More on Bisimulations for Higher Order π-Calculus*

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Abstract. In this paper, we prove the coincidence between strong/weak context bisimulation and strong/weak normal bisimulation for higher order π-calculus, which generalizes Sangiorgi's work. To achieve this aim, we introduce indexed higher order π-calculus, which is similar to higher order π-calculus except that every prefix of any process is assigned to indices. Furthermore we present corresponding indexed bisimulations for this calculus, and prove the equivalence between these indexed bisimulations. As an application of this result, we prove the equivalence between strong/weak context bisimulation and strong/weak normal bisimulation.

1 Introduction

Higher order π-calculus was proposed and studied intensively in Sangiorgi's dissertation [6]. It is an extension of the π-calculus [5] to allow communication of processes rather than names alone. In [6], some interesting bisimulations for higher order π-calculus were presented, such as barbed equivalence, context bisimulation and normal bisimulation. Barbed equivalence can be regarded as a uniform definition of bisimulation for a variety of concurrency calculi. Context bisimulation is a very intuitive definition of bisimulation for higher order π-calculus, but it is heavy to handle, due to the appearance of universal quantifications in its definition. In the definition of normal bisimulation, all universal quantifications disappeared, therefore normal bisimulation is a very economic characterisation of bisimulation for higher order π-calculus.

The main difficulty with definitions of context bisimulation and barbed equivalence that involve quantification over contexts is that they are often awkward to work with directly. It is therefore important to look for more tractable characterisations of the bisimulations. In [6, 7], the equivalence between weak normal bisimulation, weak context bisimulation and weak barbed equivalence was proved for early and late semantics respectively, but the proof method cannot be adapted to prove the equivalence between strong context bisimulation and strong normal bisimulation.

To the best of our knowledge, no paper gives the proof of equivalence between strong context bisimulation and strong normal bisimulation. In [7], this

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problem was stated as an open problem. The main difficulty is that the proof strategy for the equivalence between weak context bisimulation and weak normal bisimulation does not work for the strong case. Roughly speaking, for the case of weak bisimulations, the mapping to triggered processes will bring some redundant tau actions. Since weak bisimulations abstract from tau action, the problem is inessential. But for the case of strong bisimulations, the situation is different. We have to match these redundant tau actions to prove that two processes are bisimilar. Therefore we need some new proof strategies to solve the problem.

The main aim of this paper is to give a uniform proof for the equivalence between strong/weak context bisimulation and strong/weak normal bisimulation. Especially, we will give a proof of the coincidence between strong context bisimulation and strong normal bisimulation, which solves an open problem presented by Sangiorgi in [7]. To achieve this aim, we introduce the notion of indexed processes and define several bisimulations on indexed processes such as indexed context bisimulation and indexed normal bisimulation. Furthermore, we present indexed triggered mapping, prove an indexed factorisation theorem, and give the equivalence between these indexed bisimulations. As an application of this result, we get a uniform proof for the equivalence between strong/weak context bisimulation and strong/weak normal bisimulation.

This paper is organized as follows: Section 2 gives a brief review of syntax and operational semantics of the higher order π-calculus, then recalls the definitions of context and normal bisimulations. Section 3 introduces indexed higher order π-calculus and some indexed bisimulations. The equivalence between these indexed bisimulations also be proved. In Section 4 we give a proof for the equivalence between strong/weak context bisimulation and strong/weak normal bisimulation. The paper is concluded in section 5.

2 Higher Order π-Calculus

2.1 Syntax and Labelled Transition System of Higher Order π-Calculus

In this section we briefly recall the syntax and labelled transition system of the higher order π-calculus. Similar to [7], we only focus on a second-order fragment of the higher order π-calculus, i.e., there is no abstraction in this fragment.

We assume a set N of names, ranged over by $a, b, c, ...$ and a set Var of process variables, ranged over by $X, Y, Z, U, ...$. We use $E, F, P, Q, ...$ to stand for processes. Pr denotes the set of all processes.

We first give the grammar for the higher order π-calculus processes as follows:

$P ::= 0 \mid U \mid \pi.P \mid P_1|P_2 \mid (\nu a)P \mid !P$

π is called a prefix and can have one of the following forms:

$\pi ::= \tau \mid l \mid \overline{l} \mid a(U) \mid \overline{a}\langle P\rangle$, here τ is a tau prefix; l is a first order input prefix; $\overline{l}$ is a first order output prefix; $a(U)$ is a higher order input prefix and $\overline{a}\langle P\rangle$ is a higher order output prefix.

Table 1.

$$ALP: \frac{P \xrightarrow{\alpha} P'}{Q \xrightarrow{\alpha} Q'} P \equiv_\alpha Q, P' \equiv_\alpha Q' \qquad TAU: \tau.P \xrightarrow{\tau} P$$

$$OUT1: \bar{l}.P \xrightarrow{\bar{l}} P \qquad IN1: l.P \xrightarrow{l} P$$

$$OUT2: \overline{a}\langle E\rangle.P \xrightarrow{\overline{a}\langle E\rangle} P \qquad IN2: a(U).P \xrightarrow{a\langle E\rangle} P\{E/U\}$$

$$PAR: \frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} bn(\alpha) \cap fn(Q) = \emptyset$$

$$COM1: \frac{P \xrightarrow{\bar{l}} P' \quad Q \xrightarrow{l} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

$$COM2: \frac{P \xrightarrow{(\nu\widetilde{b})\overline{a}\langle E\rangle} P' \quad Q \xrightarrow{a\langle E\rangle} Q'}{P|Q \xrightarrow{\tau} (\nu\widetilde{b})(P'|Q')} \widetilde{b} \cap fn(Q) = \emptyset$$

$$RES: \frac{P \xrightarrow{\alpha} P'}{(\nu a)P \xrightarrow{\alpha} (\nu a)P'} a \notin n(\alpha) \qquad REP: \frac{P|!P \xrightarrow{\alpha} P'}{!P \xrightarrow{\alpha} P'}$$

$$OPEN: \frac{P \xrightarrow{(\nu\widetilde{c})\overline{a}\langle E\rangle} P'}{(\nu b)P \xrightarrow{(\nu b,\widetilde{c})\overline{a}\langle E\rangle} P'} a \neq b,\ b \in fn(E) - \widetilde{c}$$

For higher order π-calculus, the notations of free name, bound name, free variable, bound variable and etc are given in [6, 7]. The set of all closed processes, i.e., the processes which have no free variable, is denoted as Pr^c.

The operational semantics of higher order processes is given in Table 1. We have omitted the symmetric of the parallelism and communication rules.

2.2 Bisimulations in Higher Order π-Calculus

Context and normal bisimulations were presented in [6, 7] to describe the behavioral equivalences for higher order π-calculus. In the following, we abbreviate $P\{E/U\}$ as $P\langle E\rangle$.

Definition 1. A symmetric relation $R \subseteq Pr^c \times Pr^c$ is a strong context bisimulation if $P\ R\ Q$ implies:

(1) whenever $P \xrightarrow{\alpha} P'$, there exists Q' such that $Q \xrightarrow{\alpha} Q'$, here α is not a higher order action and $P'\ R\ Q'$;

(2) whenever $P \xrightarrow{a\langle E\rangle} P'$, there exists Q' such that $Q \xrightarrow{a\langle E\rangle} Q'$ and $P'\ R\ Q'$;

(3) whenever $P \xrightarrow{(\nu\widetilde{b})\overline{a}\langle E\rangle} P'$, there exist Q', F, $\widetilde{c}$ such that $Q \xrightarrow{(\nu\widetilde{c})\overline{a}\langle F\rangle} Q'$ and for all $C(U)$ with $fn(C(U)) \cap \{\widetilde{b}, \widetilde{c}\} = \emptyset$, $(\nu\widetilde{b})(P'|C\langle E\rangle)\ R\ (\nu\widetilde{c})(Q'|C\langle F\rangle)$. Here $C(U)$ is a process containing a unique variable U.

We write $P \sim_{Ct} Q$ if P and Q are strong context bisimilar.

Distinguished from strong context bisimulation, strong normal bisimulation does not have universal quantifications in the clauses of its definition. In the following, a name is called fresh in a statement if it is different from any other name occurring in the processes of the statement.

Definition 2. A symmetric relation $R \subseteq Pr^c \times Pr^c$ is a strong normal bisimulation if $P\ R\ Q$ implies:

(1) whenever $P \stackrel{\alpha}{\longrightarrow} P'$, there exists Q' such that $Q \stackrel{\alpha}{\longrightarrow} Q'$, here α is not a higher order action and $P'\ R\ Q'$;
(2) whenever $P \stackrel{a\langle \overline{m}.0\rangle}{\longrightarrow} P'$, there exists Q' such that $Q \stackrel{a\langle \overline{m}.0\rangle}{\longrightarrow} Q'$ and $P'\ R\ Q'$, here m is a fresh name;
(3) whenever $P \stackrel{(\nu\widetilde{b})\overline{a}\langle E\rangle}{\longrightarrow} P'$, there exist Q', F, $\widetilde{c}$ such that $Q \stackrel{(\nu\widetilde{c})\overline{a}\langle F\rangle}{\longrightarrow} Q'$ and $(\nu\widetilde{b})(P'|!m.E)\ R\ (\nu\widetilde{c})(Q'|!m.F)$, here m is a fresh name.

We write $P \sim_{Nr} Q$ if P and Q are strong normal bisimilar.

In the following, we use $\stackrel{\varepsilon}{\Longrightarrow}$ to abbreviate the reflexive and transitive closure of $\stackrel{\tau}{\longrightarrow}$, and use $\stackrel{\alpha}{\Longrightarrow}$ to abbreviate $\stackrel{\varepsilon}{\Longrightarrow}\stackrel{\alpha}{\longrightarrow}\stackrel{\varepsilon}{\Longrightarrow}$. By neglecting the tau action, we can get the following formal definitions of weak bisimulations:

Definition 3. A symmetric relation $R \subseteq Pr^c \times Pr^c$ is a weak context bisimulation if $P\ R\ Q$ implies:

(1) whenever $P \stackrel{\varepsilon}{\Longrightarrow} P'$, there exists Q' such that $Q \stackrel{\varepsilon}{\Longrightarrow} Q'$ and $P'\ R\ Q'$;
(2) whenever $P \stackrel{\alpha}{\Longrightarrow} P'$, there exists Q' such that $Q \stackrel{\alpha}{\Longrightarrow} Q'$, here α is not a higher order action, $\alpha \neq \tau$ and $P'\ R\ Q'$;
(3) whenever $P \stackrel{a\langle E\rangle}{\Longrightarrow} P'$, there exists Q' such that $Q \stackrel{a\langle E\rangle}{\Longrightarrow} Q'$ and $P'\ R\ Q'$;
(4) whenever $P \stackrel{(\nu\widetilde{b})\overline{a}\langle E\rangle}{\Longrightarrow} P'$, there exist Q', F, $\widetilde{c}$ such that $Q \stackrel{(\nu\widetilde{c})\overline{a}\langle F\rangle}{\Longrightarrow} Q'$ and for all $C(U)$ with $fn(C(U)) \cap \{\widetilde{b}, \widetilde{c}\} = \emptyset$, $(\nu\widetilde{b})(P'|C\langle E\rangle)\ R\ (\nu\widetilde{c})(Q'|C\langle F\rangle)$.

We write $P \approx_{Ct} Q$ if P and Q are weak context bisimilar.

Definition 4. A symmetric relation $R \subseteq Pr^c \times Pr^c$ is a weak normal bisimulation if $P\ R\ Q$ implies:

(1) whenever $P \stackrel{\varepsilon}{\Longrightarrow} P'$, there exists Q' such that $Q \stackrel{\varepsilon}{\Longrightarrow} Q'$ and $P'\ R\ Q'$;
(2) whenever $P \stackrel{\alpha}{\Longrightarrow} P'$, there exists Q' such that $Q \stackrel{\alpha}{\Longrightarrow} Q'$, here α is not a higher order action, $\alpha \neq \tau$ and $P'\ R\ Q'$;
(3) whenever $P \stackrel{a\langle \overline{m}.0\rangle}{\Longrightarrow} P'$, there exists Q' such that $Q \stackrel{a\langle \overline{m}.0\rangle}{\Longrightarrow} Q'$ and $P'\ R\ Q'$, here m is a fresh name;
(4) whenever $P \stackrel{(\nu\widetilde{b})\overline{a}\langle E\rangle}{\Longrightarrow} P'$, there exist Q', F, $\widetilde{c}$ such that $Q \stackrel{(\nu\widetilde{c})\overline{a}\langle F\rangle}{\Longrightarrow} Q'$ and $(\nu\widetilde{b})(P'|!m.E)\ R\ (\nu\widetilde{c})(Q'|!m.F)$, here m is a fresh name.

We write $P \approx_{Nr} Q$ if P and Q are weak normal bisimilar.

3 Indexed Processes and Indexed Bisimulations

3.1 Syntax and Labelled Transition System of Indexed Higher Order π-Calculus

The aim of this paper is to propose a general argument for showing the correspondence of context and normal bisimulations in both the strong and weak cases, by relying on a notion of indexed processes. Roughly, the intention is that indexed processes allow the labelled transition system semantics to record in action labels the indices of the interacting components. This mechanism is then used to filter out some tau transitions in the considered definition of bisimulation.

Now we introduce the concept of indexed processes.The index set I, w.l.o.g., will be the set of natural numbers. Intuitively, the concept of index can be viewed as the name or location of components. The class of the indexed processes IPr is built similar to Pr, except that every prefix is assigned to indices. We usually use K, L, M, N to denote indexed processes.

The formal definition of indexed process is given as follows:

$M ::= 0 \mid U \mid I\pi.M \mid M_1|M_2 \mid (\nu a)M \mid !M$

$I\pi$ is called indexed prefix and can be an indexed tau prefix or an indexed input prefix or an indexed output prefix:

$I\pi ::= \{\tau\}_{i,j} \mid \{l\}_i \mid \{\overline{l}\}_i \mid \{a(U)\}_i \mid \{\overline{a}\langle N\rangle\}_i$, i, j $\in$index set I (here N is an indexed process).

Similar to the original higher order π-calculus, in each indexed process of the form $(\nu a)M$ the occurrence of a is bound within the scope of M. An occurrence of a in M is said to be free iff it does not lie within the scope of a bound occurrence of a. The set of names occurring free in M is denoted $fn(M)$. An occurrence of a name in M is said to be bound if it is not free, we write the set of bound names as $bn(M)$. $n(M)$ denotes the set of names of M, i.e., $n(M) = fn(M) \cup bn(M)$. We use $n(M, N)$ to denote $n(M) \cup n(N)$. Indexed higher order input prefix $\{a(U)\}_i.M$ binds all free occurrences of U in M. The set of variables occurring free in M is denoted as $fv(M)$. We write the set of bound variables in M as $bv(M)$. An indexed process is closed if it has no free variable; it is open if it may have free variables. IPr^c is the set of all closed indexed processes. Indexed processes M and N are α-convertible, $M \equiv_\alpha N$, if N can be obtained from M by a finite number of changes of bound names and bound variables.

The set of all indices that occur in M, $Index(M)$, is defined inductively as follows:

(1) if $M = 0$ or U, then $Index(M) ::= \emptyset$;
(2) if $M = I\pi.M_1$, then $Index(M) ::= Index(I\pi) \cup Index(M_1)$, here $Index(I\pi)$ $::= \{i, j\}$ if $I\pi$ is in the form of $\{\tau\}_{i,j}$; $Index(I\pi) ::= \{i\} \cup Index(N)$ if $I\pi$ is in the form of $\{\overline{x}\langle N\rangle\}_i$; $Index(I\pi) ::= \{i\}$ if $I\pi$ is in the form of $\{l\}_i$ or $\{\overline{l}\}_i$ or $\{x(U)\}_i$.
(3) if $M = M_1|M_2$, then $Index(M) ::= Index(M_1) \cup Index(M_2)$;
(4) if $M = (\nu a)M_1$, then $Index(M) ::= Index(M_1)$;
(5) if $M = !M_1$, then $Index(M) ::= Index(M_1)$.

We use $Index(M, N)$ to denote $Index(M) \cup Index(N)$.

In the remainder of this paper, $\{P\}_i$ is an abbreviation for the indexed process with the same given index i on every prefix in the scope of P. The formal definition can be given inductively as follows:

(1) $\{0\}_i ::= 0$;
(2) $\{U\}_i ::= U$;
(3) $\{\tau.P\}_i ::= \{\tau\}_{i,i}.\{P\}_i$;
(4) $\{l.P\}_i ::= \{l\}_i.\{P\}_i$;
(5) $\{\overline{l}.P\}_i ::= \{\overline{l}\}_i.\{P\}_i$;
(6) $\{a(U).P\}_i ::= \{a(U)\}_i.\{P\}_i$;
(7) $\{\overline{a}\langle E\rangle.P\}_i ::= \{\overline{a}\langle\{E\}_i\rangle\}_i.\{P\}_i$;
(8) $\{P_1|P_2\}_i ::= \{P_1\}_i|\{P_2\}_i$;
(9) $\{(\nu a)P\}_i ::= (\nu a)\{P\}_i$;
(10) $\{!P\}_i ::= !\{P\}_i$.

In the labelled transition system of indexed higher order π-calculus, the label on the transition arrow is an indexed action, whose definition is given as follows:

$I\alpha ::= \{\tau\}_{i,j} \mid \{l\}_i \mid \{\overline{l}\}_i \mid \{a\langle K\rangle\}_i \mid \{\overline{a}\langle K\rangle\}_i \mid \{(\nu\widetilde{b})\overline{a}\langle K\rangle\}_i$, here $\{\tau\}_{i,j}$ is an indexed tau action, $\{l\}_i$ is an indexed first order input action, $\{\overline{l}\}_i$ is an indexed first order output action, $\{a\langle K\rangle\}_i$ is an indexed higher order input action, and $\{\overline{a}\langle K\rangle\}_i$ and $\{(\nu\widetilde{b})\overline{a}\langle K\rangle\}_i$ are indexed higher order output actions.

We write $bn(I\alpha)$ to represent the set of names bound in $I\alpha$, which is $\{\widetilde{b}\}$ if $I\alpha$ is $\{(\nu\widetilde{b})\overline{a}\langle K\rangle\}_i$ and $\emptyset$ otherwise. $n(I\alpha)$ is the set of names that occur in $I\alpha$.

Table 2.

$$ALP : \frac{M \xrightarrow{I\alpha} M'}{N \xrightarrow{I\alpha} N'} M \equiv_\alpha N, M' \equiv_\alpha N' \qquad TAU : \{\tau\}_{i,j}.M \xrightarrow{\{\tau\}_{i,j}} M$$

$$OUT1 : \{\overline{l}\}_i.M \xrightarrow{\{\overline{l}\}_i} M \qquad IN1 : \{l\}_i.M \xrightarrow{\{l\}_i} M$$

$$OUT2 : \{\overline{a}\langle K\rangle\}_i.M \xrightarrow{\{\overline{a}\langle K\rangle\}_i} M \qquad IN2 : \{a(U)\}_i.M \xrightarrow{\{a\langle K\rangle\}_i} M\{K/U\}$$

$$PAR : \frac{M \xrightarrow{I\alpha} M'}{M|N \xrightarrow{I\alpha} M'|N} bn(I\alpha) \cap fn(N) = \emptyset$$

$$COM1 : \frac{M \xrightarrow{\{\overline{l}\}_i} M' \quad N \xrightarrow{\{l\}_j} N'}{M|N \xrightarrow{\{\tau\}_{i,j}} (M'|N')}$$

$$COM2 : \frac{M \xrightarrow{\{(\nu\widetilde{b})\overline{a}\langle K\rangle\}_i} M' \quad N \xrightarrow{\{a\langle K\rangle\}_j} N'}{M|N \xrightarrow{\{\tau\}_{i,j}} (\nu\widetilde{b})(M'|N')} \widetilde{b} \cap fn(N) = \emptyset$$

$$RES : \frac{M \xrightarrow{I\alpha} M'}{(\nu a)M \xrightarrow{I\alpha} (\nu a)M'} a \notin n(I\alpha) \qquad REP : \frac{M|!M \xrightarrow{I\alpha} M'}{!M \xrightarrow{I\alpha} M'}$$

$$OPEN : \frac{M \xrightarrow{\{(\nu\widetilde{c})\overline{a}\langle K\rangle\}_i} M'}{(\nu b)M \xrightarrow{\{(\nu b,\widetilde{c})\overline{a}\langle K\rangle\}_i} M'} a \neq b, b \in fn(K) - \widetilde{c}$$

The operational semantics of indexed processes is given in Table 2. Similar to Table 1, we have omitted the symmetric of the parallelism and communication. The main difference between Table 1 and Table 2 is that the label $I\alpha$ on the transition arrow is in the form of $\{\alpha\}_i$ or $\{\tau\}_{i,j}$. If we adopt the distributed view, $\{\alpha\}_i$ can be regarded as an input or output action performed by component i, and $\{\tau\}_{i,j}$ can be regarded as a communication between components i and j.

Remark: Since $\{\tau\}_{i,j}$ and $\{\tau\}_{j,i}$ have the same meaning: a communication between components i and j, hence i,j should be considered as a set $\{i,j\}$, and not as an ordered pair. Therefore in the above labelled transition system, $\{\tau\}_{i,j}$ and $\{\tau\}_{j,i}$ are considered as the same label, i.e., $M \stackrel{\{\tau\}_{i,j}}{\longrightarrow} M'$ is viewed to be same as $M \stackrel{\{\tau\}_{j,i}}{\longrightarrow} M'$.

3.2 Indexed Context Bisimulation and Indexed Normal Bisimulation

Now we can give the concept of indexed context bisimulation and indexed normal bisimulation for indexed processes. In the remainder of this paper, we abbreviate $M\{K/U\}$ as $M\langle K\rangle$. In the following, we use $M \stackrel{\varepsilon,S}{\Longrightarrow} M'$ to abbreviate $M \stackrel{\{\tau\}_{i_1,i_1}}{\longrightarrow} \ldots \stackrel{\{\tau\}_{i_n,i_n}}{\longrightarrow} M'$, and use $M \stackrel{I\alpha,S}{\Longrightarrow} M'$ to abbreviate $M \stackrel{\varepsilon,S}{\Longrightarrow}\stackrel{I\alpha}{\longrightarrow}\stackrel{\varepsilon,S}{\Longrightarrow} M'$, here $i_1,...,i_n \in S \subseteq I$. An index is called fresh in a statement if it is different from any other index occurring in the processes of the statement. Let us see two examples. For the transition $(\nu a)((\nu b)(\{\overline{a}\}_n.0|\{\overline{b}\}_m.0|\{a\}_n.\{b\}_m.0)) \stackrel{\{\tau\}_{n,n}}{\longrightarrow}\stackrel{\{\tau\}_{m,m}}{\longrightarrow} 0$, we can abbreviate it as $(\nu a)((\nu b)(\{\overline{a}\}_n.0|\{\overline{b}\}_m.0|\{a\}_n.\{b\}_m.0)) \stackrel{\varepsilon,\{m,n\}}{\Longrightarrow} 0$. Similarly, since $(\nu a)((\nu b)(\{\overline{a}\}_n.0|\{\overline{b}\}_m.0|\{a\}_n.\{c\}_k.\{b\}_m.0)) \stackrel{\{\tau\}_{n,n}}{\longrightarrow}\stackrel{\{c\}_k}{\longrightarrow}\stackrel{\{\tau\}_{m,m}}{\longrightarrow} 0$, we can abbreviate it as $(\nu a)((\nu b)(\{\overline{a}\}_n.0|\{\overline{b}\}_m.0|\{a\}_n.\{c\}_k.\{b\}_m.0)) \stackrel{\{c\}_k,\{m,n\}}{\Longrightarrow} 0$.

This paper's main result states that strong context bisimulation coincides with strong normal bisimulation. Technically, the proof rests on the notion of indexed bisimulations... The idea is to generalize the usual notion of weak bisimulations so that tau actions can be ignored selectively, depending on a chosen set of indices S. The cases $S = \emptyset$ and $S = I$ correspond to strong and weak bisimulations respectively.

Definition 5. Let M, N be two closed indexed processes, and $S \subseteq I$ be an index set, we write $M \simeq^S_{Ct} N$, if there is a symmetric relation R and $M\ R\ N$ implies:

(1) whenever $M \stackrel{\varepsilon,S}{\Longrightarrow} M'$, there exists N' such that $N \stackrel{\varepsilon,S}{\Longrightarrow} N'$ and $M'\ R\ N'$;

(2) whenever $M \stackrel{I\alpha,S}{\Longrightarrow} M'$, there exists N' such that $N \stackrel{I\alpha,S}{\Longrightarrow} N'$ and $M'\ R\ N'$, here $I\alpha \neq \{\tau\}_{i,i}$ for any $i \in S$, $I\alpha$ is not an indexed higher order action;

(3) whenever $M \stackrel{\{a\langle K\rangle\}_i,S}{\Longrightarrow} M'$, there exists N' such that $N \stackrel{\{a\langle K\rangle\}_i,S}{\Longrightarrow} N'$ and M' $R\ N'$;

(4) whenever $M \overset{\{(\nu\widetilde{b})\overline{a}\langle K\rangle\}_{i},S}{\Longrightarrow} M'$, there exists N' such that $N \overset{\{(\nu\widetilde{c})\overline{a}\langle L\rangle\}_{i},S}{\Longrightarrow} N'$ and for any indexed process $C(U)$ with $fn(C(U))\cap\{\widetilde{b},\widetilde{c}\} = \oslash$, $(\nu\widetilde{b})(M'|C\langle K\rangle)$ R $(\nu\widetilde{c})(N'|C\langle L\rangle)$.

We say that M and N are indexed context bisimilar w.r.t. S if $M \simeq^{S}_{Ct} N$.

Definition 6. Let M, N be two closed indexed processes, and $S \subseteq I$ be an index set, we write $M \simeq^{S}_{Nr} N$, if there is a symmetric relation R and $M\ R\ N$ implies:

(1) whenever $M \overset{\varepsilon,S}{\Longrightarrow} M'$, there exists N' such that $N \overset{\varepsilon,S}{\Longrightarrow} N'$ and $M'\ R\ N'$;
(2) whenever $M \overset{I\alpha,S}{\Longrightarrow} M'$, there exists N' such that $N \overset{I\alpha,S}{\Longrightarrow} N'$ and $M'\ R\ N'$, here $I\alpha \neq \{\tau\}_{i,i}$ for any $i \in S$, $I\alpha$ is not an indexed higher order action;
(3) whenever $M \overset{\{a\langle\{\overline{m}\}_n.0\rangle\}_{i},S}{\Longrightarrow} M'$, here m is a fresh name, there exists N' such that $N \overset{\{a\langle\{\overline{m}\}_n.0\rangle\}_{i},S}{\Longrightarrow} N'$ and $M'\ R\ N'$;
(4) whenever $M \overset{\{(\nu\widetilde{b})\overline{a}\langle K\rangle\}_{i},S}{\Longrightarrow} M'$, there exists N' such that $N \overset{\{(\nu\widetilde{c})\overline{a}\langle L\rangle\}_{i},S}{\Longrightarrow} N'$, and $(\nu\widetilde{b})(M'|!\{m\}_n.K)\ R\ (\nu\widetilde{c})(N'|!\{m\}_n.L)$ with a fresh name m and a fresh index n.

We say that M and N are indexed normal bisimilar w.r.t. S if $M \simeq^{S}_{Nr} N$.

The above definitions have some geometric intuition. From a distributed view, $\{\tau\}_{i,i}$ is an internal communication in component i, and $\{\tau\}_{i,j}$, where $i \neq j$, represents an external communication between components i and j. Therefore in Definitions 5 and 6, we regard $\{\tau\}_{i,i}$ as a private event in component i, which can be neglected if i is in S, a chosen set of indices; and we view $\{\tau\}_{i,j}$ as a visible event between components i and j.

For example, by the above definition, we have $(\nu a)(\{\overline{a}\}_n.0|\{a\}_n.M) \simeq^{\{n\}}_{Ct} M$, $(\nu a)(\{\overline{a}\}_n.0|\{a\}_n.M) \not\simeq^{\emptyset}_{Ct} M$ and $(\nu a)(\{\overline{a}\}_n.0|\{a\}_n.M) \simeq^{I}_{Nr} M$.

3.3 Indexed Triggered Processes and Indexed Triggered Bisimulation

The concept of triggered processes was introduced in [6, 7]. The distinguishing feature of triggered processes is that every communication among them is the exchange of a trigger, here a trigger is an elementary process whose only functionality is to activate a copy of another process. In this section, we introduce the indexed version of triggered processes. Indexed triggered process can be seen as a sort of normal form for the indexed processes, and every communication among them is the exchange of an indexed trigger. We shall use indexed triggers to perform indexed process transformations which make the treatment of the constructs of indexed higher order processes easier.

The formal definition of indexed triggered process is given as follows:

$$M ::= 0 \mid U \mid \{\tau\}_{i,j}.M \mid \{l\}_i.M \mid \{\overline{l}\}_i.M \mid \{a(U)\}_i.M \mid (\nu m)(\{\overline{a}\langle\{\overline{m}\}_n.0\rangle\}_i.M| \\ !\{m\}_n.N) \text{ with } m \notin fn(M,N)\cup\{a\} \mid M_1|M_2 \mid (\nu a)M \mid !M.$$

The class of the indexed triggered processes is denoted as $ITPr$. The class of the closed indexed triggered processes is denoted as $ITPr^c$.

Definition 7. We give a mapping Tr^n which transforms every indexed process M into the indexed triggered process $Tr^n[M]$ with respect to index n. The mapping is defined inductively on the structure of M.

(1) $Tr^n[0] ::= 0$;
(2) $Tr^n[U] ::= U$;
(3) $Tr^n[\{\tau\}_{i,j}.M] ::= \{\tau\}_{i,j}.Tr^n[M]$;
(4) $Tr^n[\{l\}_i.M] ::= \{l\}_i.Tr^n[M]$;
(5) $Tr^n[\{\bar{l}\}_i.M] ::= \{\bar{l}\}_i.Tr^n[M]$;
(6) $Tr^n[\{a(U)\}_i.M] ::= \{a(U)\}_i.Tr^n[M]$;
(7) $Tr^n[\{\overline{a}\langle N\rangle\}_i.M] ::= (\nu m)(\{\overline{a}\langle\{\overline{m}\}_n.0\rangle\}_i.Tr^n[M]|!\{m\}_n.Tr^n[N])$, where m is a fresh name;
(8) $Tr^n[M_1|M_2] ::= Tr^n[M_1]|Tr^n[M_2]$;
(9) $Tr^n[(\nu a)M] ::= (\nu a)Tr^n[M]$;
(10) $Tr^n[!M] ::=!Tr^n[M]$.

Transformation $Tr^n[\,]$ may expand the number of $\{\tau\}_{n,n}$ steps in a process. But the behavior is otherwise the same. The expansion is due to the fact that if in M a process N is transmitted and used k times then, in $Tr^n[M]$ k additional $\{\tau\}_{n,n}$ interactions are required to activate the copies of N.

For example, let $M \stackrel{def}{=} \{\overline{a}\langle N\rangle\}_i.L|\{a(U)\}_j.(U|U)$, then $M \stackrel{\{\tau\}_{i,j}}{\longrightarrow} L|N|N \stackrel{def}{=} M'$. In $Tr^n[M]$, this is simulated using two additional $\{\tau\}_{n,n}$ interactions:

$$
\begin{aligned}
Tr^n[M] &= (\nu m)(\{\overline{a}\langle\{\overline{m}\}_n.0\rangle\}_i.Tr^n[L]|!\{m\}_n.Tr^n[N])|\{a(U)\}_j.(U|U)\\
&\stackrel{\{\tau\}_{i,j}}{\longrightarrow} (\nu m)(Tr^n[L]|!\{m\}_n.Tr^n[N]|\{\overline{m}\}_n.0|\{\overline{m}\}_n.0)\\
&\stackrel{\{\tau\}_{n,n}}{\longrightarrow}\stackrel{\{\tau\}_{n,n}}{\longrightarrow} (\nu m)(Tr^n[L]|Tr^n[N]|Tr^n[N]|!\{m\}_n.Tr^n[N])\\
&\simeq^{\emptyset}_{Ct} Tr^n[L]|Tr^n[N]|Tr^n[N] \text{ since } m \text{ is a fresh name}\\
&= Tr^n[M'].
\end{aligned}
$$

Now we can give the indexed version of triggered bisimulation as follows.

Definition 8. Let M, N be two closed indexed triggered processes, and $S \subseteq I$ be an index set, we write $M \simeq^S_{Tr} N$, if there is a symmetric relation R and M R N implies:

(1) whenever $M \stackrel{\varepsilon,S}{\Longrightarrow} M'$, there exists N' such that $N \stackrel{\varepsilon,S}{\Longrightarrow} N'$ and M' R N';
(2) whenever $M \stackrel{I\alpha,S}{\Longrightarrow} M'$, there exists N' such that $N \stackrel{I\alpha,S}{\Longrightarrow} N'$ and M' R N', here $I\alpha \neq \{\tau\}_{i,i}$ for any $i \in S$, $I\alpha$ is not an indexed higher order action;
(3) whenever $M \stackrel{\{a\langle\{\overline{m}\}_n.0\rangle\}_i,S}{\Longrightarrow} M'$, here m is a fresh name, there exists N' such that $N \stackrel{\{a\langle\{\overline{m}\}_n.0\rangle\}_i,S}{\Longrightarrow} N'$ and M' R N';
(4) whenever $M \stackrel{\{(\nu m)\overline{a}\langle\{\overline{m}\}_n.0\rangle\}_i,S}{\Longrightarrow} M'$, there exists N' such that $N \stackrel{\{(\nu m)\overline{a}\langle\{\overline{m}\}_n.0\rangle\}_i,S}{\Longrightarrow} N'$ and M' R N'.

We say that M and N are indexed triggered bisimilar w.r.t. S if $M \simeq^S_{Tr} N$.

3.4 The Equivalence Between Indexed Bisimulations

In [6, 7], the equivalence between weak context bisimulation and weak normal bisimulation was proved. In the proof, the factorisation theorem was firstly given. It allows us to factorise out certain subprocesses of a given process. Thus, a complex process can be decomposed into the parallel composition of simpler processes. Then the concept of triggered processes was introduced, which is the key step in the proof. Triggered processes represent a sort of normal form for the processes. Most importantly, there is a very simple characterisation of context bisimulation on triggered processes, called triggered bisimulation. By the factorisation theorem, a process can be transformed to a triggered process. The transform allows us to use the simpler theory of triggered processes to reason about the set of all processes. In [6, 7], weak context bisimulation was firstly proved to be equivalent to weak triggered bisimulation on triggered processes, then by the mapping from general processes to triggered processes, the equivalence between weak context bisimulation and weak normal bisimulation was proved.

In the case of strong bisimulations, the above proof strategy does not work. The main problem is that the mapping to triggered processes brings some redundant tau actions. Since weak bisimulations abstract from tau action, the full abstraction of the mapping to triggered processes holds. But in the case of strong bisimulations, the triggered mapping does not preserve the strong bisimulations, and therefore some central technical results in [6, 7], like the factorisation theorem, are not true in the strong case.

To resolve this difficulty we introduced the concept of indexed processes and the indexed version of context and normal bisimulations. Roughly, the actions of indexed processes have added indices, which are used to identify in which component or between which components an action takes place. Indexed bisimulations with respect to an indices set S neglect the indexed tau action of the form $\{\tau\}_{i,i}$ for any $i \in S$, but distinguish the indexed tau action of the form $\{\tau\}_{i,j}$ if $i \notin S$ or $j \notin S$ or $i \neq j$. One can see that the mapping from M to $Tr^n[M]$ brings redundant indexed tau actions $\{\tau\}_{n,n}$. Therefore indexed triggered mapping preserves the indexed bisimulations with respect to $S \cup \{n\}$ for any S. Similarly, we also have the indexed version of factorisation theorem. Following the proof strategy in [6, 7], we prove the equivalence between indexed context bisimulation and indexed normal bisimulation. Furthermore, when S is the empty set $\emptyset$, we discuss the relation between indexed bisimulations and strong bisimulations, and get the proposition: $P \sim_{Nr} Q \Leftrightarrow \{P\}_k \simeq^{\emptyset}_{Nr} \{Q\}_k \Leftrightarrow Tr^n[\{P\}_k] \simeq^{\{n\}}_{Tr} Tr^n[\{Q\}_k] \Leftrightarrow \{P\}_k \simeq^{\emptyset}_{Ct} \{Q\}_k \Leftrightarrow P \sim_{Ct} Q$. This solves the open problem in [7]. We also apply the proof idea to the case of weak bisimulations. When S is the full index set I, we study the relation between indexed bisimulations and weak bisimulations, and get the proposition: $P \approx_{Nr} Q \Leftrightarrow \{P\}_k \simeq^{I}_{Nr} \{Q\}_k \Leftrightarrow Tr^n[\{P\}_k] \simeq^{I}_{Tr} Tr^n[\{Q\}_k] \Leftrightarrow \{P\}_k \simeq^{I}_{Ct} \{Q\}_k \Leftrightarrow P \approx_{Ct} Q$. Therefore the proof presented here seems to be a uniform approach to the equivalence between strong/weak context bisimulation and strong/weak normal bisimulation.

Now we study the relations between the three indexed bisimulations. The main result is summarized in Proposition 8: $M \simeq^{S}_{Nr} N \Leftrightarrow Tr^n[M] \simeq^{S\cup\{n\}}_{Tr}$

$Tr^n[N] \Leftrightarrow M \simeq^S_{Ct} N$. We achieve this result by proving several propositions: including indexed factorisation theorem (Proposition 4), full abstraction of the mapping to indexed triggered processes (Proposition 5), the relation between indexed triggered bisimulation and indexed normal bisimulation (Proposition 6), and the relation between indexed triggered bisimulation and indexed context bisimulation (Proposition 7).

In the following, we first give congruence of $\simeq^S_{Ct}$ and $\simeq^S_{Tr}$.

Proposition 1 (Congruence of $\simeq^S_{Ct}$)**.** For all M, N, $K \in IPr^c$, $M \simeq^S_{Ct} N$ implies:

1. $I\pi.M \simeq^S_{Ct} I\pi.N$;
2. $M|K \simeq^S_{Ct} N|K$;
3. $(\nu a)M \simeq^S_{Ct} (\nu a)N$;
4. $!M \simeq^S_{Ct} !N$;
5. $\overline{a}\langle M\rangle.K \simeq^S_{Ct} \overline{a}\langle N\rangle.K$.

Proof : Similar to the argument of the analogous result for context bisimulation in [6, Theorem 4.2.7].

Proposition 2 (Congruence of $\simeq^S_{Tr}$)**.** For all M, N, $K \in ITPr^c$, $M \simeq^S_{Tr} N$ implies:

1. $M|K \simeq^S_{Tr} N|K$;
2. $(\nu a)M \simeq^S_{Tr} (\nu a)N$.

Proof : Similar to the argument of the analogous result for triggered bisimulation in [6, Lemma 4.6.3].

Proposition 3 states the easy part of the relation between $\simeq^S_{Ct}$ and $\simeq^S_{Nr}$.

Proposition 3. For any M, $N \in IPr^c$, $M \simeq^S_{Ct} N \Rightarrow M \simeq^S_{Nr} N$.

Proof : It is trivial by the definition of $\simeq^S_{Ct}$ and $\simeq^S_{Nr}$.

Now we give the indexed version of the factorisation theorem, which states that, by means of indexed triggers, an indexed subprocess of a given indexed process can be factorised out.

Proposition 4. For any indexed processes M and N with $m \notin fn(M, N)$, it holds that $M\{\{\tau\}_{i,j}.N/U\} \simeq^S_{Ct} (\nu m)(M\{\{\overline{m}\}_i.0/U\}|!\{m\}_j.N)$ for any S.

Proof : Similar to the proof of $P\{\tau.R/X\} \sim_{Ct} (\nu m)(P\{\overline{m}.0/X\}|!m.R)$ in [6], by induction on the structure of M.

Corollary 1. For any indexed processes M and N with $m \notin fn(M, N)$, it holds that $M\{N/U\} \simeq^{S\cup\{n\}}_{Ct} (\nu m)(M\{\{\overline{m}\}_n.0/U\}|!\{m\}_n.N)$ for any S.

Proof : It is straightforward by $\{\tau\}_{n,n}.M \simeq^{S\cup\{n\}}_{Ct} M$ and Propositions 1 and 4.

To prove the correctness of $Tr^n[\,]$, which is stated as Proposition 5, we first give the following lemma:

Lemma 1. For any M, $N \in ITPr^c$, $M \simeq_{Ct}^{S} N \Rightarrow M \simeq_{Tr}^{S} N$.

Proposition 5. For each $M \in IPr^c$,

1. $Tr^n[M]$ is an indexed triggered process;
2. $Tr^n[M] \simeq_{Ct}^{S\cup\{n\}} M$;
3. $Tr^n[M] \simeq_{Tr}^{S\cup\{n\}} M$, if M is an indexed triggered process.

Proof : 1. It is straightforward.
2. It can be proved by induction on the structure of M and using Corollary 1.
3. By Lemma 1 and Case 2.

Proposition 6 below states the relation between $\simeq_{Nr}^{S}$ and $\simeq_{Tr}^{S\cup\{n\}}$:

Proposition 6. For any M, $N \in IPr^c$, $M \simeq_{Nr}^{S} N \Rightarrow Tr^n[M] \simeq_{Tr}^{S\cup\{n\}} Tr^n[N]$, here $n \notin Index(M, N)$.

The following Lemma 2 and Lemma 3 are necessary to the proof of Proposition 7.

Lemma 2. For any M, $N \in IPr^c$, $Tr^n[M] \simeq_{Tr}^{S\cup\{n\}} Tr^n[N] \Rightarrow M \simeq_{Ct}^{S\cup\{n\}} N$, here $n \notin Index(M, N)$.

Lemma 3. For any M, $N \in IPr^c$, $M \simeq_{Ct}^{S\cup\{n\}} N \Rightarrow M \simeq_{Ct}^{S} N$, here $n \notin Index(M, N)$.

Proof : It is clear since $n \notin Index(M, N)$.
Now we get the relation between $\simeq_{Tr}^{S\cup\{n\}}$ and $\simeq_{Ct}^{S}$ as follows:

Proposition 7. For any M, $N \in IPr^c$, $Tr^n[M] \simeq_{Tr}^{S\cup\{n\}} Tr^n[N] \Rightarrow M \simeq_{Ct}^{S} N$, here $n \notin Index(M, N)$.

Proof : By Lemmas 2 and 3.
The following proposition is the main result of this section, which states the equivalence between indexed context bisimulation, indexed normal bisimulation and indexed triggered bisimulation.

Proposition 8. For any M, $N \in IPr^c$, $M \simeq_{Nr}^{S} N \Leftrightarrow Tr^n[M] \simeq_{Tr}^{S\cup\{n\}} Tr^n[N] \Leftrightarrow M \simeq_{Ct}^{S} N$, here $n \notin Index(M, N)$.

Proof : By Propositions 3, 6 and 7.

For indexed triggered processes, the above proposition can be simplified as Corollary 2.

Lemma 4. For any M, $N \in ITPr^c$, $M \simeq_{Tr}^{S\cup\{n\}} N \Rightarrow M \simeq_{Tr}^{S} N$, here $n \notin Index(M, N)$.

Proof : It is clear since $n \notin Index(M, N)$.

Corollary 2. For any M, $N \in ITPr^c$, $M \simeq_{Nr}^S N \Leftrightarrow M \simeq_{Tr}^S N \Leftrightarrow M \simeq_{Ct}^S N$.

Proof : By Proposition 8, $M \simeq_{Nr}^S N \Leftrightarrow Tr^n[M] \simeq_{Tr}^{S\cup\{n\}} Tr^n[N] \Leftrightarrow M \simeq_{Ct}^S N$, here $n \notin Index(M, N)$. Since M, $N \in ITPr^c$, $M \simeq_{Tr}^{S\cup\{n\}} Tr^n[M] \simeq_{Tr}^{S\cup\{n\}} Tr^n[N] \simeq_{Tr}^{S\cup\{n\}} N$. By Lemma 4, we have $M \simeq_{Tr}^S N$.

Sangiorgi [6] proved that barbed equivalence coincides with context bisimulation. We generalize this result to our indexed process calculus. In the following, we first present an indexed variant of barbed equivalence called indexed reduction bisimulation and then give the equivalence between indexed reduction bisimulation, indexed context bisimulation and indexed normal bisimulation. This result shows that all our indexed bisimulations are same and capture the essential of equivalence of indexed processes.

Definition 9. Let M, N be two indexed processes, and $S \subseteq I$ be an index set, we write $M \simeq_{Rd}^S N$, if there is a symmetric relation R and $K\ R\ L$ implies:

(1) $K|M\ R\ L|M$ for any indexed process M;
(2) whenever $K \overset{\varepsilon,S}{\Longrightarrow} K'$, there exists L' such that $L \overset{\varepsilon,S}{\Longrightarrow} L'$ and $K'\ R\ L'$;
(3) whenever $K \overset{\{\tau\}_{i,j},S}{\Longrightarrow} K'$, here $(i,j) \notin \{(k,k)|k \in S\}$, there exists L' such that $L \overset{\{\tau\}_{i,j},S}{\Longrightarrow} L'$ and $K'\ R\ L'$.

We say that M and N are indexed reduction bisimilar w.r.t. S if $M \simeq_{Rd}^S N$. Since $\simeq_{Ct}^S$ is equivalent to $\simeq_{Nr}^S$, the following proposition states that $\simeq_{Ct}^S$, $\simeq_{Nr}^S$ and $\simeq_{Rd}^S$ are same.

Proposition 9. For any M, $N \in IPr^c$, $M \simeq_{Ct}^S N \Rightarrow M \simeq_{Rd}^S N \Rightarrow M \simeq_{Nr}^S N$.

In [1], the concept of indexed reduction bisimulation was used to give a uniform equivalence for different process calculi.

4 The Equivalence Between Bisimulations in Higher Order π-Calculus

4.1 Strong Context Bisimulation Coincides with Strong Normal Bisimulation

The equivalence between strong context bisimulation and strong normal bisimulation can be derived by the mapping to indexed triggered process and the equivalence between indexed bisimulations.

For example, let us see the following two processes:

$P = (\nu a)(\overline{a}\langle\overline{b}.0\rangle.0|a(X).X)$;
$Q = (\nu a)(\overline{a}\langle 0\rangle.0|a(X).\overline{b}.0)$.

They are clearly strong context bisimilar. However, their triggered mappings are not strong triggered bisimilar. Indeed, the mapping of Q is $(\nu a)((\nu m)$

$(\overline{a}\langle\overline{m}.0\rangle.0|!m.0)|a(X).\overline{b}.0)$, after the communication between $\overline{a}$ and a, the residual process can perform action $\overline{b}$ without using silent tau actions, whereas the mapping of P is $(\nu a)((\nu m)(\overline{a}\langle\overline{m}.0\rangle.0|!m.\overline{b}.0)|a(X).X)$, and to match this behavior, one has to go through a trigger and this therefore requires some form of weak transition. Hence the proof strategy in [6, 7] cannot be generalized to the case of strong bisimulation.

In our approach, we first consider the indexed version of P and Q:

$$\{P\}_0 = (\nu a)(\{\overline{a}\langle\{\overline{b}\}_0.0\rangle\}_0.0|\{a(X)\}_0.X);$$
$$\{Q\}_0 = (\nu a)(\{\overline{a}\langle 0\rangle\}_0.0|\{a(X)\}_0.\{\overline{b}\}_0.0).$$

It is clearly $\{P\}_0 \simeq^{\emptyset}_{Ct} \{Q\}_0$. Now the indexed triggered mapping of $\{P\}_0$ is $Tr^n[\{P\}_0] = (\nu a)((\nu m)(\{\overline{a}\langle\{\overline{m}\}_n.0\rangle\}_0.0|!\{m\}_n.\{\overline{b}\}_0.0)|\{a(X)\}_0.X)$, and the indexed triggered mapping of $\{Q\}_0$ is $Tr^n[\{Q\}_0] = (\nu a)((\nu m)(\{\overline{a}\langle\{\overline{m}\}_n.0\rangle\}_0.0|$ $!\{m\}_n.0)|\{a(X)\}_0.\{\overline{b}\}_0.0)$. Unlike the un-indexed case, $Tr^n[\{P\}_0]$ and $Tr^n[\{Q\}_0]$ are indexed triggered bisimilar w.r.t. $S = \{n\}$. For example, let us consider the transition: $Tr^n[\{P\}_0] \stackrel{\{\tau\}_{0,0}}{\longrightarrow} (\nu a)((\nu m)(0|!\{m\}_n.\{\overline{b}\}_0.0|\{\overline{m}\}_n.0)) \stackrel{\{\tau\}_{n,n}}{\longrightarrow} (\nu a)((\nu m)$ $(0|\{\overline{b}\}_0.0|!\{m\}_n.\{\overline{b}\}_0.0|0)) \stackrel{\{\overline{b}\}_0}{\longrightarrow} (\nu a)((\nu m)(0|0|!\{m\}_n.\{\overline{b}\}_0.0|0))$. Since we neglect indexed tau action of the form $\{\tau\}_{n,n}$ in the definition of $\simeq^{\{n\}}_{Tr}$, we have a matching transition $Tr^n[\{Q\}_0] \stackrel{\{\tau\}_{0,0}}{\longrightarrow} (\nu a)((\nu m)(0|!\{m\}_n.0|\{\overline{b}\}_0.0)) \stackrel{\{\overline{b}\}_0}{\longrightarrow} (\nu a)((\nu m)$ $(0|!\{m\}_n.0|0))$. Hence $Tr^n[\{P\}_0]$ and $Tr^n[\{Q\}_0]$ are bisimilar. Formally, we have $Tr^n[\{P\}_0] \simeq^{\{n\}}_{Tr} Tr^n[\{Q\}_0]$. Similarly, we can further build the relation between $\simeq^{\{n\}}_{Tr}$ and $\simeq^{\emptyset}_{Nr}$: $Tr^n[\{P\}_0] \simeq^{\{n\}}_{Tr} Tr^n[\{Q\}_0] \Leftrightarrow \{P\}_0 \simeq^{\emptyset}_{Nr} \{Q\}_0$.

In this section, we will show that $P \sim_{Nr} Q \Rightarrow \{P\}_0 \simeq^{\emptyset}_{Nr} \{Q\}_0$ and $\{P\}_0 \simeq^{\emptyset}_{Ct}$ $\{Q\}_0 \Rightarrow P \sim_{Ct} Q \Rightarrow P \sim_{Nr} Q$. Since $\{P\}_0 \simeq^{\emptyset}_{Ct} \{Q\}_0 \Leftrightarrow \{P\}_0 \simeq^{\emptyset}_{Nr} \{Q\}_0$ by Proposition 8, the equivalence between $P \sim_{Nr} Q$ and $P \sim_{Ct} Q$ is obvious.

Now we prove that strong context bisimulation and strong normal bisimulation coincide, which was presented in [7] as an open problem.

Firstly, we introduce the concept of strong indexed context equivalence, strong indexed normal equivalence and strong indexed triggered equivalence.

Definition 10. Strong indexed context equivalence.
Let $P, Q \in Pr^c$, we write $P \sim^i_{Ct} Q$, if $\{P\}_k \simeq^{\emptyset}_{Ct} \{Q\}_k$ for some index k. As we defined before, here $\{P\}_k$ denotes indexed process with the same given index k on every prefix in P.

Definition 11. Strong indexed normal equivalence.
Let $P, Q \in Pr^c$, we write $P \sim^i_{Nr} Q$, if $\{P\}_k \simeq^{\emptyset}_{Nr} \{Q\}_k$ for some index k.

Definition 12. Strong indexed triggered equivalence.
Let $P, Q \in Pr^c$, we write $P \sim^{i,\{n\}}_{Tr} Q$, if $Tr^n[\{P\}_k] \simeq^{\{n\}}_{Tr} Tr^n[\{Q\}_k]$ for some index k with $k \neq n$.

The following lemma states that strong normal bisimulation implies strong indexed normal equivalence.

Lemma 5. For any $P, Q \in Pr^c$, $P \sim_{Nr} Q \Rightarrow P \sim^i_{Nr} Q$.

Now, the equivalence between $\sim_{Nr}$ and $\sim_{Ct}$ can be given.

Proposition 10. For any $P, Q \in Pr^c$ and any index n, $P \sim_{Nr} Q \Leftrightarrow P \sim^i_{Nr} Q \Leftrightarrow P \sim^{i,\{n\}}_{Tr} Q \Leftrightarrow P \sim^i_{Ct} Q \Leftrightarrow P \sim_{Ct} Q$.

Proof : Firstly, it is easy to prove $P \sim^i_{Ct} Q \Rightarrow P \sim_{Ct} Q \Rightarrow P \sim_{Nr} Q$. By Lemma 5, $P \sim_{Nr} Q \Rightarrow P \sim^i_{Nr} Q$. Hence $P \sim^i_{Ct} Q \Rightarrow P \sim_{Ct} Q \Rightarrow P \sim_{Nr} Q \Rightarrow P \sim^i_{Nr} Q$. By Proposition 8, we have $P \sim^i_{Nr} Q \Leftrightarrow P \sim^{i,\{n\}}_{Tr} Q \Leftrightarrow P \sim^i_{Ct} Q$ for any index n. Therefore the proposition holds.

Moreover, we can define strong indexed reduction equivalence $\sim^i_{Rd}$ as follows: let $P, Q \in Pr^c$, we write $P \sim^i_{Rd} Q$, if $\{P\}_k \simeq^{\emptyset}_{Rd} \{Q\}_k$ for some index k. By Propositions 9 and 10, we know that $\sim^i_{Rd}$ coincides with $\sim_{Nr}$ and $\sim_{Ct}$.

4.2 Weak Context Bisimulation Coincides with Weak Normal Bisimulation

Based on the equivalence between indexed bisimulations, we can give an alternative proof for the equivalence between weak context bisimulation and weak normal bisimulation.

Definition 13. Weak indexed context equivalence.
Let $P, Q \in Pr^c$, we write $P \approx^i_{Ct} Q$, if $\{P\}_k \simeq^I_{Ct} \{Q\}_k$ for some index k, here I is the full index set.

Definition 14. Weak indexed normal equivalence.
Let $P, Q \in Pr^c$, we write $P \approx^i_{Nr} Q$, if $\{P\}_k \simeq^I_{Nr} \{Q\}_k$ for some index k, here I is the full index set.

Definition 15. Weak indexed triggered equivalence.
Let $P, Q \in Pr^c$, we write $P \approx^i_{Tr} Q$, if $Tr^n[\{P\}_k] \simeq^I_{Tr} Tr^n[\{Q\}_k]$ for some indices k and n, here $k \neq n$ and I is the full index set.

Lemma 6. For any $P, Q \in Pr^c$, $P \approx_{Nr} Q \Rightarrow P \approx^i_{Nr} Q$.

Proposition 11. For any $P, Q \in Pr^c$, $P \approx_{Nr} Q \Leftrightarrow P \approx^i_{Nr} Q \Leftrightarrow P \approx^i_{Tr} Q \Leftrightarrow P \approx^i_{Ct} Q \Leftrightarrow P \approx_{Ct} Q$.

Proof : By Proposition 8, it is easy to get $P \approx^i_{Nr} Q \Rightarrow P \approx^i_{Tr} Q \Rightarrow P \approx^i_{Ct} Q \Rightarrow P \approx_{Ct} Q \Rightarrow P \approx_{Nr} Q$. By Lemma 6, $P \approx_{Nr} Q \Rightarrow P \approx^i_{Nr} Q$, therefore the proposition holds.

Similarly, we can define weak indexed reduction equivalence $\approx^i_{Rd}$ as follows: let $P, Q \in Pr^c$, we write $P \approx^i_{Rd} Q$, if $\{P\}_k \simeq^I_{Rd} \{Q\}_k$ for some index k. By Propositions 9 and 11, $\approx^i_{Rd}$ coincides with $\approx_{Nr}$ and $\approx_{Ct}$.

In [6, 7], the proposition: $P \approx_{Nr} Q \Leftrightarrow Tr[P] \approx_{Tr} Tr[Q] \Leftrightarrow P \approx_{Ct} Q$ was proved, where $Tr[\ ]$ is the triggered mapping and $\approx_{Tr}$ is the weak triggered bisimulation. In fact, this proposition can be get from Proposition 11. Firstly by

Proposition 11, we have $P \approx_{Nr} Q \Leftrightarrow Tr^n[\{P\}_k] \simeq^I_{Tr} Tr^n[\{Q\}_k] \Leftrightarrow P \approx_{Ct} Q$. Secondly, we can prove that $Tr[P] \approx_{Tr} Tr[Q] \Leftrightarrow Tr^n[\{P\}_k] \simeq^I_{Tr} Tr^n[\{Q\}_k]$. Hence $P \approx_{Nr} Q \Leftrightarrow Tr[P] \approx_{Tr} Tr[Q] \Leftrightarrow P \approx_{Ct} Q$ is a corollary of Proposition 11. But for the strong case, the claim: $P \sim_{Nr} Q \Leftrightarrow Tr[P] \sim_{Tr} Tr[Q] \Leftrightarrow P \sim_{Ct} Q$ does not hold. For example, let $P = (\nu a)(\overline{a}\langle\overline{b}.0\rangle.0|a(X).X)$ and $Q = (\nu a)(\overline{a}\langle 0\rangle.0|a(X).\overline{b}.0)$, then $P \sim_{Nr} Q$, $P \sim_{Ct} Q$ and $Tr[P] \not\sim_{Tr} Tr[Q]$. Hence $P \sim_{Nr} Q \not\Leftrightarrow Tr[P] \sim_{Tr} Tr[Q] \not\Leftrightarrow P \sim_{Ct} Q$. This also shows that we cannot prove the equivalence between strong context bisimulation and strong normal bisimulation by the original technique of triggered mapping.

5 Conclusions

To prove the equivalence between context bisimulation and normal bisimulation, this paper proposed an indexed higher order π-calculus. In fact, this indexed calculus can also be viewed as a model of distributed computing, where indices represent locations, indexed action $\{\alpha\}_i$ represents an input/output action α performed in location i, and $\{\tau\}_{i,j}$ represents a communication between locations i and j. There are a few results on bisimulations for higher order π-calculus. In [6], context bisimulation and normal bisimulation were compared with barbed equivalence. In [3], authors proved a correspondence between weak normal bisimulation and a variant of barbed equivalence, called contextual barbed equivalence. In [4] an alternative proof of the correspondence between context bisimulation and barbed equivalence was given. It would be interesting to understand whether our concept of indexed processes and indexed bisimulations can be helpful to study the relation between bisimulations in the framework of other higher order concurrency languages.

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