An Inverse Kinematics Model For Post-operative Knee

Ligament Parameters Estimation From Knee Motion

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Abstract. A motion-based Inverse Kinematics Knee (IKK) model was developed for Total Knee Replacement (TKR) joints. By tracking a sequence of passive knee motion, the IKK model estimated ligament properties such as insertion locations. The formulation of the IKK model embedded a Forward Kinematics Knee (FKK) [1] model in a numerical optimization algorithm known as the Unscented Kalman Filter [2]. Simulation results performed on a semi-constrained TKR design suggested that ligament insertions could be accurately estimated in the medial-lateral (ML) and the proximal-distal (PD) directions, but less reliably in the anterior-posterior (AP) direction for the tibial component. However, the forward kinematics produced by both the true and estimated ligament properties were nearly identical, suggesting that the IKK model recovered a *kinematically equivalent* set of ligament properties. These results imply that it may not be necessary to use a patient-specific CT or MRI scan to locate ligaments, which considerably widens potential applications of kinematic-based total knee replacement.

1 Introduction

We previously introduced [3] and validated [1] a Forward Kinematics Knee (FKK) model for postoperative knees. Given a set of joint parameters such as ligament insertion locations, our FKK model *predicts* the location of the femorotibial contact for each joint angle using the principle of ligament strain minimization [4]. Knee motion can be reconstructed by finding successive femorotibial contact from full extension to full flexion.

We now introduce an Inverse Kinematics Knee (IKK) model that performs the opposite: by tracking a sequence of knee motion, the IKK model decomposes the motion into a set of *actual* joint angles and the corresponding femorotibial contact locations. The observed joint angles are used by the FKK model to produce a set of *predicted* femorotibial contacts that are contrasted with the observed contacts. Together, the *predictor-actual* pair of contacts are used in the Unscented Kalman Filter [5] to estimate the joint parameters that would lead to the observed knee motion.

R. Larsen, M. Nielsen, and J. Sporring (Eds.): MICCAI 2006, LNCS 4190, pp. 313–320, 2006.

2 Method

Articular surfaces of a size-3 Sigma Knee (Johnson & Johnson) were laser-scanned at a resolution of $0.4mm$, resulting in two point clouds of approximately $31,000$ and $19,000$ points for the femoral (F) and tibial (T) components, respectively. Joint coordinate systems [6] were assigned to the components. The *absolute*, space-fixed coordinate system was associated with the tibial component, and the *relative*, body-fixed coordinate system was associated with the femoral component. Without loss of generality, the Z-axes were aligned with the anatomical axis of the lower limb. The X-axes were perpendicular to the Z-axes lying on the sagittal plane with the anterior direction being positive. The Y-axes were derived as the cross product of the two: $Y = Z \times X$. The centroid of the point clouds were chosen as the origin of the coordinate systems.

These two coordinate systems were related by a homogeneous transformation. If $\bar{p}^f$ is a 3×1 column vector that measures the coordinate of a point in the femoral system, then its corresponding tibial location $\bar{p}^t$ can be expressed as:

$$\begin{bmatrix} \bar{p}^t \\ 1 \end{bmatrix} = \begin{bmatrix} R \; \bar{d} \\ 0 \; 1 \end{bmatrix} \begin{bmatrix} \bar{p}^f \\ 1 \end{bmatrix} \tag{1}$$

where R is a 3×3 rotation matrix and $\bar{d}$ is a 3×1 translation vector. Because the tibial was assumed to be fixed, R and $\bar{d}$ represented the relative joint angle and position of the femoral component. The rigid-body transformation of a point is a rotation (R) to the space-fixed coordinate system followed by a linear displacement ($\bar{d}$).

2.1 Forward Kinematics Knee Model

The FKK for the passive knee was developed based on the principle of ligament strain minimization[4]: In a passive knee where no external force and torque is present, the knee would rest into an equilibrium position where the strain stored in the knee ligaments is minimized. The passive FKK model can be formalized as:

$$\bar{d} = FKK(F, T, R, \bar{L}) \tag{2}$$

where F and T are the geometry of the femoral and the tibial TKR components, respectively, R is the femorotibial joint angle, and $\bar{L}$ is a state vector representing the mechanical properties of knee ligaments. $\bar{L}$ is a concatenation of n filament state vectors, each with the following format:

$$\bar{l}^i = [p^f_x, p^f_y, p^f_z, p^t_x, p^t_y, p^t_z, K, B]^T \tag{3}$$

where $\bar{l}^i$ represents the i^{th} filament of the ligament, $[p^f_x, p^f_y, p^f_z]^T$ and $[p^t_x, p^t_y, p^t_z]^T$ are the 3D femoro- and the tibial- insertion locations, respectively, and K and B are the optional spring constants for the filament. The filaments are modeled as springs that store no compressive force, with the strain energy of each filament calculated as:

$$E = \begin{cases} .5 \times K \times (L' - \tilde{L})^2 + B \times (L' - \tilde{L}), & \text{if } L' \geq \tilde{L} \\ 0, & \text{if } L' < \tilde{L} \end{cases} \tag{4}$$

where $\tilde{L}$ is the neutral length of the filament, L' is the Euclidean distance between filament attachment points, and K and B are the spring constants.

The vector $\bar{d}$ in Eqn. (2) represents the femorotibial contact location where the total ligament strain energy stored in $\bar{L}$ is minimized [3]; thus, the 4×4 homogeneous matrix $\begin{bmatrix} R & \bar{d} \\ 0 & 1 \end{bmatrix}$ completely specifies the femorotibial pose. Knee kinematics can be reconstructed by varying R from full extension to full flexion and computing the corresponding displacement vectors $\{\bar{d}\}$.

2.2 FKK Model Validation

A spring-ligament apparatus mimicking passive TKR knee was constructed as a validation tool for the FKK model [1]. The apparatus is composed of the laser-

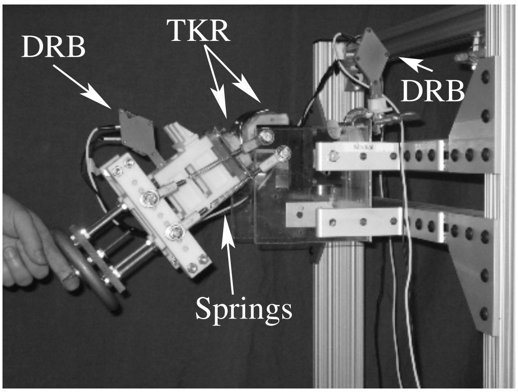

Fig. 1. A spring-ligament apparatus simulating the kinematics of passive knee

scanned TKR components held in contact by the tensile forces stored within a set of mechanical springs. The mechanical properties needed to satisfy Eqn. (3) were determined for each spring.

A Dynamic Reference Body (DRB) was rigidly attached to each TKR component, allowing the motion of the components to be tracked with an optical tracker (OPTOTRAK 3020, NDI, Canada). Let P_f and P_t be the poses of the DRBs, in the OPTOTRAK coordinate system, attached to the femoral and tibial components, respectively, and let Q_f and Q_t be the 4×4 registration matrices that register the scanned femoral and the tibial point clouds to their respected components. Then for a given tracked instance, the relative femorotibial pose, in the scanned tibial point cloud coordinate system, is given by Eqn. (5):

$$\begin{bmatrix} R' & \bar{d}' \\ 0 & 1 \end{bmatrix} = Q_t^{-1} P_t^{-1} P_f Q_f \tag{5}$$

where R' is the *actual* femorotibial angle and the $\bar{d}'$ is the *actual* femoral displacement. Thus, knowing the spring ligament properties of the apparatus, the observed *actual* femorotibial angle R' can be used as the input to Eqn. (2); the computed virtual displacement vector $\bar{d}$ be compared to the actual displacement vector $\bar{d}'$.

Given accurate inputs needed by Eqn.(2), the FKK model was found [1] to predict the femorotibial contact locations (and the displacement vector $\bar{d}$) with sub-millimeter accuracy. Simulation results suggested that, when a semi-constrained TKR (such as the Sigma) is used, a simple single-bundle ligament model is sufficient to generate accurate knee kinematics: it is better to get a good rough estimate of knee-ligament geometry rather than to toil for a other aspect of a complex ligament model.

2.3 Unscented Kalman Filter

The Square-Root Unscented Kalman Filter [5] (SQ-UKF) was implemented as the numerical optimization algorithm for its numerical stability and efficiency. Kalman and related filters operate in a *predictor-corrector* and *iterative* fashion. In the Kalman filter paradigm, the state of a stochastic process is represented by a set of variables that evolves through time according to a *process* model. The state variables are related to a set of observable variables through a *measurement* model. When an new observation is available, the current state vector is advanced in time through the process model and a *predicted* measurement is generated through the measurement model accordingly. The difference between the actual and the predicted measurements, called the *innovation*, is used to correct the state vector so the state vector better fits the observation.

We apply the UKF to ligament parameter estimation in which the ligament properties were treated as the unknown. The *process* and *measurement* models needed for UKF are:

$$\bar{L}_{k+1} = \bar{L}_k + v_k \tag{6}$$

$$\bar{d}_k = FKK(F, T, R, \bar{L}_k,) + n_k \tag{7}$$

That is, the process model for parameter estimation is the *identity* function, because the quantities to be estimated are assumed to remain constant over time. The measurement model is the FKK model itself, where the quantity to be measured, as a function of the ligament state vector, is the displacement vector $\bar{d}$ of a given femorotibial angle R. The noise vectors v_k and n_k refer to the uncertainties of the process and measurement models, respectively.

A set of 1089 *synthetic* displacement vectors $\bar{d}$ of varying femorotibial angles was precomputed using a 3-ligament (PCL, MCL, and LCL) FKK model. The single-bundle ligament geometry was adapted from our previous study [3]: the single-bundle ligament was artifically generated by taking the geometrical mean of the ligament insertions in the multi-bundle ligament configuration, using the summed spring constants. The femorotibial angle ranged from 0° to 120° in flexion, −1° to 1° in varus, and −1° to 1° in internal angulation, all at 1° intervals. This set of *synthetic* displacement vectors is destinated as the *true* observations since it was calculated using the known and correct ligament geometry.

Due to the computational complexity, our simulations were performed using a 2-ligament state vector (MCL plus LCL) in which the ligament properties of the PCL were assumed known. Furthermore, the spring constants needed for Eqn.(3) were assumed to be known because the formulation of Eqn. (4) did not require the absolute value of the spring constants, only the relative stiffness between the ligaments [7]. Hence the state vector for the IKK model has a dimensionality of 12, in which the femoral and the tibial insertions of both the MCL/LCL were the unknowns.

The iterative process of UKF parameter estimation proceeded as follows. First, an initial guess of each of the ligament insertion was given and the state vector $\bar{L}$ need for Eqn. (2) was constructed according to Eqn. (3). At each itera-

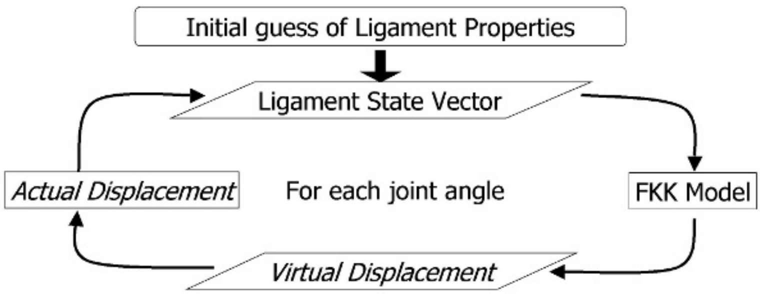

Fig. 2. The iterative process of UKF parameter estimation. When a new observation becomes available, the current estimate of the state vector is used to predict the femorotibial contact locations using the FKK model. The difference in the predicted and the observed femorotibial contact locations, call the *innovation*, is used to correct the state vector estimation.

tion when a new observation (i.e. an actual displacement $\bar{d}'$) became available, a *predicted* measurement (i.e. $\bar{d}$ using Eqn. (2)) was computed using the estimated state vector. The *innovation* term was calculated and propagated through the UKF formulation [5], and the state vector was updated accordingly.

3 Results

Figure 3 depicts a typical result of the ligament parameter estimation using the UKF paradigm. Each of the 4 ligament insertion locations was erroneously and intentionally chosen at a location $10mm$ away from the true insertion.

Figure 3(a) shows the convergence of each of the MCL-femoral, MCL-tibial, LCL-femoral, and LCL-tibial insertion from their respective initial guess. During the UKF iterations, all 4 insertion estimates converged to a steady-state after about 200 observations. After this point, the LCL-tibial insertion estimate remained about $4.0mm$ away from the true location, and the MCL-tibial insertion estimate remained about $4.7mm$ away from the true location. The femoral insertion estimates for both ligaments had errors about $2.0mm$

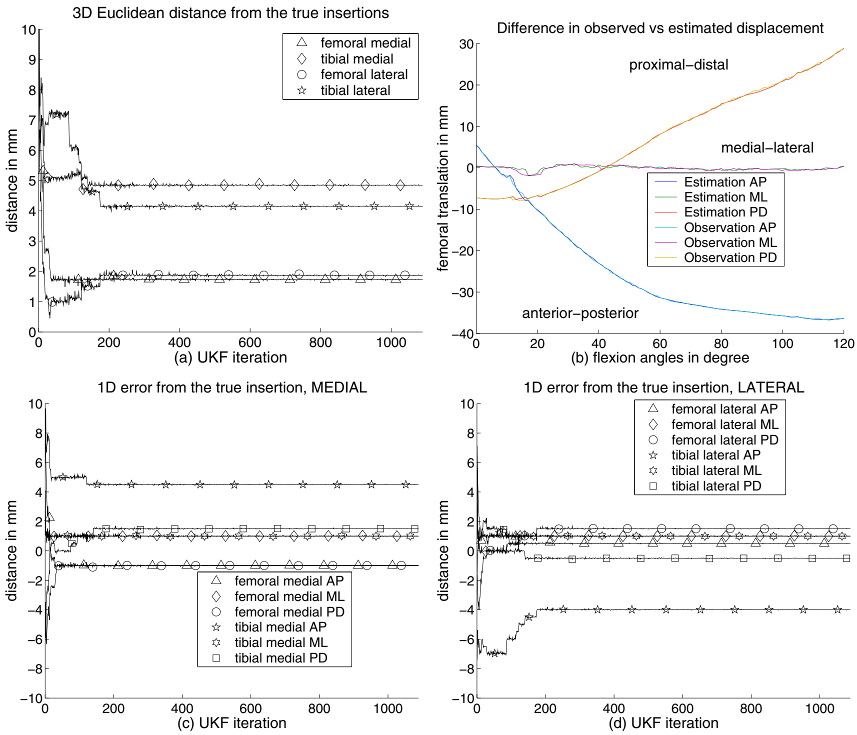

Fig. 3. Simulation results for ligament parameter estimation using UKF. Insertion locations of both MCL and LCL were initialized to be $10mm$ away from the true location.

After the ligament estimates reach a steady state, they were used in the FKK model to produce a sequence of passive knee motion from full extension $(0°)$to full flexion $(120°)$ at $1°$ intervals. Figure 3(b) depicts a visualization of knee kinematics by plotting the (x, y, z) components of the displacement vector $\{\bar{d}\}$: Both the true and the estimated ligament insertions produced near identical passive forward kinematics.

Figure 3(c) and (d) show the convergence of the ligament insertions of the MCL and the LCL ligament, respectively, in each of the AP, ML, and PD directions. For MCL ligament estimation (Figure 3(c)), the largest error was the tibial AP (x-axis) direction, which had a magnitude of about $4mm$. Errors in the other two directions had a magnitude of $2mm$ or smaller. For LCL ligament estimation (Figure 3(d)), the largest error was the tibial AP (x-axis) direction, with a magnitude of about $4mm$. Errors in other directions were $2mm$ or smaller in magnitude.

4 Discussion

The complexity of the SQ-UKF parameter estimation algorithm is $O(n^2)$ [5], where n is the dimension of the state vector $\bar{L}$. Using a 2GHz computer with sufficient memory, the IKK model took about $5-7$ days to iterate through 1089 observations. It should be noted that the IKK process can be terminated as soon as the state vector reaches a steady-state: on the example shown above, the IKK model provided a steady-state estimate in about 1 day. In addition, the UKF can be implemented with ease in a multi-threaded environment: up to $(2n+1)$ computers can be used to obtain a linear speed-up.

In the example shown above, the parameter estimation process was not able to fully recover all components in the state vector. In particular, the tibial anterior-posterior component for both MCL and LCL insertion sites had much higher error compared to other two directions. However, the resulting forward kinematics generated by both the true and the estimated ligament insertions were nearly identical. This explains why the IKK could not converge any further: the *innovation* vectors needed to correct the state vector were almost zero and thus had negligible effect on updating the state vector. One can also think of this phenomena as reaching a local minima, where there exist multiple solutions to the IKK model. Thus, the IKK model has recovered a *kinematics-equivalent* ligament properties.

The sensitivity of the surgical placement on the resulting kinematics can be attributed to the geometry of the TKR components. Most TKR designs are *more* congruent on the coronal plane than on the sagittal plane; thus a small misplacement of the TKR components in the coronal plane would greatly influence the elongation of the knee ligaments (see Eqn. (4)) changing the femorotibial contact location. For the same reason, the surgical placement of the femoral component would have a more profound effect on the knee ligament because is is more convex than the tibial component.

5 Conclusion

We introduced an Inverse Kinematics Knee model that embeds a Forward Kinematics Knee model in the Unscented Kalman Filter paradigm. Given an observed knee motion and an initial guess of the ligament properties, the IKK model recovers a set of *kinematics-equivalent* ligament properties with a semi-constrained TKR design. Simulation results suggested that:

– Surgeons have some freedom in the surgical placement of the tibial component in the anterior-posterior direction without changing the postoperative passive kinematics,
– Precise placement of the femoral component is indicated as the resulting kinematics are sensitive to the placement,
– There is a direct, and yet complex, relationship between the TKR design, the surgical placement, and the resulting knee kinematics: The less congruent the TKR geometry is in a given direction, the greater freedom the surgeon has in the surgical placement of the component in that direction.

This study is limited by the number of ligaments examined, the configuration of the ligaments, and the number of prosthesis designs included in the study. Future works include applying the IKK model to more TKR designs, for postoperative TKR assessment, and extending the IKK model to natural knees.

We believe that the IKK model may provide a new paradigm for TKR surgical planning; the ligament insertions may be estimated prior to TKR surgery. These results may suggest that it is not necessary to use a patient-specific CT or MRI scan to locate ligaments, which considerably widens potential applications of kinematic-based total knee replacement. Further inquiry into this subject is indicated.

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