Efficient Shape Matching Using Weighted Edge Potential Functions

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Abstract. An efficient approach to shape matching in digital images is presented. The method, called Weighted Edge Potential Function, is a significant improvement of the EPF similarity measure, which models the image edges as charged elements in order to generate a field of attraction over similarly shaped objects. Experimental results and comparisons demonstrate that WEPF enhances the properties of EPF and outperforms traditional similarity metrics in shape matching applications, in particular in the presence of noise and clutter.

1 Introduction

The task of automatically matching shapes in digital images is a fundamental problem in pattern recognition. Applications of shape matching include industrial processes, robotics, video surveillance, and many others. Several approaches have been proposed in the literature to solve this problem in different application domains. For a thorough survey on the matter, please refer to [7]. The above mentioned applications are concerned with two different problems: (i) how to match objects, and (ii) how to measure the similarity among them. The first focuses on the matching procedure between an object and a model, while the latter concentrates on the problem of defining when a target is řeasonablyšimilar to a query object. Although the two aspects are often strongly connected, the definition of effective similarity measures is gaining increasing attention, also due to emerging applications such as content-based image indexing and retrieval.

As far as the evaluation of the similarity is concerned, several metrics have been developed. Traditional metrics include Minskowski, Euclidean, and Mahalanobis. More recently, similarity measures based on fuzzy logic have also been proposed. For a thorough survey on the matter, please refer to [8]. An important class of methods is the one based on distance transforms. Chamfer Matching [2] and Hausdorff Distance [5] are the current reference approaches in the field, and ensure very good performance even in the presence of complex images, distortion (e.g., affine transforms), occlusion, noise, and clutter. In [3], an alternative approach was proposed called Edge Potential Function (EPF). This method mimics the attraction field generated by charged particles [6], and differs from traditional point-to-set distances in the fact that it exploits the all edge instead

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of just nearest neighbors. This allows reinforcing the effect of coherent contours as compared to noise.

This paper reports a significant improvement of EPF, the Weighted Edge Potential Function (WEPF), which shows interesting properties for efficient shape matching. The paper is organized as follow: in Sect. 2 the concepts of EP and WEP are outlined and motivated. In Sect. 3 the use of WEP to shape matching under distortion conditions is presented. In Sect. 4, a set of selected test results is proposed, showing the performance of the proposed approach in different application conditions, and comparing it to other established approaches.

2 Weighted Edge Potential Functions

The concept of EPF can be summarized as follows (see [3] for details):

Definition 1: Given a test point q and a set of edge points $A = \{a_1, \ldots, a_m\}$, the edge potential EP generated in q by A is:

$$EP\left(q, A\right) = \begin{cases} \frac{1}{\epsilon}\sum_{i=1}^{m}\frac{1}{\|q-a_i\|}, \forall a_i : a_i \neq q \\ \gamma, \exists a_k : a_k = q \end{cases} \tag{1}$$

where γ is a peak value replacing the singularity point of the potential, and is a permittivity constant that controls the slope of the potential.

Definition 2: Given two finite point sets $A = \{a_1, \ldots, a_m\}$ and $B = \{b_1, \ldots, b_n\}$ the EP Function (EPF) of set B with respect to set A is defined as:

$$EPF\left(B, A\right) = \frac{1}{|B|}\sum_{b_i \in B} EP\left(b_i, A\right) \tag{2}$$

Given $\gamma = 1$, EPF(B,A) is in the range [0:1] where 0 reflects complete similarity between B and A, while 1 indicates a perfect match, thus providing a normalized similarity measure. It is important noting that EPF is asymmetric. This means that if for instance B is a subset of A, we find a perfect matching of B over A (EPF=1), but not of A over B. This allows matching correctly fragments or subparts of shapes.

Although EPF was demonstrated to be a very efficient similarity measure, it can be improved in two respects:

- EP depends on the values γ and ϵ, which are heuristically chosen.
- Though the high slope helps the convergence to be faster and isolates noise spots, it makes the EPF fall suddenly when the set A misses some points due to occlusion, clutter or deformation.

The second problem can be particularly critical in the presence of inaccurate preprocessing and edge-extraction, where several contour pixels may get lost. In order to overcome these problems, a weighted version of the EP is proposed, by introducing an adaptation (weighting) parameter:

Definition 3: Given a test point q and a set of edge points $A = \{a_1, \ldots, a_m\}$, the weighted edge potential WEP generated in q by A is:

$$WEP\left(q,A\right) = w_q EP\left(q,A\right) = w_q \left(\frac{1}{\epsilon}\sum_{i=1}^{m}\frac{1}{\|q-a_i\|}\right) \tag{3}$$

where $w_q = \epsilon min_{a_i \in A}\|q-a_i\|$ is the weighting factor. It is easy to verify that WEP is independent of ϵ. To this purpose, let's define $r_{qm} = \|q-a_m\| = min_{a_i \in A}\|q-a_i\|$

After a few simple passages we obtain:

$$WEP\left(q,A\right) = 1 + r_{qm}\sum_{i=1}^{m-1}\frac{1}{\|q-a_i\|} \tag{4}$$

If WEP is computed at every image point, a surface is generated with zero height in correspondence of each edge point. It is to be observed that when q is far from its nearest neighbor, the weighting factor becomes negligible. In order to better compare WEP with EP, WEP is normalized and remapped, according to Eq. 5

$$NWEP\left(q,A\right) = f\left(WEP\left(q,A\right)-1\right) \tag{5}$$

Theorem: Given a finite point set $A = \{a_1, \ldots, a_m\}$ and a point q, if we add to A a finite point set B made up of k points to create a finite point set $C = A \cup B$, then $WEP(q,C) \geq WEP(q,A)$.

Proof: Let's consider $r_{qm} = \|q-a_m\| = min_{a_i \in A}\|q-a_i\| \leq min_{b_i \in B}\|q-bi\|$ if $r_{qm} = 0$ then the theorem is evidenced. If $r_{qm} \neq 0$, then we have:

$$WEP\left(q,C\right) = min_{c_i \in C}\|q-c_i\|\sum_{i=1}^{m+k-1}\frac{1}{\|q-c_i\|} = r_{qm}\sum_{i=1}^{m+k-1}\frac{1}{\|q-c_i\|}$$

$$= r_{qm}\left(\sum_{i=1}^{m-1}\frac{1}{\|q-a_i\|}\right) + r_{qm}\left(\sum_{j=1}^{k}\frac{1}{\|q-b_j\|}\right)$$

$$WEP\left(q,C\right) = WEP\left(q,A\right) + WEP\left(q,B^{'}\right), B^{'} = B \cap \{a_m\}$$

Thus, the theorem is evidenced. In the case $min_{a_i \in A}\|q-a_i\| \geq min_{b_i \in B}\|q-b_i\|$, A and B are permuted, and the theorem is evidenced as well.

This theorem introduces an important advantage of the weighted function, consisting in a higher robustness to noise and clutter. As a matter of facts, in the presence of dot noise the weighting factor produces an automatic increase of the slope of WEP surface, which tends to isolate noise spots. On the contrary, in the presence of clutter or scattered contours, WEP automatically decreases the slope, thus producing a reasonably continuous potential function even along discontinuities in the contour.

Definition 4: Given two finite point sets $A=\{a_1,\ldots,a_m\}$ and $B=\{b_1,\ldots,b_n\}$, the Weighted EPF (WEPF) and the Normalized WEPF (NWEP) of B over A are defined as:

$$WEPF\,(B,A) = \frac{1}{n}\sum_{i=1}^{n} WEP\,(b_i,A) \tag{6}$$

or $NWEPF\,(B,A) = \frac{1}{n}\sum_{i=1}^{n} NWEP\,(b_i,A)$. Also in this case, $NWEPF\,(B{,}A) \neq NWEPF\,(A,B)$, and the larger the NWEPF value the higher the similarity of set B with respect to set A.

3 WEPF and Shape Matching

Several interesting pattern recognition problems can be addressed by using the proposed methodology. Here, we will consider the problem in its more general form, while specific applications have been already proposed in previous works with application to image retrieval[3] and video indexing[4]. The objective is to determine if the target image contains an object whose shape is similar to a model after an affine transform. The affine transform produces an instance of the model by taking into account an operator $c = (t_x, t_y, \theta, t_w, t_h)$ where θ is rotation; t_x, and t_y are the translation along x-axis and y-axis, respectively; and t_w, and t_h are the scaling along x-axis and y-axis, respectively. The matching process consists in determining the operator c that maximizes the similarity of the relevant instance of the model and the target. As far as the definition of a suitable metric is concerned, the matching function defined in Eq. 6 can be rewritten as:

$$WEPF\,(c_k) = \frac{1}{n^{(c_k)}}\sum_{i=1}^{n^{(c_k)}} WEP\left(b_i^{(c_k)},A\right) \tag{7}$$

or $NWEPF\,(c_k) = \frac{1}{n^{(c_k)}}\sum_{i=1}^{n^{(c_k)}} NWEP\left(b_i^{(c_k)},A\right)$, where $n^{(c_k)}$ is the number of pixels of the c_k-th instance of the sketch contour, $b_i^{(c_k)}$ is the i^{th} pixel of the c_k instance.

Eq. 7 can be considered a highly nonlinear multivariate fitness function to be globally maximized. This process can be solved in different ways by taking into account convergence and speed criteria. Multi-resolution and hierarchical approaches should be used to this purpose, as well as statistical methods (e.g., simulated annealing, genetic algorithms). In the present work, a Genetic Algorithm (GA) was implemented and customized to this purpose, providing a very efficient optimization tool in terms of speed and reliability.

4 Experimental Results and Conclusions

In this section, the results achieved by WEPF are analyzed and compared with other established methods. First, a comparison is proposed in terms of performance of the similarity measure (i.e., capability to correctly catch the similarity/differences among two shapes in complex/noisy scenarios). Then, a possible application of WEPF to sketch-based image matching is assessed.

As far as the comparative analysis is concerned, two well known approaches are used as a reference: Chamfer Matching and Hausdorff distance. To this purpose, the following schemes are taken into consideration:

- (DT, CM): Chamfer Matching using Distance transform [1]
- (DT, HD): Hausdorff using Distance transform.
- (EPF): Edge Potential Function.
- (NWEPF): Normalized Weighted Edge Potential function.

Two test cases are investigated to show the beneficial attributes of EPF: in both cases a complex scenario is considered, with various shapes mixed to different color textures. In the first test case, images are processed under severe noise conditions. In the second one, edges are artificially damaged.

Test 1: Fig. 1 shows a sketch-based shape matching: images (a) and (b) are the query and the target images, while (d) is a noisy version of the target image. (c) and (e) show the edge maps extracted from (b) and (d) by a Canny-Rothwell detector.

To demonstrate the effectiveness of the proposed similarity measure, an exhaustive search was performed by making the transformation operator to vary in a large range. The goal was to verify if the peak of the similarity function corresponds to the optimum matching, and if there are suboptimal or wrong solutions achieving near fitness.

Fig. 2 shows the histogram of the similarity values achieved for each examined position: it should be noticed that distance-transform-based approaches generate a spike in the histogram near the maximum similarity values, thus meaning that numerous solutions assume values near to the maximum. This makes the

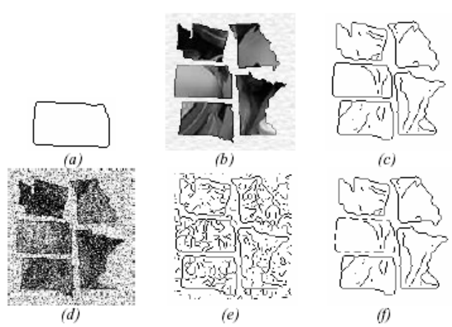

Fig. 1. Testbed 1

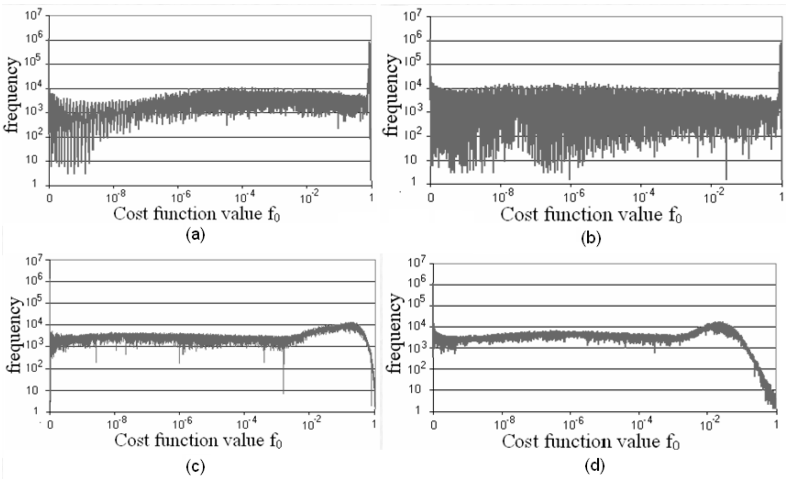

Fig. 2. Exhaustive search results - noisy environment (a) f(x)=(DT,CM) (b) f(x) = (DT, HD) (c) f(x) = (EPF) (d) f(x) = NWEPF. Frequency = Number of trial solutions for which f(x) = f_0

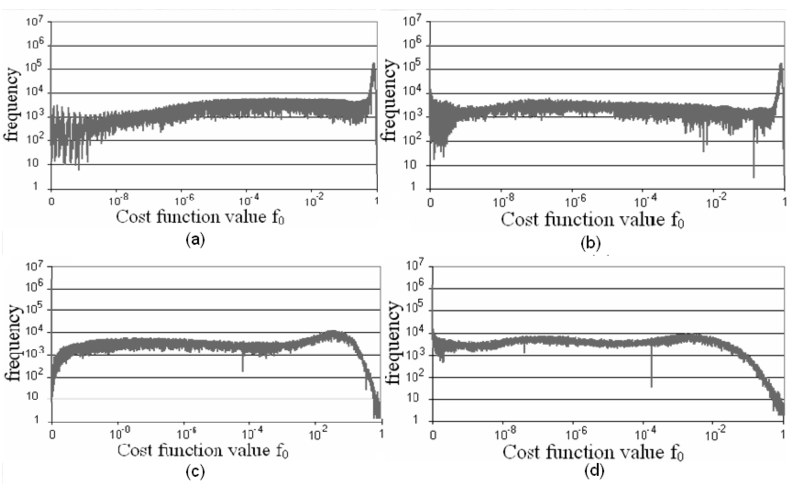

Fig. 3. Exhaustive search results - clutter environment (a) f(x)=(DT,CM) (b) f(x) = (DT, HD) (c) f(x) = (EPF) (d) f(x) = NWEPF. Frequency = Number of trial solutions for which f(x) = f_0

convergence more difficult, as confirmed by the first part of Fig. 4, where the number of false positives (wrong positions showing a computed fitness higher than the correct solution) is analysed. In the example only NWEPF is able to detect the best match with no false positives.

Test 2: Let us consider again the case of Fig. 1. This time, we would like to compare NWEPF with competing algorithms in the case of imperfect edge

Test Example	Method	The largest value of similarity measure	Frequency	Position status
Ex. 1 Noisy environment	(DT, CM)	0.998590	3	1 right, 2 wrong
	(DT, HD)	0.998588	3	1 right, 2 wrong
	(EP, EPF)	0.980825	2	1 right, 1 wrong
	(NWEP, NWEPF)	0.916814	1	1 right
Ex. 2 Clutter environment	(DT, CM)	0.997416	4	1 right, 3 wrong
	(DT, HD)	0.997407	4	1 right, 3 wrong
	(EP, EPF)	0.933193	1	1 right
	(NWEP, NWEPF)	0.914820	1	1 right

Fig. 4. Comparing the efficiency and effectiveness among (DT, CM), (DT, HD), (EPF) and (NWEPF) methods

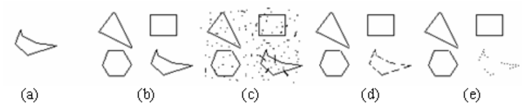

Fig. 5. Testbed 2

extraction. To this purpose, we simulate the loss of contour points by randomly erasing 50% of the contour points on the target object, i.e., the object that matches the query (f). Also in this case an exhaustive search is performed.

The resulting histogram of similarity values (Fig. 3) demonstrates the higher selectivity of EP-based measures, and the relevant matching results proposed in the second part of Fig. 4 confirm once again the ability of EPF and particularly NWEPF to achieve almost perfect detection.

Sketch-based image matching: As an additional performance test, WEPF has been introduced as a similarity measure in a content-based image retrieval (CBIR) scheme based on a genetic matching [3]. The comparative performance analysis was carried out by substituting different similarity metrics in the same matching scheme.

In particular, the following scenario is investigated: detection of the presence of a user-given sketch within a binary image representing some object shapes with added noise and clutter. Moreover, comparisons with state of the art approaches are provided to show the effectiveness of the proposed approach. To demonstrate the robustness of EPF with respect to the matching strategy adopted and to the parameter setting, all the tests shown in this section are performed by using the same matching procedure, based on a Genetic Algorithm optimization, and a fixed set of parameters.

The scenario considered concerns the detection of a object shape under several noise conditions such as additive random noise and contour losses (with loss ratio ranging from 20% to 70%). Fig. 5 shows a typical example: Fig. 5a is the query model, which is applied to the target image in Fig. 5b. Fig. 5c shows the relevant noisy image, while Figs. 5d-e show the result of a 20% and 70% loss, respectively.

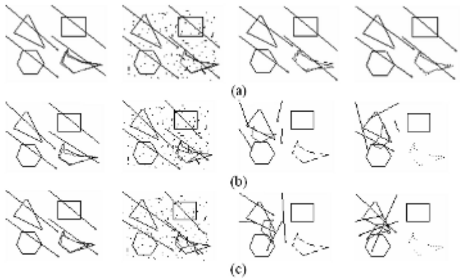

Fig. 6. (a) using (NWEPF) (b) using (DT,CM) (c) using (DT, HD)

Figs. 6a, b, and c illustrate the result when using NWEPF, DT-CM, and DT-HD, respectively, to perform the matching. By analysing these figures, it is possible to clearly state that NWEPF achieves better performance in all the situations. In particular, the charts that show the GA performance make evident that the probability of falling in a local minima corresponding to a wrong object location is pretty high when using DT-HD and DT-CM, thus achieving a wrong positioning even in the presence of a high fitness value.

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