NEW ATTACKS ON DOL EQUIVALENCE PROBLEM

EQUIVALENCE OF L-SYSTEMS

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This paper summarizes some results concerning decidability of various kinds of equivalence problems for classes of L-systems - primarily DOL-systems. The reader is assumed to be familiar with some standard definitions and notations from the theory of L-systems.

For any finite alphabet, $\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$, let π_Σ denote the mapping that associates with each word from Σ^* its corresponding Parikh-vector, i.e., for every word $x \in \Sigma^*$, $\pi_\Sigma(x)$ is the vector, in which the i'th component is the number of occurrences of σ_i in x.

For any class of deterministic L-systems, you may consider equivalence with respect to

WL(WS) : the set (sequence) of words generated

PL(PS) : the set (sequence) of Parikh-vectors associated with the words generated

NL(NS) : the set (sequence) of lengths of the words generated

Note that WL-, PL-, and NL-equivalence are also well-defined for nondeterministic L-systems.

The decidability of the corresponding six equivalence problems for DOL-systems is considered in [70]. The following result is proved ($|\Sigma|$ denotes the cardinality of Σ):

Theorem 1

For any two DOL-systems over some alphabet Σ, generating sequences of words $\{w_i\}$ and $\{v_i\}$ respectively:

1) $\forall$ i, $0 \le i$: $\quad \pi_\Sigma(w_i) = \pi_\Sigma(v_i)$

iff

2) $\forall$ i, $0 \le i \le |\Sigma|$: $\quad \pi_\Sigma(w_i) = \pi_\Sigma(v_i)$

A direct consequence of Theorem 1 is

Theorem 2

PS-equivalence is decidable for DOL-systems.

Furthermore, the following two theorems are proved in [70]:

Theorem 3

PL-equivalence is decidable for DOL-systems.

Theorem 4

WL-equivalence is decidable for DOL-systems iff WS-equivalence is decidable for DOL-systems.

These theorems leave open one of the main open questions in the theory of L-systems, namely the decidability of WL- and WS-equivalence for DOL-systems. It is known that these equivalence problems are decidable for some subclasses of DOL, e.g., ([41]):

Theorem 5

WL-equivalence is decidable for any class of unary L-systems (systems over a one-letter alphabet).

The following result is fairly easy to prove (a stronger version of the theorem has been proved by P. Johansen):

Theorem 6

WS-equivalence is decidable for locally catenative ([95]) DOL-systems.

Furthermore, G. Rozenberg has proved:

Theorem 7

WS-equivalence is decidable for DOL-systems with polynomial growth ([75]).

On the other hand, WL- and WS-equivalence are also known to be undecidable for some classes of systems, that include DOL. The following two theorems are proved in [80], [84], and [99]:

Theorem 8

WL-equivalence is undecidable for POL-systems.

Theorem 9

WL-equivalence is undecidable for PDTOL-systems.

Furthermore, using an idea suggested by P. Vitanyi (originally to prove undecidability of NS-equivalence) you can prove:

Theorem 10

All six equivalence problems considered in this paper are undecidable for D1L-systems.

It seems likely, however, that WS- and thereby WL-equivalence is decidable for DOL-systems. The following two results which are somewhat related to the problems are proved in [70]:

Theorem 11

There exists an algorithm that will produce for any reduced ([70]) DOL-system over an alphabet Σ, all (finitely many) systems over Σ, which are PL-(PS-) equivalent to the given system.

Theorem 12

Let S_1 and S_2 be two WS-equivalent DOL-systems over an alphabet Σ, for which the first $|\Sigma|$ generated Parikh-vectors are linearly independent, then $S_1 = S_2$.

(Note that the property "reduced" is decidable for OL-systems, but not for 1L-systems ([33])).

The following conjecture is suggested:

Conjecture

There exists a computable function f, mapping integers to integers, such that for any two DOL-systems over some alphabet Σ, generating sequences of words $\{w_i\}$ and $\{v_i\}$:

1) $\forall$ i, $0 \leq i$: $w_i = v_i$

iff

2) $\forall$ i, $0 \leq i \leq f(|\Sigma|)$: $w_i = v_i$

This conjecture implies, of course, the decidability of WS-equivalence for DOL-systems. Note the similarity between the conjecture and Theorem 1, which states that for sequences of Parikh-vectors generated, the conjecture is true with f as the identity-function. That this is not the case for sequences of words generated, is seen from the following example.

Example

Consider the two DOL-systems, S_1 and S_2, over the alphabet $\Sigma = \{a_i, b_i \mid 1 \leq i \leq n\}$.

	S_1	S_2
axiom	$a_1\ b_1$	$a_1\ b_1$
productions	$a_1 \rightarrow a_2$	$a_1 \rightarrow a_2$
	$b_1 \rightarrow b_2$	$b_1 \rightarrow b_2$
	.	.
	.	.
	.	.
	.	.
	$a_{n-1} \rightarrow a_n$	$a_{n-1} \rightarrow a_n$
	$b_{n-1} \rightarrow b_n$	$b_{n-1} \rightarrow b_n$
	$a_n \rightarrow a_1\ b_1\ b_1$	$a_n \rightarrow a_1\ b_1\ b_1\ a_1\ a_1\ b_1\ b_1$
	$b_n \rightarrow a_1\ a_1\ b_1\ b_1\ a_1$	$b_n \rightarrow a_1$

It is easy to verify that the sequences of words generated by these two systems coincide until the 3n'th generated word and no longer. This implies that if the above conjecture is true, then $f(i) > 1\frac{1}{2} \cdot i$ for every integer i.

Finally, concerning length-equivalence of DOL-systems [75]:

<u>Theorem 13</u>

NS-equivalence is decidable for DOL-systems.

Decidability of NL-equivalence is still an open question for DOL-systems. In [70] the following theorem was proved.

<u>Theorem 14</u>

NL-equivalence is decidable for PDOL-systems.

But the proof of Theorem 14 builds essentially on the propagating property of the systems, and furthermore, J. Karhumaki has shown that there exists a DOL-system for which the range of its growth-function is not the range of the growth-function of any PDOL-system.