# A New Computer Algorithm Approach to Identification of Continuous-Time Batch Bioreactor Model Parameters

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**Abstract.** The performance of a continuous-time Recursive Least Squares (CRLS) and a discrete-time Recursive Least Squares (DRLS) algorithms are examined for the growth medium temperature control of a cooling batch bioreactor in which *Saccharomyces cerevisiae* growth at aerobic condition by using Continuous-time Generalised Predictive Control (CGPC) algorithm. MATLAB programme was utilized for recursive parameter identification algorithms (CRLS and DRLS). The success or otherwise of these algorithms are estimated using parameter norm criterion for the various order of models and several input signals. There is a considerable improvement of identification algorithms with the reduced order of models. It has been shown that the performance of a DRLS algorithm is as successful as the other recursive parameter identification of a continuous-time system model.

# **1** Introduction

In bioprocesses, an organic compound is converted to a valuable product or products using enzymes or microorganisms called biocatalysts. Since the biocatalysts are very sensitive to changes occuring in their environment it is inevitable to control the operating parameters such as temperature, pH, dissolved oxygen concentration and substrate concentration for maintenance of optimal conditions for product formation in the complex environment in a bioreactor. Temperature is a fundamental parameter regulating microorganism growth, kinetics and overall product yield. For this reason temperature control systems are an integral part of biochemical processes that regulate the quality and the rate at which can be produced. About 40 % to 50 % of the energy stored in a carbon and energy source is converted to biological energy (ATP) during aerobic metabolism, and the rest of the energy is released as heat. Thus heat evolution is directly related to microbial growth [1].

Batch processes are extensively used to produce specialty chemicals, biotechnology, pharmaceutical and agricultural products. *S.cerevisiae* microorganism which is known as Baker's yeast is produced by using batch or fed-batch operation under aerobic conditions [2]. Although setting up, operating and modeling are

available in the literature for batch processes, controlling them is quite challenging. In addition, these processes exhibit time variant dynamic behavior and recharacterized by complex, nonlinear physiologial phenomenon that are difficult to model [3,4].

A model may be obtained by examining the internal structure of the system, although it is often the case that a complete picture cannot be achieved due to unknown factor, an element which is not directly measurable or an extremely complicated process [5]. The class and accuracy of a particular model is dependent on its required application. System identification is an effective procedure for the modelling of the systems. The model structures must therefore be defined and evaluations made of the parameters contained within each models. A certain model structure should approximate the system to a chosen degree and contain all the known information about operating conditions. It must also be flexible and lead to fast parameter estimation procedures [6].

Identification of process parameters for control purposes must often be done using discrete-time computation, from samples of input-output observations. On the other hand, the process is usually of continuous-time nature, and modelled in terms of differential equations. In the previously published literatures, several approaches have been developed for the identification of continuous-time model parameters [7,8,9,10].

The major objective of this study is to identify the recursive parameters of certain models in MATLAB for control of the temperature in the growth medium. DRLS and CRLS algorithms are realized for this purpose.

## 2 Materials and Method

Temperature disturbances range was chosen by taking into account the real temperature change in the bioreactor during the *Saccharomyces cerevisiae* growth at aerobic condition. *Saccharomyces cerevisiae* yeast (NRRL Y-567)utilized in this study was obtained from the ARS culture collection (Northern Regional Research Center, Peoria, IL, U.S.A.). Stock cultures were maintained on agar slants containing (in g/L): Glucose (20), yeast extract (6), K<sub>2</sub>HPO<sub>4</sub> (3), (NH<sub>4</sub>)<sub>2</sub>SO<sub>4</sub> (3.35), NaH<sub>2</sub>PO<sub>4</sub> (3.76), MgSO<sub>4</sub>.7H<sub>2</sub>O (0.52), CaCl<sub>2</sub>.4H<sub>2</sub>O (0.01) and agar (20) (pH 5). The cells growing on the newly prepared slants were inoculated in to the same liquid medium (without agar) and cultivated at 32 °C for 24 h in an incubator-shaker.

The bioreactor given in Fig. 1 was modelled in MATLAB. In this experimental system, bioreactor temperature is measured by a thermocouple. A 2 liters bioreactor has a cooling jacket. Also sensors were placed in this to measure pH and DO in the culture medium. Cooling water was continuously fed into the jacket at changing rates as an input type. Agitation was supplied using a turbine impeller at 600 rpm. An immersed heater for heating the culture medium to the desired operating temperature was also placed in the bioreactor. Air was supplied to the bioreactor by passing through a rotometer and microbiological filter at 1 vvm.

For on-line data acquisition VISIDAQ package programme was utilized. This programming package consists of Task Designer and Display Designer. The on-line computer was used in experimental studies. In the theoretical model identification work was realized by using MATLAB.



Fig. 1. Experimental system

### **3** Continuous-Time Recursive Least Squares Estimation (CRLS)

Continuous-time Generalised Predictive Control (CGPC) was based upon a continuous-time system model [11]. To be useful in control applications, the system model parameters should be iteratively estimated. For control purposes, a continuous-time single input-single output system model was utilized to identify the system. The model in the laplace domain is then given by

$$A(s)Y(s) = B(s)U(s) + E(s)$$
<sup>(1)</sup>

$$A(s) = a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n$$
(2)

$$B(s) = b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m$$

Eq. 1 can be rewritten as differential equation;

$$a_0 \frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_n y(t) = b_0 \frac{d^m u(t)}{dt^m} + b_1 \frac{d^{m-1} u(t)}{dt^{m-1}} + \dots + b_m u(t) + e(t)$$
(3)

The value of the parameter is given as  $a_n = 1$ , the estimation process can be given as linear in the parameters model;

$$y(t) = \phi^{T}(t)\theta + e(t)$$
(4)

$$\phi^{T}(t) = \left[ -\frac{d^{n} y(t)}{dt^{n}}, -\frac{d^{n-1} y(t)}{dt^{n-1}}, ..., -\frac{dy(t)}{dt}, \frac{d^{m} u(t)}{dt^{m}}, \frac{d^{m-1} u(t)}{dt^{m-1}}, ..., \frac{du(t)}{dt}, u(t) \right]$$
(5)  
$$\theta = \left[ a_{0}, a_{1}, ..., a_{n-1}, b_{0}, b_{1}, ..., b_{m} \right]^{T}$$

The data vector includes the derivative of input-output data and thus derivation increases the noise, the new data was obtained by filtering the data vector with a polynomial. For this purpose, Y(s) was added to (Eq. 1) and than this equation divided by T(s) polynomial. The rearrange equation was given as follows:

$$\frac{Y(s)}{T(s)} = \frac{B(s)U(s)}{T(s)} + \left[1 - A(s)\right]\frac{Y(s)}{T(s)} + \frac{E(s)}{T(s)}$$
(6)

where the conditions are  $deg(T) \ge deg(A)$  and  $a_n = 1$ . The rearranged linear model with the parameters is given as

$$Y_f(s) = \phi^T(s)\theta + \varepsilon(s) \tag{7}$$

the new parameter and data vectors are given below respectively;

$$\boldsymbol{\theta} = [b_0, b_1, \dots, b_m, -a_0, -a_1, \dots, -a_{n-1}]^T$$
(8)

$$\phi^{T}(s) = \frac{1}{T(s)}([s^{m}, s^{m-1}, ..., 1]U(s) \quad [s^{n}, s^{n-1}, ..., s]Y(s))$$
(9)

As it is accepted that the parameter estimation vector at time t are given in (Eq. 4), the estimated output and the prediction error are given in (Eq. 10 and 11) respectively.

$$\hat{y}(\tau) = \phi^T(\tau)\hat{\theta}(t) \qquad , \tau \le t$$
(10)

$$\varepsilon(t,\tau) = y(\tau) - \hat{y}(\tau) = y(\tau) - \phi^T(t)\hat{\theta}(t)$$
(11)

The aim is to choose the current estimate  $\hat{\theta}(t)$  such that thus error is minimum over the range  $0 \le \tau \le t$ . Least squares estimation method considers a cost function of the following form to achieve this objective. The cost function for CRLS algorithm is defined as [12];

$$J(\hat{\theta}(t),t) = \frac{1}{2}e^{-\beta_{c}t}\left(\hat{\theta}(t) - \hat{\theta}_{0}\right)^{T}S_{0}\left(\hat{\theta}(t) - \hat{\theta}_{0}\right) + \frac{1}{2}\int_{0}^{t}e^{-\beta_{c}(t-\tau)}\varepsilon^{2}(t,\tau)d\tau$$
(12)

Where  $\beta_c \ge 0$ , initial information matrix  $(S_0)$  is a possitive definite symmetric matrix,  $\hat{\theta}_0(t)$  is the initial estimate of  $\theta$ . The first term is the cost allow us to include a prior estimate in the algorithm. The second brings in the measured data into the criterion.  $\beta_c$  is the forgetting factor. As time t increases, the effect of old data at time  $\tau \langle t$  is discounted exponentially with the elapsed time  $t - \tau$ .

The main equations for CRLS algorithm are given as;

$$S(t+T) = e^{-\beta_c t} S(t) + \int_{t}^{t+T} e^{-\beta_c(t+T-\tau)} \phi(\tau) \phi^T(\tau) d\tau$$
(13)

$$\hat{\theta}(t+T) = \hat{\theta}(t) + S^{-1}(t+T) \int_{t}^{t+T} e^{-\beta_{c}(t+T-\tau)} \phi(\tau) [y(\tau) - \phi^{T}(\tau)\hat{\theta}(t)] d\tau$$
(14)

In the present work, it is noted that by filtering input-output data with a certain polynomial, the DRLS algorithm can also be used for continuous-time model parameters. The main equation for this method were given in the previously published work [13,14].

#### 4 Results and Discussion

A CRLS algorithm with MATLAB programme was utilized for recursive parameter identification of a continuous-time system model. A DRLS algorithm was also used succesfully for the same purpose. Performance of the both identification algorithm with the estimated models of the system given in Table 1 were investigated theoretically for square wave input signal. These results were given in (Fig. 2-5).

Table 1. Continuous-time models for examining the DRLS and CRLS algorithms

	Model	Poles	Zeros
Model 1	$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s}{a_0 s + a_1} = \frac{0.8s}{s + 0.5}$	-0.5	0
Model 2	$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^2 + b_1 s}{a_0 s^2 + a_1 s + a_2} = \frac{0.8s^2 + 1.2s}{s^2 + 1.5s + 0.5}$	-0.5 -1.0	0 -1.5

Affects of model order on performance criteria, parameter error norm  $(\|\theta - \hat{\theta}\| / \|\theta\|)$ ,

were investigated. It is noted that increasing model order reduces the performance of the both CRLS and DRLS algorithms (Table 2).

The important parameters of the CRLS and DRLS algorithms such as initial parameter vector, covariance matrix, information matrix, forgetting factor and sampling period were determined by using trial and error method. Their values are given 0, 10000, 0.0000001, 0.99, 0.1 respectively.

It is noted that as a new approach, discrete-time identification algorithm can be used acceptably for evaluation of a continuous-time model parameters. When the certain values of the parameter such as forgetting factor, filter polynomial etc., which affect the identification performances was choosen most effectively, the widely used DRLS algorithm in previously published work has been used successfully for continuous-time model identification.



Fig. 2. DRLS identification with Model 1

Fig. 3. DRLS identification with Model 2



Fig. 4. CRLS identification with Model 1

Fig. 5. CRLS identification with Model 2

The results of parameter identification in CRLS and DRLS for Model 1and several input signals are shown in Fig. 6-9. It is noted that the performance of a CRLS algorithm was more successfull than the performance of a DRLS algorithm (Table 2 and 3).

Table 2. Identification performance of DRLS and CRLS for the square wave input

Algorithm	Model	Parameter error norm
	Model 1	0.0109
DKLS	Model 2	0.4110
CDIS	Model 1	0.0081
CKLS	Model 2	0.2896



**Fig. 6.** DRLS identification with Model 1 and ternary input

**Fig. 7.** DRLS identification with Model 1 and PRBS input



**Fig. 8.** CRLS identification with Model 1 and ternary input

Fig. 9. CRLS identification with Model 1 and PRBS input

Table 3. Identification performance of DRLS and CRLS for various input type

Algorithm	Model	Input signal	Parameter error norm
	Model 1	Square wave	0.0109
DRLS		PRBS	0.1936
		Ternary	0.0066
		Square wave	0.0081
CRLS	Model 1	PRBS	0.0588
		Ternary	0.0080

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## Nomenclature

е	Noise
u, U	Input
y,Y	Actual output
ŷ	Predicted output
Greek Letters	
$\theta$	Actual parameter vector
^	

 $\theta$  Estimated parameter vector

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