73
ORTHES	2.5	3.5	3.5	3.6
ORTRAN	.73	.79	.77	.77
ORTBAK	.71	1.3	1.2	1.2
HQR2	28	45	36	36
HQR	12	18	13	13
INVIT	4.7	6.0	6.2	6.4
PATHS				
RG (BALANC, ELMHES, ELTRAN, HQR2, BALBAK)	29	46	37	37
RG (BALANC, ELMHES, HQR)	13	19	14	14
BALANC, ELMHES, HQR, INVIT, ELMBAK, BALBAK	13+.25M	19+.17M	14+.12M	14+.09M

TABLE 36

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: CDC 6600/6400, RUN Compiler
THE UNIVERSITY OF TEXAS

	ORDER OF MATRIX			
SUBROUTINE	N=20	N=40	N=60	N=80
Time Unit (Sec)	.033	.21	.63	1.4
TRED1	1	1	1	1
TRED2	1.9	2.1	2.1	2.1
TRED3	1.8	1.6	1.7	1.7
TRBAK1	1.7	1.8	1.8	1.9
TRBAK3	2.0	2.1	2.2	2.3
HTRIDI	2.9	3.1	3.3	3.4
HTRID3	3.5	3.5	3.7	3.9
HTRIBK	3.7	4.5	4.8	5.1
HTRIB3	4.2	4.9	5.3	5.6
FIGI	-	-	-	-
FIGI2	-	-	.02	.02
BAKVEC	-	-	-	.02
TQL2	4.4	4.3	4.2	4.2
IMTQL2	4.6	4.8	4.8	4.5
TQLRAT	.90	.39	.29	.21
TQL1	1.2	.84	.60	.44
IMTQL1	1.7	.92	.70	.53
BISECT	12	6.8	4.9	3.8
TRIDIB	12	6.8	4.8	3.8
IMTQLV	2.3	1.1	.79	.59
RATQR	3.6	2.5	1.9	1.5
TINVIT	1.1	.64	.51	.47
TSTURM	13	7.6	5.5	4.3
PATHS				
RS (TRED2,TQL2)	6.5	6.3	6.3	6.3
RS (TRED1,TQLRAT)	2.0	1.5	1.3	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+.73M	1+.23M	1+.12M	1+.08M
TRED1,BISECT	1+.59M	1+.17M	1+.08M	1+.05M
CH (HTRIDI,TQL2,HTRIBK)	11	12	12	13
CH (HTRIDI,TQLRAT)	3.8	3.6	3.5	3.6
HTRIDI,BISECT,TINVIT,HTRIBK	2.9+.83M	3.1+.30M	3.3+.17M	3.4+.12M
HTRIDI,BISECT	2.9+.59M	3.1+.17M	3.3+.08M	3.4+.05M

TABLE 37

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: Honeywell 6070 Fortran-Y(SR-F) Optimized
BELL LABORATORIES

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.041	.31	2.5	8.6	20
CBAL	.16	.07	.03	.02	.02
CBABK2	.07	.03	.01	.01	.01
COMHES	1	1	1	1	1
COMBAK	.46	.54	.56	.55	.55
CORTH	2.4	2.5	2.6	2.5	2.5
CORTB	1.1	1.1	1.2	1.1	1.1
COMQR2	14	13	11	11	11
COMLR2	8.2	6.7	5.7	5.7	5.4
COMQR	6.2	4.7	3.7	3.5	3.4
COMLR	4.5	2.9	2.1	2.0	1.8
CINVIT	4.3	3.1	2.4	2.3	2.2
PATHS					
CG (CBAL,CORTH,COMQR2,CBABK2)	17	15	14	14	13
CG (CBAL,CORTH,COMQR)	8.8	7.3	6.3	6.0	6.0
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	8.7+.54M	7.3+.22M	6.3+.09M	6.1+.06M	5.9+.04M

TABLE 38

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: Honeywell 6070 Fortran-Y(SR-F) Optimized
BELL LABORATORIES

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.009	.070	.54	1.8	4.3
BALANC	.32	.12	.06	.04	.03
BALBAK	.15	.06	.03	.02	.01
ELMHES	1	1	1	1	1
ELTRAN	.16	.07	.03	.02	.01
ELMBAK	.52	.54	.56	.56	.56
ORTHES	2.9	3.0	3.1	3.1	3.1
ORTRAN	.86	.81	.80	.79	.79
ORTBAK	1.2	1.2	1.2	1.2	1.2
HQR2	15	12	11	10	10
HQR	8.6	6.1	4.3	3.9	3.5
INVIT	4.6	3.6	3.0	2.7	2.6
PATHS					
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	16	14	12	11	11
RG (BALANC,ELMHES,HQR)	10	7.2	5.4	5.0	4.5
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	10+.53M	7.2+.21M	5.4+.09M	5.0+.05M	4.5+.04M

TABLE 39

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: Honeywell 6070 Fortran-Y(SR-F) Optimized
BELL LABORATORIES

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.013	.078	.53	1.6	3.6
TRED1	1	1	1	1	1
TRED2	1.4	1.8	1.9	2.0	2.0
TRED3	1.1	1.2	1.2	1.2	1.3
TRBAK1	.87	1.1	1.3	1.4	1.4
TRBAK3	1.0	1.3	1.5	1.6	1.6
HTRIDI	2.6	3.0	3.3	3.4	3.5
HTRID3	2.7	3.1	3.5	3.6	3.7
HTRIBK	2.9	3.8	4.5	4.7	4.9
HTRIB3	2.9	3.8	4.5	4.8	4.9
FIGI	.10	.03	.01	.00	.00
FIGI2	.14	.05	.02	.01	.01
BAKVEC	.12	.06	.03	.02	.02
TQL2	4.1	4.4	4.2	4.3	4.2
IMTQL2	4.2	4.6	4.6	4.6	4.6
TQLRAT	1.1	.60	.29	.21	.15
TQL1	1.7	1.1	.59	.42	.32
IMTQL1	1.8	1.2	.65	.46	.34
BISECT	5.2	3.1	1.6	1.1	.84
TRIDIB	5.3	3.2	1.6	1.1	.81
IMTQLV	1.9	1.1	.65	.45	.35
RATQR	3.0	2.1	1.4	1.0	.82
TINVIT	1.0	.67	.39	.31	.33
TSTURM	6.2	3.9	2.0	1.4	1.2
PATHS					
RS (TRED2,TQL2)	5.5	6.4	6.0	6.4	6.2
RS (TRED1,TQLRAT)	2.0	1.6	1.3	1.2	1.1
TRED1,BISECT,TINVIT,TRBAK1	1+.71M	1+.24M	1+.08M	1+.05M	1+.03M
TRED1,BISECT	1+.52M	1+.15M	1+.04M	1+.02M	1+.01M
CH (HTRIDI,TQL2,HTRIBK)	9.9	11	12	12	13
CH (HTRIDI,TQLRAT)	3.7	3.6	3.6	3.6	3.6
HTRIDI,BISECT,TINVIT,HTRIBK	2.6+.91M	3.0+.38M	3.3+.16M	3.4+.10M	3.5+.08M
HTRIDI,BISECT	2.6+.52M	3.0+.15M	3.3+.04M	3.4+.02M	3.5+.01M

TABLE 40

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: Univac 1110, Fortran V(9) Compiler
THE UNIVERSITY OF WISCONSIN

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.025	.18	1.3	4.4	10
CBAL	.16	.08	.04	.02	.02
CBABK2	.08	.03	.01	.01	.01
COMHES	1	1	1	1	1
COMBAK	.48	.53	.57	.58	.58
CORTH	2.3	2.6	2.8	2.8	2.9
CORTB	1.0	1.1	1.1	1.2	1.2
COMQR2	13	12	11	11	11
COMLR2	9.4	7.5	6.3	6.3	6.1
COMQR	6.2	4.8	3.9	3.9	3.6
COMLR	5.5	3.6	2.5	2.4	2.2
CINVIT	4.7	3.4	2.7	2.4	2.3
PATHS					
CG (CBAL,CORTH,COMQR2,CBABK2)	16	15	14	14	14
CG (CBAL,CORTH,COMQR)	8.7	7.4	6.7	6.7	6.5
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	8.7+.58M	7.4+.23M	6.7+.10M	6.7+.06M	6.6+.04M

TABLE 41

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: Univac 1110, Fortran V(9) Compiler
THE UNIVERSITY OF WISCONSIN

SUBROUTINE	ORDER OF MATRIX N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.009	.062	.45	1.5	3.4
BALANC	.39	.17	.08	.05	.04
BALBAK	.11	.05	.02	.02	.01
ELMHES	1	1	1	1	1
ELTRAN	.16	.08	.03	.02	.02
ELMBAK	.54	.57	.58	.59	.59
ORTHES	3.2	3.6	3.8	4.0	4.0
ORTRAN	.72	.74	.76	.77	.78
ORTBAK	1.1	1.1	1.2	1.2	1.2
HQR2	15	13	11	11	11
HQR	8.5	6.3	4.7	4.3	3.9
INVIT	5.6	4.6	4.0	3.8	3.7
PATHS					
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	16	14	13	12	12
RG (BALANC,ELMHES,HQR)	9.9	7.5	5.8	5.4	4.9
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	9.9+.63M	7.5+.26M	5.8+.12M	5.4+.07M	4.9+.05M

TABLE 42

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: Univac 1110, Fortran V(9) Compiler
THE UNIVERSITY OF WISCONSIN

SUBROUTINE	ORDER OF MATRIX N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.012	.065	.40	1.2	2.7
TRED1	1	1	1	1	1
TRED2	1.5	1.7	1.9	1.9	2.0
TRED3	1.3	1.7	2.0	2.2	2.3
TRBAK1	.81	1.1	1.3	1.4	1.5
TRBAK3	1.4	2.0	2.5	2.7	2.9
HTRIDI	2.3	2.5	2.7	2.9	2.9
HTRID3	2.1	2.4	2.7	2.8	2.9
HTRIBK	2.2	3.0	3.8	4.1	4.3
HTRIB3	2.2	3.1	3.8	4.1	4.3
FIGI	.08	.02	.01	.00	.00
FIGI2	.09	.04	.02	.01	.01
BAKVEC	.08	.05	.03	.02	.01
TQL2	3.4	3.8	4.0	4.1	4.2
IMTQL2	3.6	4.0	4.4	4.4	4.3
TQLRAT	.99	.63	.36	.26	.19
TQL1	1.4	.96	.58	.41	.31
IMTQL1	1.5	1.0	.64	.45	.34
BISECT	6.3	3.9	2.2	1.6	1.2
TRIDIB	6.2	3.8	2.2	1.5	1.1
IMTQLV	1.6	1.1	.67	.47	.35
RATQR	3.3	2.7	1.8	1.4	1.2
TINVIT	1.2	.82	.51	.39	.38
TSTURM	7.4	4.6	2.7	1.9	1.6
PATHS					
RS (TRED2,TQL2)	4.9	5.5	5.8	6.0	6.2
RS (TRED1,TQLRAT)	2.0	1.6	1.4	1.2	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+.83M	1+.29M	1+.10M	1+.06M	1+.04M
TRED1,BISECT	1+.63M	1+.19M	1+.06M	1+.03M	1+.01M
CH (HTRIDI,TQL2,HTRIBK)	8.1	9.3	11	11	11
CH (HTRIDI,TQLRAT)	3.3	3.1	3.1	3.1	3.1
HTRIDI,BISECT,TINVIT,HTRIBK	2.3+.97M	2.5+.39M	2.7+.16M	2.9+.10M	2.9+.07M
HTRIDI,BISECT	2.3+.63M	2.5+.19M	2.7+.06M	2.9+.03M	2.9+.01M

TABLE 43

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: DEC PDP-10, F40 Compiler
YALE UNIVERSITY

	ORDER OF MATRIX			
SUBROUTINE	N=10	N=20	N=30	N=40
Time Unit (Sec)	.29	2.4	8.3	20
CBAL	.16	.07	.05	.04
CBABK2	.05	.03	.01	.01
COMHES	1	1	1	1
COMBAK	.57	.57	.58	.59
CORTH	2.0	1.9	2.0	2.0
CORTB	1.1	1.1	1.1	1.1
COMQR2	13	11	11	11
COMLR2	8.8	7.6	7.2	6.7
COMQR	5.1	3.7	3.4	3.2
COMLR	4.3	3.1	2.6	2.3
CINVIT	3.4	2.7	2.6	2.5
PATHS				
CG (CBAL,CORTH,COMQR2,CBABK2)	15	13	13	13
CG (CBAL,CORTH,COMQR)	7.2	5.7	5.4	5.2
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	7.2+.46M	5.7+.19M	5.4+.12M	5.2+.09M

TABLE 44

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: DEC PDP-10, F40 Compiler
YALE UNIVERSITY

SUBROUTINE	ORDER OF MATRIX N=10	N=20	N=30	N=40
Time Unit (Sec)	.10	.80	2.7	6.2
BALANC	.27	.12	.08	.06
BALBAK	.12	.04	.03	.02
ELMHES	1	1	1	1
ELTRAN	.21	.08	.06	.04
ELMBAK	.54	.58	.59	.59
ORTHES	1.8	1.9	1.9	1.9
ORTRAN	.75	.73	.73	.72
ORTBAK	1.1	1.1	1.1	1.1
HQR2	12	11	9.9	9.7
HQR	6.7	5.0	4.1	3.8
INVIT	3.5	2.9	2.7	2.6
PATHS				
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	14	12	11	11
RG (BALANC,ELMHES,HQR)	7.9	6.2	5.2	4.9
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	8.0+.42M	6.2+.17M	5.2+.11M	4.9+.08M

TABLE 45

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: DEC PDP-10, F40 Compiler
YALE UNIVERSITY

SUBROUTINE	ORDER OF MATRIX N=10	N=20	N=30	N=40
Time Unit (Sec)	.10	.64	2.0	4.7
TRED1	1	1	1	1
TRED2	1.8	2.0	2.0	2.1
TRED3	.85	.87	.88	.88
TRBAK1	1.2	1.5	1.6	1.7
TRBAK3	1.1	1.4	1.5	1.5
HTRIDI	3.2	3.6	3.7	3.8
HTRID3	2.6	3.1	3.2	3.3
HTRIBK	4.4	5.5	5.9	6.2
HTRIB3	4.4	5.5	5.9	6.2
FIGI	.09	.01	.01	.01
FIGI2	.09	.05	.03	.02
BAKVEC	.12	.07	.04	.03
TQL2	3.7	4.2	4.1	4.0
IMTQL2	3.9	4.5	4.2	4.3
TQLRAT	.73	.42	.26	.20
TQL1	1.0	.65	.42	.33
IMTQL1	1.1	.75	.48	.39
BISECT	3.9	2.1	1.4	1.0
TRIDIB	3.9	2.1	1.3	.99
IMTQLV	1.2	.79	.51	.41
RATQR	2.1	1.4	1.1	.87
TINVIT	1.2	.70	.50	.39
TSTURM	5.0	2.8	1.8	1.4
PATHS				
RS (TRED2,TQL2)	5.5	6.2	6.2	6.1
RS (TRED1,TQLRAT)	1.8	1.4	1.3	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+.62M	1+.22M	1+.12M	1+.08M
TRED1,BISECT	1+.39M	1+.10M	1+.05M	1+.03M
CH (HTRIDI,TQL2,HTRIBK)	12	13	14	14
CH (HTRIDI,TQLRAT)	3.9	4.0	4.0	4.0
HTRIDI,BISECT,TINVIT,HTRIBK	3.2+.94M	3.6+.41M	3.7+.26M	3.8+.19M
HTRIDI,BISECT	3.2+.39M	3.6+.10M	3.7+.05M	3.8+.03M

TABLE 46

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
COMPLEX GENERAL MATRICES

MACHINE: Amdahl 470V/6, Fortran H, OPT=2
UNIVERSITY OF MICHIGAN

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.005	.040	.33	1.1	2.8
CBAL	.12	.05	.02	.01	.01
CBABK2	.07	.03	.02	.01	.01
COMHES	1	1	1	1	1
COMBAK	.46	.50	.54	.58	.60
CORTH	2.4	2.3	2.2	2.2	2.1
CORTB	1.3	1.3	1.2	1.2	1.1
COMQR2	18	15	13	13	12
COMLR2	9.4	8.1	6.7	6.5	6.2
COMQR	8.0	5.7	4.5	4.2	3.9
COMLR	5.1	3.4	2.4	2.2	2.0
CINVIT	4.8	3.3	2.5	2.2	2.3
PATHS					
CG (CBAL,CORTH,COMQR2,CBABK2)	20	18	16	15	14
CG (CBAL,CORTH,COMQR)	11	8.1	6.7	6.4	6.0
CBAL,CORTH,COMQR,CINVIT, CORTB,CBABK2	11+.62M	8.1+.23M	6.7+.09M	6.4+.06M	6.0+.04M

TABLE 47

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS FOR
REAL GENERAL MATRICES

MACHINE: Amdahl 470V/6, Fortran H, OPT=2
UNIVERSITY OF MICHIGAN

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.002	.014	.12	.41	.98
BALANC	.24	.09	.04	.02	.02
BALBAK	.12	.05	.02	.02	.01
ELMHES	1	1	1	1	1
ELTRAN	.16	.07	.03	.02	.02
ELMBAK	.54	.57	.57	.58	.59
ORTHES	2.4	2.2	2.1	2.0	1.9
ORTRAN	.81	.71	.65	.62	.60
ORTBAK	1.2	1.1	.98	.93	.90
HQR2	20	16	12	12	11
HQR	12	7.7	5.2	4.5	3.9
INVIT	4.5	3.4	2.7	2.4	2.3
PATHS					
RG (BALANC,ELMHES,ELTRAN, HQR2,BALBAK)	21	17	14	13	12
RG (BALANC,ELMHES,HQR)	13	8.8	6.2	5.5	4.9
BALANC,ELMHES,HQR,INVIT, ELMBAK,BALBAK	13+.51M	8.8+.20M	6.3+.08M	5.5+.05M	5.0+.04M

TABLE 48

SUMMARY OF EXECUTION TIMES FOR THE
EISPACK SUBROUTINES INCLUDED IN THE PATHS THAT
REDUCE FULL MATRICES TO REAL SYMMETRIC TRIDIAGONAL FORM

MACHINE: Amdahl 470V/6, Fortran H, OPT=2
UNIVERSITY OF MICHIGAN

	ORDER OF MATRIX				
SUBROUTINE	N=10	N=20	N=40	N=60	N=80
Time Unit (Sec)	.002	.013	.090	.29	.70
TRED1	1	1	1	1	1
TRED2	1.6	1.8	2.0	2.0	2.0
TRED3	.99	.98	1.0	.98	.96
TRBAK1	1.2	1.4	1.6	1.6	1.6
TRBAK3	1.0	1.2	1.4	1.4	1.4
HTRIDI	2.7	3.1	3.4	3.6	3.7
HTRID3	2.6	3.1	3.4	3.6	3.6
HTRIBK	3.3	4.4	5.1	5.4	5.5
HTRIB3	3.5	4.6	5.3	5.5	5.5
FIGI	.09	.03	.01	.00	.00
FIGI2	.15	.06	.03	.02	.01
BAKVEC	.10	.06	.03	.02	.02
TQL2	4.9	5.3	5.1	5.0	4.6
IMTQL2	5.0	5.2	5.3	5.0	4.8
TQLRAT	1.4	.81	.42	.27	.19
TQL1	2.3	1.5	.86	.57	.42
IMTQL1	2.3	1.5	.88	.58	.43
BISECT	11	6.8	3.7	2.5	1.8
TRIDIB	11	7.0	3.8	2.6	1.9
IMTQLV	2.4	1.6	.89	.59	.43
RATQR	3.4	2.6	1.7	1.2	.93
TINVIT	1.0	.67	.40	.29	.30
TSTURM	12	7.5	4.2	2.9	2.2
PATHS					
RS (TRED2,TQL2)	6.5	7.1	7.1	7.0	6.7
RS (TRED1,TQLRAT)	2.4	1.8	1.4	1.3	1.2
TRED1,BISECT,TINVIT,TRBAK1	1+1.3M	1+.44M	1+.14M	1+.07M	1+.05M
TRED1,BISECT	1+1.1M	1+.34M	1+.09M	1+.04M	1+.02M
CH (HTRIDI,TQL2,HTRIBK)	11	12	13	14	14
CH (HTRIDI,TQLRAT)	4.2	3.9	3.8	3.9	3.9
HTRIDI,BISECT,TINVIT,HTRIBK	2.7+1.5M	3.1+.59M	3.4+.23M	3.6+.14M	3.7+.10M
HTRIDI,BISECT	2.7+1.1M	3.1+.34M	3.4+.09M	3.6+.04M	3.7+.02M

TABLE 49

EXECUTION TIMES FOR SELECTED EISPACK DRIVER SUBROUTINES
OVER VARIOUS COMPUTER SYSTEMS MEASURED ON MATRICES OF ORDER 40
BOTH WITH EIGENVECTORS COMPUTED (1) AND WITHOUT (0)

MACHINE	CG(1)	CG(0)	RG(1)	RG(0)	RS(1)	RS(0)	CH(1)	CH(0)
Time Unit (Sec)	1.7	.77	.66	.31	.22	.055	.44	.14
IBM 370/195 (ARGONNE NATIONAL LABORATORY)	1	1	1	1	1	1	1	1
IBM 360/75 (UNIVERSITY OF ILLINOIS)	16	15	11	11	15	12	13	12
IBM 370/168 (UNIVERSITY OF MICHIGAN)	5.8	5.2	3.9	3.7	5.6	3.9	5.0	4.5
IBM 370/165 (THE UNIVERSITY OF TORONTO)	3.5	3.2	2.9	2.6	3.3	2.9	3.2	2.9
BURROUGHS 6700 (UNIVERSITY OF CALIFORNIA, SAN DIEGO)	118	108	95	92	125	82	122	100
CDC 6600 (KIRTLAND AIR FORCE BASE)	5.0	5.3	4.9	4.9	5.5	5.0	3.7	3.0
CDC 6600 (NASA LANGLEY RESEARCH CENTER)	4.5	4.3	5.5	5.6	5.3	5.0	4.2	3.8
CDC 7600 (NATIONAL CENTER FOR ATMOSPHERIC RESEARCH)	1.1	1.0	1.2	1.1	1.0	1.1	.81	.83
CDC 6400 (NORTHWESTERN UNIVERSITY)	22	22	19	18	21	18	17	17
CDC 6400/6500 (PURDUE UNIVERSITY)	31	32	42	38	24	20	22	21
CDC 6600/6400 (THE UNIVERSITY OF TEXAS)	8.2	8.9	17	15	6.1	5.9	5.6	5.6
HONEYWELL 6070 (BELL LABORATORIES)	21	20	9.9	9.4	15	13	14	14
UNIVAC 1110 (THE UNIVERSITY OF WISCONSIN)	11	11	8.9	8.4	11	11	9.8	9.2
DEC PDP-10 (YALE UNIVERSITY)	155	135	104	98	132	106	146	139
AMDAHL 470V/6 (UNIVERSITY OF MICHIGAN)	3.1	2.9	2.4	2.3	2.9	2.4	2.7	2.6

TABLE 50

SUMMARY OF EXECUTION TIMES FOR SELECTED PROBLEMS
USING THE EISPAC CONTROL PROGRAM

MACHINE: IBM 370/195, Fortran H, OPT=2
ARGONNE NATIONAL LABORATORY

Time Unit: 1 second

PROBLEM CLASS	EIGEN-VALUES	EIGEN-VECTORS	TRANSFOR-MATIONS	ORDER OF MATRIX N=10	N=20	N=40	N=60	N=80
CG	All	All	Unitary	.044	.25	1.7	5.7	13
CG	All	Some (M=N)	Unitary	.047	.22	1.3	4.2	9.6
CG	All	All	Elementary	.032	.16	1.0	3.3	7.6
CG	All	Some (M=N)	Elementary	.041	.17	.92	2.7	6.7
RG	All	All	Elementary	.023	.10	.63	2.0	4.5
RG	All	Some (M=N)	Elementary	.024	.095	.51	1.5	3.4
RG	All	All	Orthogonal	.026	.12	.72	2.3	5.2
RG	All	Some (M=N)	Orthogonal	.026	.10	.59	1.8	4.0
RS	All	All		.011	.040	.22	.65	1.5
RS	All	None		.007	.016	.059	.15	.33
RS	Some (M=N)	Some (M=N)		.025	.080	.33	.81	1.6
RS	Some (M=N)	None		.022	.066	.24	.56	1.0
CH	All	All		.016	.07	.44	1.4	3.4
CH	All	None		.009	.026	.14	.42	1.0
CH	Some (M=N)	Some (M=N)		.029	.11	.52	1.5	3.3
CH	Some (M=N)	None		.023	.08	.33	.83	1.8

Section 4.2

REPEATABILITY AND RELIABILITY OF THE MEASURED EXECUTION TIMES

In the multi-program environment of modern computers, it is often very difficult to reliably measure the execution time of a program. Significant variations can occur depending on the load on the machine and the amount of I/O interference. For the EISPACK subroutines in particular, allowance has to be made also for the dependence of execution times upon the matrix (see Section 4.3).

Towards improving the usefulness of the timing information for the various machines, each subroutine was run a number of times on different matrices and the average time reported. Also, the timing programs were structured to perform no output until all the timings had been collected. For the most part, the timing routines used to measure execution time factor out external interrupts which may occur. The reported times, therefore, although not to be interpreted as absolute bounds, should be useful in giving a feeling for the execution time required by the individual subroutines or paths.

Section 4.3

DEPENDENCE OF THE EXECUTION TIMES UPON THE MATRIX

To characterize the dependence of the execution times of the EISPACK subroutines upon the matrix, it is helpful to divide the subroutines into three categories: first, the non-iterative reduction and back transformation subroutines; second, the iterative reduced-form eigenvalue-eigenvector subroutines; and third, the semi-iterative subroutines which balance a matrix and back transform the eigenvectors of the balanced matrix.

For the first category of subroutines, the execution times for all non-sparse matrices of a given order are approximately constant, close to those given in the tables of Section 4.1. For sparse matrices, some of the reduction transformations may be skipped and the arithmetic unit may consume less time to manipulate the many zero operands; however, except for special sparse matrices, the decrease in execution time of these subroutines is marginal.

For the second category of subroutines, their execution times depend greatly upon the structure of the input matrix (e.g., diagonally dominant, blocked, sparse), the closeness of the eigenvalues, and the defectiveness of the matrix. Based upon our experiments on the IBM 370/195, random matrices tend to produce slower execution of category two subroutines, and therefore the execution times given in the tables of Section 4.1 appear to be near the maximum for these subroutines.

For the third category of subroutines, their execution times are somewhat variable but, in general, are negligible compared with the execution times of paths in which they are included. For example, the algorithms BALANC and BALBAK together require about 0.3% of the execution time in the path BALANC-ELMHES-ELTRAN-HQR2-BALBAK used to find the complete eigensystem of the order 80 real general matrix of Table 5, and CBAL and CBABK2 together require an even smaller percentage of the time for the corresponding complex matrix of Table 4. In contrast, we have contrived a matrix for which the combination of BALANC and BALBAK requires 7% of the normal path time, but this matrix represents an extreme case. More commonly, the execution times for these subroutines are only a few percent of the total path times.

Section 4.4

EXTRAPOLATION OF TIMING RESULTS TO OTHER MACHINES AND COMPILERS

The extrapolation of timing results to other machines and compilers may appear straightforward but can be grossly inaccurate if characteristics of the particular machines and compilers are not carefully considered. To illustrate this point, we make two comparisons: first, the execution times of EISPACK on an IBM 360/75 and 370/195, and second, the execution times of EISPACK compiled with the CDC 6600 Fortran RUN and FTN, OPT=1 compilers.

The eigensystems of several random matrices of order 80 were determined using identical object modules on the IBM 360/75 and 370/195. The ratios of the IBM 360/75 times to the 370/195 times show large variations from subroutine to subroutine (compare Tables 4-6 with Tables 7-9). Many of the ratios are between 11 and 14, but at opposite extremes the ratio for BISECT is 6.7 whereas for TQL2, 19.4.

These large variations in the ratios of execution times among the subroutines are attributable to the special architecture of the IBM 370/195; namely, the buffered or two-level memory, the pipe line or parallel arithmetic units, and the instruction stack. Some of the EISPACK subroutines are able to take advantage of these special features of the 370/195 better than others.

The compiler can also affect the relative efficiencies of EISPACK subroutines. On two comparable CDC 6600 computers, one employing the RUN compiler and the other the FTN, OPT=1 compiler, the ratios of the execution times for the EISPACK subroutines were determined (compare Tables 22-24 with Tables 34-36). Many of the ratios are close to 1, but extremes as large as 2.2 for INVIT, 2.7 for HQR, and 3.0 for HQR2 obtain for real general matrices of order 60.

It is clear from these examples that the relative efficiency of an algorithm can be very machine and compiler dependent, and hence that the extrapolation of our timing results to other systems must necessarily include these considerations.

Section 4.5

THE SAMPLE MATRICES FOR THE TIMING RESULTS

In this section, Fortran listings of the program segments which generate the sample matrices for the timing results are provided. The matrix elements are pseudo-random integers sampled from a uniform distribution on the interval $(-2^{15},+2^{15})$. A listing of the auxiliary subroutine RANDOM which generates the random integers is also provided.

```
C     COMPLEX GENERAL MATRIX
C
      DO 20 I = 1,N
         DO 10 J = 1,N
            CALL RANDOM (INIT, AR(I,J))
            CALL RANDOM (INIT, AI(I,J))
   10    CONTINUE
   20 CONTINUE


C     COMPLEX HERMITIAN MATRIX
C
      IF (N .EQ. 1) GO TO 30
      DO 20 I = 2,N
         CALL RANDOM (INIT, AR(I-1,I-1))
         AI(I-1,I-1) = 0.0
         DO 10 J = I,N
            CALL RANDOM (INIT, AR(I-1,J))
            AR(J,I-1) = AR(I-1,J)
            CALL RANDOM (INIT, AI(I-1,J))
            AI(J,I-1) = -AI(I-1,J)
   10    CONTINUE
   20 CONTINUE
   30 CALL RANDOM (INIT, AR(N,N))
      AI(N,N) = 0.0


C     COMPLEX HERMITIAN PACKED MATRIX
C
      DO 20 I = 1,N
         DO 10 J = 1,N
            CALL RANDOM (INIT, A(I,J))
   10    CONTINUE
   20 CONTINUE


C     REAL GENERAL MATRIX
C
      DO 20 I = 1,N
         DO 10 J = 1,N
            CALL RANDOM (INIT, A(I,J))
   10    CONTINUE
   20 CONTINUE


C     REAL SYMMETRIC MATRIX
C
      DO 20 I = 1,N
         DO 10 J = I,N
            CALL RANDOM (INIT, A(I,J))
            A(J,I) = A(I,J)
   10    CONTINUE
   20 CONTINUE
```

```
C     REAL SYMMETRIC PACKED MATRIX
C
      NNN = N*(N+1)/2
      DO 10 I = 1,NNN
         CALL RANDOM (INIT, A(I))
   10 CONTINUE
```

```
C     REAL SYMMETRIC TRIDIAGONAL MATRIX
C
      CALL RANDOM (INIT, A(1,2))
      IF (N .EQ. 1) GO TO 30
      DO 20 I = 2,N
         DO 10 J = 1,2
            CALL RANDOM (INIT, A(I,J))
   10    CONTINUE
   20 CONTINUE
   30 CONTINUE
```

```
C     SPECIAL REAL NON-SYMMETRIC TRIDIAGONAL MATRIX
C
      DO 20 I = 1,N
         DO 10 J = 1,3
            CALL RANDOM (INIT, A(I,J))
   10    CONTINUE
   20 CONTINUE
      IF (N .EQ. 1) GO TO 40
      DO 30 I = 2,N
         A(I,1) = SIGN(A(I,1), A(I-1,3))
   30 CONTINUE
   40 CONTINUE
```

Subroutine RANDOM:

```
      SUBROUTINE RANDOM (INIT,X)
      REAL X
      INIT = MOD(3125*INIT, 65536)
      X = INIT - 32768
      RETURN
      END
```

Subroutine RANDOM produces pseudo-random integer elements X between -2^{15} and 2^{15} from a starting integer INIT. It was designed so that it could be implemented in Fortran and would produce the same set of pseudo-random numbers on different machines. As a result it has some shortcomings as a random number generator, including a rather short period of 2^{14} numbers.