SHAPE OPTIMAL CONCEPTION OF ANTENNA ARRAYS

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Abstract The synthesis of an array antenna is an inverse problem. The solution of this problem is the optimization of the complex excitation law of the antenna's radiating elements and the shape of the antenna which provides a specified radiation pattern.

The general character of this method permits to find numerous application in civil and military areas (satellites, telecommunication for earth mobiles, radar antennas, etc ...).

keywords: Shape optimisation, array antenna, Newton method.

1. Introduction

An array antenna is made of many elementary antennas located on a surface in R^3 so as to direct the radiated power towards a desired angular sector. As a general rule, we can consider many different shapes for the array geometry like cylindrical, spherical or conformal surface. However, in order to reduce the difficulty of the problem we consider in this paper planar array antennas. Consequently we note $\zeta \in R^{2M}$ the geometry parameter where M is the number of elements. Finally we call "weight" the amplitude and the phase of an element that we note $\omega \in C^M$.

On one hand, the basic property of an array is that the relative displacements of the antenna elements with respect to each other introduce relative phase shifts in the radiation vectors, which can then add constructively in some directions or destructively in others.
On the other hand, once an array has been designed to focus towards a particular direction, it becomes a simple matter to steer it towards some other direction by changing the relative phases of the array elements. Moreover it is also possible to modify the size of a desired angular sector by changing the amplitudes of the array elements.

The difficult problem is the dependence of the weights of the array elements towards the array's geometry, which we note henceforth $\zeta(\omega)$. On that account,

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the method which consists in optimizing both elements positions and weights is divided into sub-problems owing to the fact that each array's geometry is associated with a weight's optimization procedure.

We should note that for the general problem joining together the optimization of elements positions and weights, only stochastic methods such as genetics algorithms (GA) (see [8],[6]), simulated annealing (see [7]) or Integer linear programming (see [8]) were used in the literature. However, the problem which consists in optimizing elements weights for a fixed geometrical arrangement, is commonly solved by analytical methods. For example, the least squares technique which minimize the difference between desired and synthesized radiation pattern.

In this framework, this paper proposes a new analytical model , based on a *bicriterion* optimization method, which takes advantage of the derivative of functional expressed in term of min or max, see [4],[1],[3],[5].

2. Mathematical Formulation

We consider a planar array antenna Ω shown in Figure (1), whose M elements are located on the (x, y) plane according to the notation:

$$\zeta_m = (x_m, y_m) \ , \ \forall m \in [1..M].$$

In this paper, we are interested in fields that have radiated to large distances from the antenna. The *far field* approximation assumes that the field point r is very far from the antenna ($r >> \zeta_m \ , \ \forall m \in [1..M]$). In this case, the electromagnetic wave radiation in the far field is a spherical wave and the radiation pattern is independent of the field point distance $\|r\|$. Consequently we consider the field point $\hat{r}$ on the unit sphere in R^3 noted by S^2, where $\hat{r}$ gives the angular position in the field point.

2.1 Radiation Pattern and Directive Gain:

The radiation pattern factor of the array antenna is defined as follows:

$$| E(\zeta, \omega, \hat{r}) |^2 = \Big| \sum_{m=1}^{M} \omega_m \ g_m(\hat{r}) \ e^{j \frac{2\pi}{\lambda} \hat{r}.\zeta_m} \Big|^2$$

where λ is the wavelength, ω_m is the weight coefficient of element m and $g_m(\hat{r})$ is the radiation pattern of element m.

The *Directive Gain* of an antenna towards a given direction $\hat{r}$ is the radiation pattern nomalized by the corresponding *isotropic radiation intensity*:

$$D(\zeta, \omega, \hat{r}) = \frac{|E(\zeta, \omega, \hat{r})|^2}{\frac{1}{4\pi}\int_{S^2} |E(\zeta, \omega, u)|^2 ds(u)} \tag{1}$$

We note that the *Directive Gain* is often expressed in dB, that is $D_{dB} = 10\log_{10} D$.

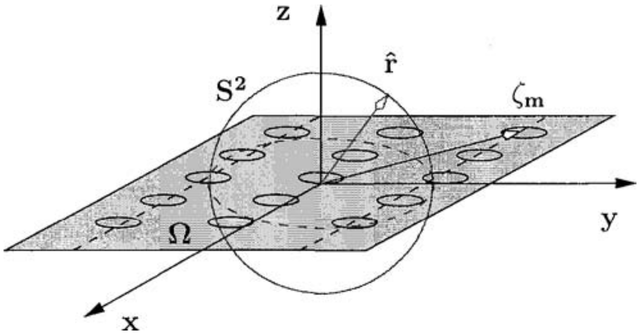

Figure 1. Representation of the planar array antenna Ω

2.2 Spatial Sampling and Grating Lobes :

If we consider a uniformly spaced array antenna, we can apply the *Nyquist-Shannon* sampling theorem (used in signal processing) to spatial sampling. This criterion implies that in order to avoid spatial aliasing the spacial sampling interval must be $d \leq \frac{\lambda}{2}$.

Consequently, the array elements should have interelement spacing of $\frac{\lambda}{2}$ or less in order to remove the grating lobes in the visible region.
The shape optimization method will create a non-periodical geometry in order to reduce the grating lobes. Indeed the presence of grating lobes has an adverse effect on the peak of the main lobe. When a grating lobe appears, energy is taken away from the main lobe and placed into the grating lobe resulting in a loss of *Directive Gain*.

3. Shape Optimization Method

The design problem consists in optimizing geometrical arrangement (ζ) and relative weights (ω) of the array elements so as to focus towards an angular pattern centered at the particular direction $u_p \in S^2$ noted $\mathcal{V}_\lambda(u_p)$.

The optimization criteria have to create beampatterns with main lobe conforms to the desired angular sector $\mathcal{V}_\lambda(u_p)$ and low sidelobe elsewhere in the spherical area $S^2/\mathcal{V}_\lambda(u_p)$. Thus in a mathematical formulation this *bicriterion* becomes:

- **Maximize the Directive Gain** in the angular pattern $\mathcal{V}_\lambda(u_p)$: using the minimisation of the functional

$$R_{in}(\zeta,\omega) = ||D(\zeta,\omega)||^{-1}_{L^\infty(\mathcal{V}_\lambda(u_p))}$$

- **Minimize the Directive Gain** in the angular pattern $S^2/\mathcal{V}_\lambda(u_p)$: using the minimisation of the functional

$$R_{out}(\zeta,\omega) = ||D(\zeta,\omega)||_{L^\infty(S^2/\mathcal{V}_\lambda(u_p))}$$

The Shape Optimization method should find an optimal geometrical arrangement (ζ^*) and its optimal weigth (ω^*)(i.e. the shortand notation for $\omega^*(\zeta^*)$), which satisfy:

$$(\zeta^*,\omega^*) = arg\left[\min_{(\zeta,\omega)\in\mathcal{U}} \left[R_{in}(\zeta,\omega)\,,\, R_{out}(\zeta,\omega)\right]^T\right] \qquad (2)$$

where the feasible set is defined as follows:

$$\mathcal{U} = R^{2M}\times\mathcal{K} \quad \text{and} \quad \mathcal{K} = \{\omega\in C^M \quad | \quad ||\omega||_{C^M} = 1\ \}$$

ASSUMPTION 1 *A vector $(\zeta^*,\omega^*) \in \mathcal{U}$ is Edgeworth-Pareto optimal for problem (2) if and only if there exists no vector $(\zeta,\omega) \in \mathcal{U}$ such that:*

$$R_{in}(\zeta,\omega) \leq R_{in}(\zeta^*,\omega^*) \quad \textit{and} \quad R_{out}(\zeta,\omega) \leq R_{out}(\zeta^*,\omega^*) \qquad (3)$$

where at least one of them is strict.

A commonly used procedure to optimize several criteria simultaneously is to combine them. We choose the functional which is the product of the two criteria: $J(\zeta,\omega) = R_{in}(\zeta,\omega)\,R_{out}(\zeta,\omega)$. The global optimization problem is

$$(\mathcal{P}) \quad \min_{\zeta\in R^{2M}}\ \min_{\omega\in\mathcal{K}}\ J(\zeta,\omega) \qquad (4)$$

For a fixed geometry $\zeta \in R^{2M}$, the weight optimization problem is

$$(\mathcal{P}_1) \quad \min_{\omega\in\mathcal{K}} J(\zeta,\omega) = \min_{\substack{\omega\in C^M \\ ||\omega||_{C^M}=1}} J(\zeta,\omega) \qquad (5)$$

3.1 Tangential Newton Method to Solve the Problem $(\mathcal{P}_1)$

A necessary condition to solve the problem $(\mathcal{P}_1)$ is to solve the vector equation:

$$\nabla_{\mathcal{K}} J(\zeta, \omega) = 0. \tag{6}$$

where $\nabla_{\mathcal{K}} J(\omega)$ is the tangential gradient on the manifold $\mathcal{K}$. Tus, this zero finding problem (6) can by solve with the *Newton-Raphson* method, which consists in solving the Newton equation through the algorithm:

1 Given ω such that $\omega^t \omega = 1$.

2 Compute Δ solution of the newton equation:

$$\nabla_{\mathcal{K}} J(\zeta, \omega) + D_{\mathcal{K}} [\nabla_{\mathcal{K}} J(\zeta, \omega)] \Delta = 0 \tag{7}$$

(where $D_{\mathcal{K}} [\nabla_{\mathcal{K}} J(\zeta, \omega)]\Delta$ denotes the directional tangential derivate of $\nabla_{\mathcal{K}} J(\zeta, \omega)$ in the direction of Δ.)

3 Compute the update $\quad \omega_+ = \omega + \Delta$.

ASSUMPTION 2 *Let* $\Pi_\omega = (Id_{2M} - \omega \otimes \omega)$ *be the orthogonal projection onto the tangent space at* $\omega \in \mathcal{K}$. *We have :*

$$\nabla_{\mathcal{K}} J(\zeta, \omega) = Pi_\omega \nabla J(\zeta, \omega) \quad \textit{and} \quad D_{\mathcal{K}} [\nabla_{\mathcal{K}} J(\zeta, \omega)] = D [\nabla_{\mathcal{K}} J(\zeta, \omega)] \Pi_\omega. \tag{8}$$

Remarks:

i) The function $J(\zeta, \omega)$ is homogeneous of degree zero:

$$\forall \omega \in C^M \quad \forall \lambda \in C \qquad J(\zeta, \lambda \omega) = \lambda J(\zeta, \omega).$$

On that account the solutions of (6) are not isolated in C^M, namely, if ω is a solution, then all the elements of the equivalence class $\{\lambda\omega \mid \forall \lambda \in C\}$ are solutions too, therefore the feasible set of the weigth is the unit sphere in C^M:

$$\mathcal{K} = \{\omega \in C^M \mid ||\omega||_{C^M} = 1 \}$$

ii) The solution of the Newton equation (7), when unique, is $\Delta = -\omega$. So any point ω is mapped to $\omega_+ = 0$. This is clearly a solution of the equation (6) but it is a trivial solution. A way to avoid it consists in constraining Δ to belong to the *horizontal space* orthogonal to ω defined as follows:

$$H_\omega = \{y \in C^M \mid y^t \omega = 0\} \tag{9}$$

With this constraint on Δ, the solution Δ of $\nabla_{\mathcal{K}} J(\zeta, \omega + \Delta) = 0$ become isolated. Moreover, the Newton equation (7) has in general no solution in H_ω, so the Newton equation must be relaxed. Therefore we project the Newton equation onto H_ω and replace it by the system:

$$\begin{cases} \Pi_\omega \left(\nabla_{\mathcal{K}} J(\zeta,\omega) \ + \ D_{\mathcal{K}} [\nabla_{\mathcal{K}} J(\zeta,\omega)] \, \Delta \right) = 0 \\ \omega^t \, \Delta \ = \ 0 \end{cases} \tag{10}$$

Implementation: Numerically the computation of the optimal weight is done by a *Newton-Raphson* under constraints method preceded by a projected conjugate gradient method. The goal of this *Newton-Raphson* method is of course to find the optimal weight $\omega^*(\zeta)$, solution of the problem $(\mathcal{P}_1)$ but especially to solve the vector equation (6) as we can see in the next paragraph.

3.2 Method to Solve the Problem $(\mathcal{P})$:

Let us consider $J(\zeta, \omega^*(\zeta)) = \min_{\omega \in \mathcal{K}} \ J(\zeta, \omega)$, then we can rewrite the problem $(\mathcal{P})$ as follows: $\min_{\zeta \in R^{2M}} \ J(\zeta, \omega^*(\zeta))$. A necessary condition to solve the problem $(\mathcal{P})$ is to solve the vector equation $\frac{d}{d\zeta} \ J(\zeta, \omega(\zeta)) = 0$, which by a chain rule becomes:

$$\nabla_\zeta J(\zeta, \omega(\zeta)) + \nabla_\omega J(\zeta, \ \omega(\zeta)) . \frac{\partial}{\partial \zeta} \omega(\zeta) \ = \ 0 \tag{11}$$

However, since we solved previously the problem $(\mathcal{P}_1)$ by the *Newton-Raphson* method, we have obtain the optimal weight $\omega^*(\zeta)$. In this way we avoid the fastidious calculation of the term $\frac{\partial}{\partial \zeta} \omega(\zeta)$. Indeed, we have:

$$\begin{aligned} &\omega^*(\zeta) \ \text{solution of} \ (\mathcal{P}_1) \ \Rightarrow \ \nabla_\omega J(\zeta, \ \omega^*(\zeta)) = 0 \\ &\Rightarrow \ \frac{d}{d\,\zeta} J(\zeta, \omega^*(\zeta)) \ = \ \nabla_\zeta J(\zeta, \omega^*(\zeta)). \end{aligned}$$

3.3 Problem $(\mathcal{P})$ with Geometrical Constraints:

In the shape optimization of an antenna, we often need to constrain the antenna geometry. As a consequence we replace the problem $(\mathcal{P})$ by the problem under constraints :

$$(\mathcal{P}') \ \ \min_{\zeta \in \mathcal{C}} \ J(\zeta, \omega^*(\zeta)) \qquad \text{where } \mathcal{C} \ \text{ is the feasible set} \tag{12}$$

In this article, we consider two different kinds of constraints concerning the geometry of the antenna in order to control the expansion or the contraction of its surface. Thus in order to manage the feasible geometry we ought to:

- constrain the maximum size of the antenna by including the realizable geometries in a circle of radius R, therefore we have for the feasible set :

$$C = \Big\{\zeta \in R^{2M} \quad | \quad \forall p \in [1..M] \quad g_p(\zeta) = \|\zeta_p\|_2 - R \ \leq 0 \ \Big\}.$$

- constrain the overlap of each array element not to have a superposition of elements between them. Thus for each element we defined its neighbors as follows:

$$\forall p \in [1..M] \qquad F_p = \Big\{q \in [1..M] \ \mid \ \|\zeta_p - \zeta_q\|_2 \leq d_{min}\Big\}.$$

and the feasible set becomes:

$$\begin{aligned} C &= \Big\{\zeta \in R^{2M} \quad | \quad \forall p \in [1..M]\,, \forall q \in F_p \\ g_{p,q}(\zeta) &= d_{min} - \|\zeta_p - \zeta_q\|_2 \leq 0\Big\}. \end{aligned}$$

4. Numerical Results

To assess the effectiveness of the proposed approach, we consider two different planar antenna arrays, where we approximate the single element pattern by : $EP(\hat{r}) = (\cos\theta)^2$. In the first example, we consider a sparse array with a fixed maximum size of the antenna. In the second example, we consider a planar antenna array constituted of a great number of elements. However, in order to reduce the size of the optimization parameter in the problem, we use the well-known idea which consists in dividing the array into groups of subarrays. Thus, each subarray m is fed through a single weight (ω_m) and (ζ_m) represents its center. In this example we use the shape optimization method under geometrical constraints to manage the overlap of the subarrays and obtain a feasible geometry.

4.1 Example 1 : Shape Optimization for a Sparse Array

We consider a sparse hexagonal array where the interelement spacing is $d = 2\lambda$, according to the *Nyquist-Shannon* criterion (2.2) this geometry creates grating lobes in the visible region as we can see in the Figure (3). We apply the shape optimization method under geometrical constraint in order to solve the problem $(\mathcal{P}')$. We compare the result of the optimal array to the hexagonal array in Figure (2).

4.2 Example 2 : Shape Optimization Antenna Using Subarrays

In this example we assume that all the M subarrays are identical and refer to each one as a *primary array*. Each subarray is made of a uniformly square

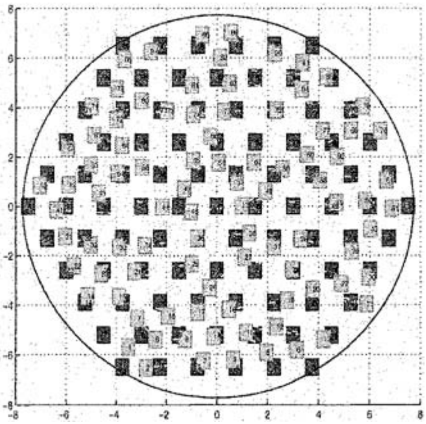

Figure 2. Geometrical representation of the hexagonal antenna array (dark) and the optimal antenna array (clear).

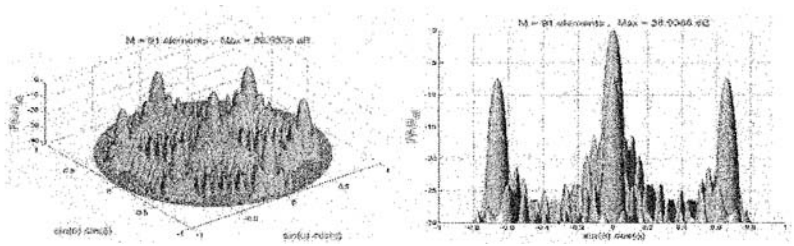

Figure 3. *Directive Gain* in dB for the antenna array with an hexagonal geometry.

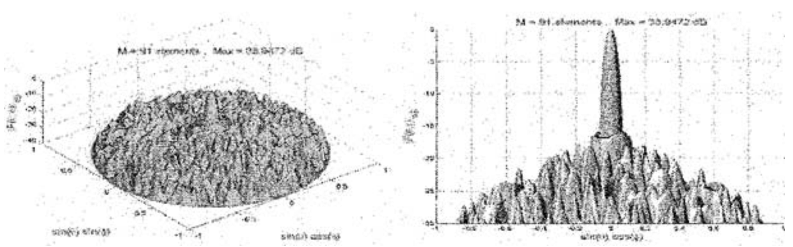

Figure 4. *Directive Gain* in dB for the antenna array with an optimal geometry.

array with N^2 elements where the interelement spacing is $d = \frac{\lambda}{2}$. The radiation pattern of each subarray is defined as follows: $|\, g_m(\hat{r})\,|^2 \; = \; |\, g(\hat{r})\,|^2$ is

$$EP(\hat{r})\,|^2 \left| \frac{\sin\left[\frac{N\pi}{2}(\,\sin\theta\cos\varphi)\right]}{N\sin\left[\frac{\pi}{2}(\,\sin\theta\cos\varphi)\right]} \frac{\sin\left[\frac{N\pi}{2}(\,\sin\theta\sin\varphi)\right]}{N\sin\left[\frac{\pi}{2}(\,\sin\theta\sin\varphi)\right]} \right|^2 \qquad \forall m \in [1..M]\,,$$

where (θ, φ) is the parametrization of the field point $\hat{r}$ on the unit sphere S^2 and $EP(\hat{r})$ is the element pattern. The array of the primary array is called the *secondary array*. The secondary array is a sparse square array where the interelement spacing is $d = \frac{N\lambda}{2}$, according to the *Nyquist-Shannon* criterion (2.2) this geometry creates grating lobes in the visible region as we can see in the Figure (1). The combined array factor will be equal to the product of these two array factors such that

$$|\, d(\zeta, \omega, \hat{r})\,|^2 \;=\; |\, g(\hat{r})\,|^2 \left| \sum_{m=1}^{M} \omega_m \; e^{j\,\frac{2\pi}{\lambda}\,\hat{r}.\zeta_m} \right|^2$$

We desire to center the angular pattern $\mathcal{V}_\lambda(u_p)$ at the particular direction $u_p = (\theta_p, \varphi_p) = (9^o, 9^o)$ and we choose $M = 100$, $N = 4$. We compare the result between the *Weights Optimization method* from the problem $(\mathcal{P}_1)$ and the *Shape Optimization under constraints method* from the problem $(\mathcal{P}')$ which optimizes both elements positions and weights. The comparison between the optimal array and the initial array is shown in Figure (5).

4.3 Weights Optimization versus Shape Optimization under constraints

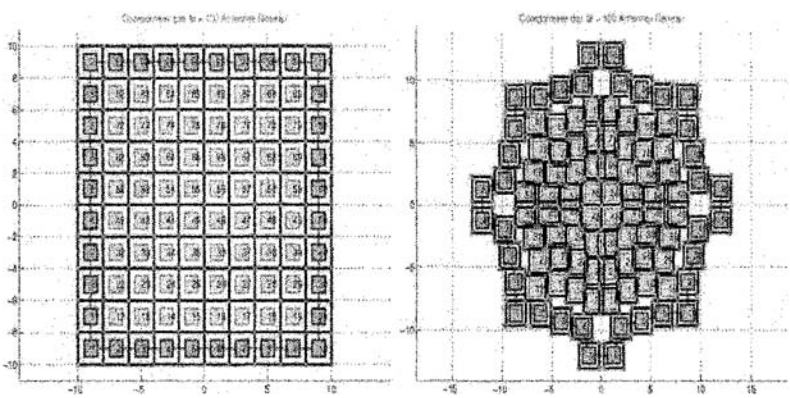

Figure 5. Geometrical representation of the initial antenna (on the left) and the optimal antenna (on the right).

From now on, all the figures on the left side will result from the **Weights Optimization** method (with the initial antenna) , and all those on the right side will result from the **Shape Optimization under constraints** method.

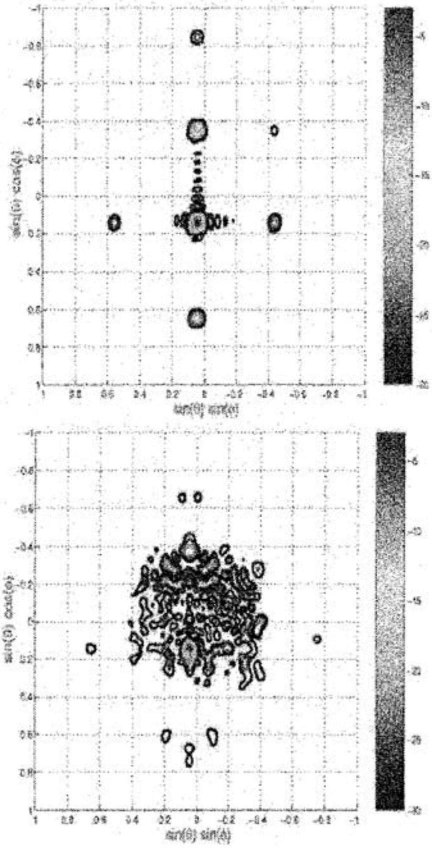

Figure 6. Isovalues of *Directive Gain* in dB.

Results from the initial antenna and the Weights Optimization method.

	Directive Gain mean	*Directive Gain* maximum
in the zone $\mathcal{V}_\lambda(u_p)$	**29.78 dB**	**35.33 dB**
in the zone $S^2/\mathcal{V}_\lambda(u_p)$	**3.27 dB**	**27.02 dB**

difference between the main lobe and the highest grating lobe : $8.31\, dB$.

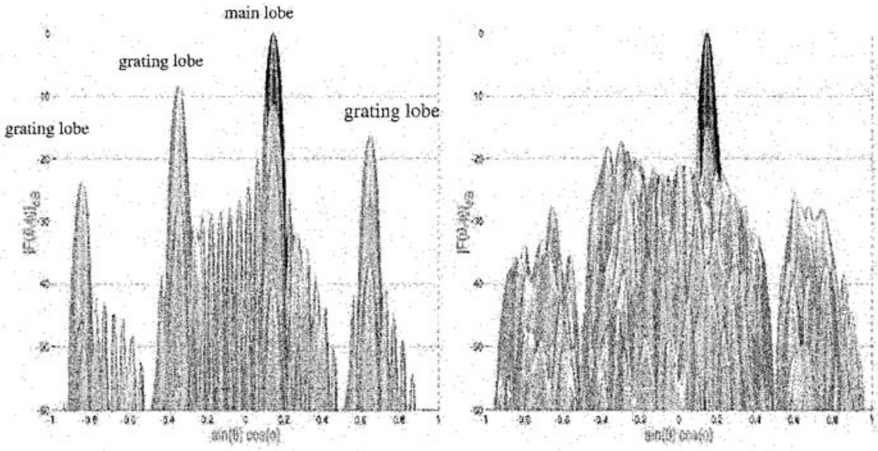

Figure 7. *Directive Gain* in dB in the areas $\mathcal{V}_\lambda(u_p)$ (dark) and $S^2/\mathcal{V}_\lambda(u_p)$ (clear).

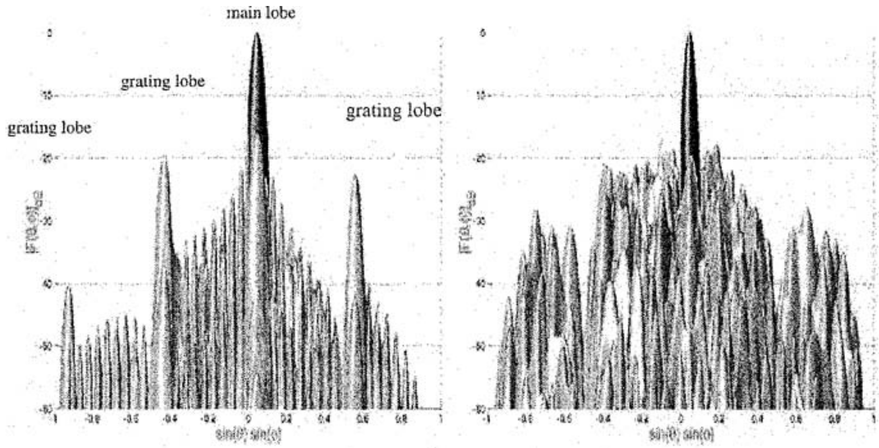

Figure 8. *Directive Gain* in dB in the areas $\mathcal{V}_\lambda(u_p)$ (dark) and $S^2/\mathcal{V}_\lambda(u_p)$ (clear).

Results from the optimal antenna and the Shape Optimization method.

	Directive Gain mean	*Directive Gain* maximum
in the zone $\mathcal{V}_\lambda(u_p)$	**29.05 dB**	**35.48 dB**
in the zone $S^2/\mathcal{V}_\lambda(u_p)$	**3.60 dB**	**18.28 dB**

difference between the main lobe and the highest grating lobe : $17.20\,dB$.

5. Conclusion

A computational method has been successfully designed for the shape optimization of an array antenna. Intuitively the non-periodicity of the array antenna is a necessary condition to avoid the grating lobes because the hypotheses of the Nyquyst's theorem are not satisfied any longer. Nevertheless, the optimization method is essential in order to definitively prevents the grating lobes and reduces the sidelobes. This analysis can be generalized for array antenna distributed on a surface by making use of intrinsic geometry approach induced by the oriented distance function [2].

References

[1] M. Delfour and J.P. Zolésio "Shape and Geometry" Advances in Design and Control,04, SIAM, 2001

[2] Delfour, Michel C. and Zolésio, Jean-Paul "Oriented distance function and its evolution equation for initial sets with thin boundary". SIAM J. Control Optim. 42 (2004), no. 6, 2286–2304

[3] Delfour, M. C. and Zolésio, J.-P., *Shape sensitivity analysis via min max differentiability*, SIAM J. Control Optim. , 1988, 26, 4, 834-862.

[4] J.P. Zolésio "In *Optimization of Distributed Parameter structures*", vol.II, (E. Haug and J. Céa eds.), Adv. Study Inst. Ser. E: Appl. Sci., 50, Sijthoff and Nordhoff, Alphen aan den Rijn, 1981 :

i) The speed method for Shape Optimization, 1089-1151 . ii) Domain Variational Formulation for Free Boundary Problems, 1152-1194. iii) Semiderivative of reapeted eigenvalues , 1457-1473.

[5] M. Cuer and J.P. Zolésio "Control of singular problem via differentiation of a min-max". Systems Control Lett. 11 (1988), no. 2, 151–158.

[6] Donelli, M., Caorsi, S., De Natale F., Pastorino M. and Massa A. "Linear antenna synthesis with a hybrid genetic algorithm" Progress In Electromagnetics Research, PIER 49, 1-22,2004

[7] Murino, V.,A. Trucco, and C.S. Regazzoni "Synthesis of unequally spaced arrays by simulated annealing" IEEE Trans.Signal Processing, Vol. 44,119-123, 1996

[8] S. Holm, A. Austeng, K. Iranpour and J. F. Hopperstad, "Sparse sampling in array processing", chapter 19 "in Sampling theory and practice" (F. Marvasti Ed.) Plenum, New York, 2000.