A Variational PDE Based Level Set Method for a Simultaneous Segmentation and Non-rigid Registration

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Abstract. A new variational PDE based level set method for a simultaneous image segmentation and non-rigid registration using prior shape and intensity information is presented. The segmentation is obtained by finding a non-rigid registration to the prior shape. The non-rigid registration consists of both a global rigid transformation and a local non-rigid deformation. In this model, a prior shape is used as an initial contour which leads to decrease the numerical calculation time. The model is tested against two chamber end systolic ultrasound images from thirteen human patients. The experimental results provide preliminary evidence of the effectiveness of the model in detecting the boundaries of the incompletely resolved objects which were plagued by noise, dropout, and artifact.

1 Introduction

Segmentation and registration play an important role in image processing, image analysis, and computer vision. There have been several different approaches to solve image segmentation and registration problems. In the past, solutions of these two problems have been studied separately from each other.

Segmentation techniques have been developed to capture the object boundary by several different approaches; edge-based methods mainly using active contour models, region-based methods, or the combination of the two by using Geodesic Active Region models. However, these methods may not be able to accommodate all types of imaging difficulties. The prior shape and intensity information has been incorporated into the geodesic active contour model [13, 12, 1, 4, 3, 2] to make the resulting segmentation more accurate. Recently, the prior shape and intensity information has been also incorporated into deformable models [10, 9].

Image registration is the process of overlaying two or more images of the same scene taken at different times, and/or by different sensors. Area based methods or feature based methods have been developed to match given images. For the details of the image registration techniques, please refer to [29]. Several approaches have been also made in establishing the application of mutual information for non-rigid registration [7, 16, 14, 8, 24]. Instead of intensity information, a feature based model combines region matching and segmentation with free boundary conditions in [23], but the model in [23] can be applied only in the single regions of interest.

However, segmentation or registration technique alone, dependence of segmentation on registration methods, or dependence of registration on segmentation methods

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do not solve the image segmentation and registration problems completely. A joint segmentation and registration methods have been developed through active contours [27, 1, 4, 2, 3, 26]. An explicit combination of registration with segmentation has been developed in a variational framework through active contours [27]. The algorithms of [27] was extended in [15] to joint segmentation and of an object in two images. The morphing active contours algorithm is combined with the joint segmentation and registration with an application to CT scans for radiotherapy treatment planning [26].

In this paper, a new variational partial differential equation (PDE) based level set method for a simultaneous image segmentation and registration is presented with an application to two chamber human heart end systole ultra sound images. The purpose of the model is to segment the endocardial borders of the given images. The segmentation is obtained by finding a non-rigid registration to the prior shape. The model performs a simultaneous segmentation and registration in a similar way from [27, 1, 3] through active contours. But the model differs from [27, 1, 3] by using a shape prior as an initial contour. Since the shape prior is used as an initial contour, the level set form of the model is fixed which does not require the re-initialization process during the numerical calculation. The model in this paper is similar to [12, 3, 2] by utilizing both prior shape and prior intensity information to get better segmentation, but the segmentation in this model is obtained by finding the non-rigid deformation to the prior shape. This paper follows the idea from [22, 21, 25, 10, 9] for matching nonequivalent shapes by the combination of a global rigid transformation and a local non-rigid deformation. However, the model differs from [22, 21] by having an intensity matching term in the energy function which provides the difference of the intensity variation across the prior shape and the shape of interest. Minimizing the difference of the gradients of the intensity between smoothed prior image and the smoothed given image is more accurate than minimizing the gradient term in the given image, due to the loss of the information of the gradient near the edge of the desired object. This paper is organized as follows: In section 2, the model is proposed for a simultaneous segmentation and non-rigid registration and the level set form of the proposed model is described. In section 3, numerical methods pertaining to the model are explained. Experimental results of the model which were applied to human heart two chamber end systole ultrasound images are also showed in this section. In section 4, the conclusion follows and future work is stated.

2 Description of the Proposed Model

2.1 A Simultaneous Segmentation and Non-rigid Registration

A new variational PDE based level set method for a simultaneous image segmentation and non-rigid registration is presented in this section. The purpose of the model is to segment endocardial borders of the given image using a prior shape and intensity information. The segmentation is obtained by finding a non-rigid registration to the prior shape. Only small non-rigid deformation is considered in the registration. The notion of shape is independent of scaling, rotation, and translation in the model.

Let $C^*(p) = (x(p), y(p))(p \in [0, 1])$ be a differentiable parameterized curve, called shape prior, in an associated prior image I^*. The domain $N(C^*)$ is the neighborhood of the prior shape. Let $G_\sigma(x) = \frac{1}{\sigma} * e^{-\frac{|x|^2}{4\sigma}}$ and $g(|\nabla I|)$ can be chosen as $g(|\nabla I|) = \frac{1}{1+\beta*|\nabla(G_\sigma * I)|^2}$, where $\beta > 0$ is a parameter. The model is aimed to find u, v, a, μ, R, T by minimizing the energy functional:

$$
\begin{aligned}
E(u, v, a, \mu, R, T) = \lambda_1 \int_{N(C^*)} |\nabla(G_\sigma * I^*)(x, y) - a\nabla(G_\sigma * I)(\tilde{u}, \tilde{v})|^2 d\bar{x} \\
+\lambda_2 \int_{N(C^*)} (|u|^2 + |v|^2) d\bar{x} + \lambda_3 \int_{N(C^*)} (|\nabla u|^2 + |\nabla v|^2) d\bar{x} \\
+\lambda_4 \int_0^1 g(|\nabla I|)(\tilde{u}, \tilde{v})(C^*(p))|(C^*)'(p)|dp,
\end{aligned} \tag{1}
$$

and the vector $\begin{bmatrix} \tilde{u}(x, y) \\ \tilde{v}(x, y) \end{bmatrix} = \mu * R * \begin{bmatrix} x \\ y \end{bmatrix} + T + \begin{bmatrix} u(x, y) \\ v(x, y) \end{bmatrix}$, where μ is a scaling, R is a rotation matrix with respect to an angle θ, and T is a translation. A prior image I^*, a novel image I, the prior shape C^*, and the neighborhood $N(C^*)$ of C^* as a domain are given and $\lambda_i > 0 (i = 1, 2, 3, 4)$ are parameters balancing the influences from four terms in the model.

The first term minimizes the difference of the intensity variation across the prior shape and the shape of interest. Minimizing the difference of gradients of the intensity between smoothed prior image and the smoothed given image is more accurate than minimizing the gradient term in the given image, due to the loss of the information of the gradient near the edge of the desired object. Since images may have different levels of intensity, the model is also minimized over the scale factor variable a. The second term is minimizing the magnitude of the non-rigid deformation term $\begin{bmatrix} u \\ v \end{bmatrix}$. Smoothing the non-rigid term $\begin{bmatrix} u \\ v \end{bmatrix}$ is in the third term. Final term is the energy functional of active contours [5] and [11], that measures the amount of high gradient under the trace of the curve and increases the attraction of the evolving curve toward the boundary of the object. Since the prior shape C^* is used as an initial contour and the domain is taken as the neighborhood of the prior shape, it helps the numerical solutions converge faster. In addition, when the model is formulated into the level set form in Equation (2), the level set form of the active contour in this model is fixed. Therefore, re-initializing process in the calculation is not needed which decreases the numerical calculation time. In the model, a simultaneous segmentation and non-rigid registration performs. The segmentation is obtained by finding a non-rigid registration to the prior shape. A registration consists of global rigid transformation and local non-rigid deformation.

2.2 Level Set Formulation of the Proposed Model

Level set methods of [17] are extensively used in the problems of curve evolution including snakes and active contours, since they allow the curve to develop cusps, corners, and topological changes. Now, a variational level set approach is given from [28] and [6]. Let the contour C^* be the zero level set of a Lipschitz function w such that $\{z|w(z) > 0\}$ is the set inside C^*. Let $H(z)$ be the Heaviside function, that is $H(z) = 1$ if $z \geq 0$, and $H(z) = 0$ if $z < 0$, and $\delta = H'(z)$(in the sense of distribution) be the Dirac measure. Then, the length of the zero level set of w in the conformal metric $ds = g(|\nabla I|)|(C^*)'(p)|dp$ can be measured by $\int_{N(C^*)} g(|\nabla I|)|\nabla H(w)| =$

$\int_{N(C^*)} \delta(w)g(|\nabla I|)|Dw|$, where $N(C^*)$ is the given image domain. Therefore, the level set formulation of the model is to find u, v, a, μ, R, T by minimizing the energy functional:

$$\begin{aligned} E(u,v,a,\mu,R,T) = \lambda_1 \int_{N(C^*)} |\nabla(G_\sigma * I^*)(x,y) - a\nabla(G_\sigma * I)(\tilde{u},\tilde{v})|^2 d\bar{x} \\ +\lambda_2 \int_{N(C^*)} (|u|^2 + |v|^2) d\bar{x} + \lambda_3 \int_{N(C^*)} (|\nabla u|^2 + |\nabla v|^2) d\bar{x} \\ +\lambda_4 \int_{N(C^*)} \delta(d_{C^*}) g(|\nabla I|)(\tilde{u},\tilde{v}) |\nabla d_{C^*}| d\bar{x}, \end{aligned} \tag{2}$$

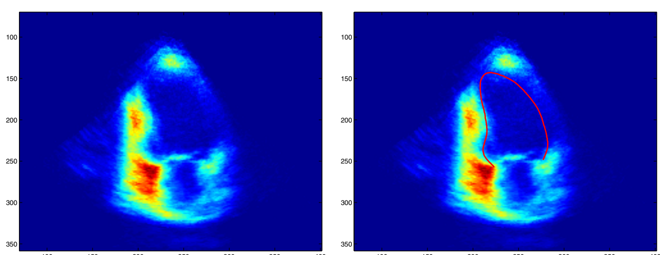
where d_{C^*} is the distance function from the prior shape C^*, $g(|\nabla I|)$ can be chosen as $g(|\nabla I|) = \frac{1}{1+\beta * |\nabla(G_\sigma * I)|^2}$, $\beta > 0$ is a parameter, $G_\sigma(x) = \frac{1}{\sigma} * e^{-\frac{|x|^2}{4\sigma}}$, and in a similar way, the vector $\begin{bmatrix} \tilde{u}(x,y) \\ \tilde{v}(x,y) \end{bmatrix} = \mu R \begin{bmatrix} x \\ y \end{bmatrix} + T + \begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix}$. A prior image I^*, a novel image I, and the prior shape C^* are given and λ_1,λ_2, λ_3, and λ_4 are positive parameters.

3 Numerical Methods and Experimental Results

In this section, numerical methods to solve the Equation (2) are explained and the results of applications to the thirteen human heart two chamber end systole ultrasound images are showed. The Equation (2) was solved by finding a steady state solution of the evolution equations. The evolution equations are associated with the Euler-Lagrange equations of the Equation (2). Following the approach from [17], H and δ are replaced by the regularized versions of them during the numerical calculation. A finite difference scheme and the gradient descent method is applied to discretize the evolving equations.

Prior shape C^* and prior intensity I^* of 13 Human heart two chamber end systole ultra sound images which have $0.62mm$ resolution in each pixel were generated by the method from [2]. The prior intensity information and a prior shape are in Figure 1 (a) and (b). The Figure 1 (a) represents the average image as prior intensity information. The solid red line in Figure 1 (b) is the average curve as a prior shape.

To show the effectiveness of the local non-rigid deformation term in the model, a global rigid transformation model is created without non-rigid deformation term $\begin{bmatrix} u \\ v \end{bmatrix}$

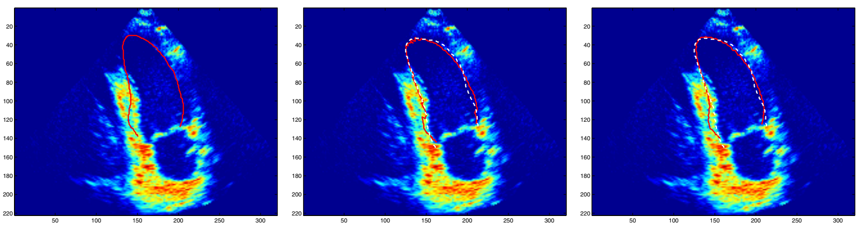


(a) (b)

Fig. 1. (a). An average image as a prior intensity information. (b). An average curve with a solid line as a prior shape.

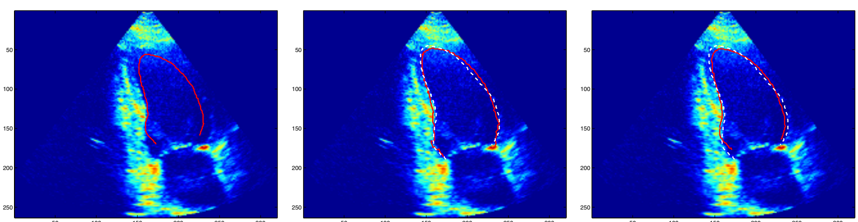
in Equation (2). Numerical calculation is done without non-rigid term. To validate experimental results of the model, distance measurements to the results of the model from expert's borders from Figure 2 to Figure 4 are followed in the Table 1. Table 1 provides the effectiveness of the suggested model which is better than the global rigid transformation model.

From Figure 2 to Figure 4, A solid red line in (a) is the initial contour in a novel image. A prior shape is used as an initial contour. The solid red line in (b) is the model's result with the local non-rigid deformation term. The expert endocardium border is compared as a dotted white line. The solid red line in (c) is the result of the model without a local non-rigid deformation term in the Equation (2). In a similar way, the expert endocardium border is compared as a dotted white line. From the Figures and the distance measurement results from the expert' border in Table 1, it is easily seen that the performance of the suggested model is better than the model without a local non-rigid deformation in the Equation (2). In Figure 2, the distance result from the model with non-rigid term is $1.8442(1.1434mm)$ which is better than the distance result $2.2160(1.3739mm)$ from the global rigid transformation model. In Figure 3, the distance result from the model with non-rigid term is $2.2301(1.3826mm)$ which is better than the distance result $2.8891(1.7912mm)$ from the global rigid transformation model. In Figure 4, the distance result from the model with non-rigid term is $2.0686(1.2825mm)$ which is better than the distance result $3.0470(1.8891mm)$ from



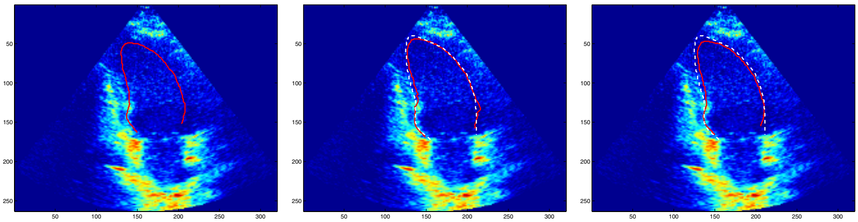
(a) (b) (c)

Fig. 2. (a). Solid line is an initial contour in the novel image. (b). The results from the model with non-rigid term (solid) and expert's border (dotted) in an image. (c). The results from the model without non-rigid term (solid) and expert's border (dotted) in an image.



(a) (b) (c)

Fig. 3. (a). Solid line is an initial contour in the novel image. (b). The results from the model with non-rigid term (solid) and expert's border (dotted) in an image. (c). The results from the model without non-rigid term (solid) and expert's border (dotted) in an image.



(a) (b) (c)

Fig. 4. (a). Solid line is an initial contour in the novel image. (b). The results from the model with non-rigid term (solid) and expert's border (dotted) in an image. (c). The results from the model without non-rigid term (solid) and expert's border (dotted) in an image.

Table 1. Distance measurements of the result of the model from expert's border from Figure 2 to Figure 4

Figures	(a)Initial	(b)Result with non-rigid term	(c)Result with rigid term only
Figure 2	25.1354 (15.5839mm)	1.8442(1.1434mm)	2.2160 (1.3739mm)
Figure 3	107.2969 (66.5240mm)	2.2301 (1.3826mm)	2.8891 (1.7912mm)
Figure 4	4.4458 (2.7563mm)	2.0686 (1.2825mm)	3.0470 (1.8891mm)

the global rigid transformation model. Since the local deformation is assumed small in the model, the distance difference between the model with non-rigid term and the global rigid transformation only model is not big. However, the Table 1 shows the effectiveness of the suggested model.

4 Conclusions and Future Work

A new variational method for a simultaneous segmentation and non-rigid registration that incorporates the shape and intensity priors is proposed. Numerical results showed the effectiveness of the model to capture the boundary of the given images in detail.

From Figure 2 to Figure 4, the model performance to the given images which is compared to expert borders is showed. The global rigid transformation model was calculated in the Equation (2) without $\begin{bmatrix} u \\ v \end{bmatrix}$ term. The distance measurements to the results of the model from the expert's borders by using the distance function are followed in the Table 1. The results show the effectiveness of the model to capture the boundary of the given image, which is better than the model with global rigid transformation term only. These results were obtained through Matlab and the computational time in each iteration was about 1 second utilizing a Pentium 4 CPU running at 2.4 GHZ with 512 MB of RAM. Windows XP Home Edition was used as the operating system.

Due to the inhomogeneity, dropout, and loss of some information near the edges in heart images, parameters are adjusted in each image to get desired results. Some of the images with a diminishing problem which depends on the parameters are observed in the experiments. Since the shape prior is used as an initial contour, the level set form of the model is fixed which does not require the re-initialization process during the numerical calculation. The intensity matching term in the energy function of the model provides the difference of the intensity variation across the prior shape and the shape of interest. Minimizing the difference of the gradients of the intensity between smoothed prior image and the smoothed given image is more accurate than minimizing the gradient term in the given image, due to the loss of the information of the gradient near the edge of the desired object. Moreover, this model can be extended to 3-d cases and any other types of images. Generalization of the model to global non-rigid deformation is needed in the future work.

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