

## **Errata**

# **Matrix Algebra From a Statistician's Perspective**

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The paperback and online versions of the book contain some errors, and the corrections to these versions are given on the following pages.

## Preface

Page vi (Preface), line 16. Delete the comma after the word And.

## 8

### Inverse Matrices

Page 87, line 2. Delete the parentheses enclosing  $\mathbf{a}_{k_1}, \mathbf{a}_{k_2}, \dots, \mathbf{a}_{k_n}$ .

Page 87, line 13. Delete the parentheses enclosing  $\mathbf{a}'_{k_1}, \mathbf{a}'_{k_2}, \dots, \mathbf{a}'_{k_n}$ .

Page 87, line 14. Replace  $k$  (in the condition  $k = 3$ ) with  $n$ .

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The online version of the original chapter can be found at  
[http://dx.doi.org/10.1007/978-0-387-22677-4\\_8](http://dx.doi.org/10.1007/978-0-387-22677-4_8)

## 13

### Determinants

Page 194, line 9. Replace  $a_{ij}$  with  $a_{i'j}$ .

Page 194, line 16. Replace the phrase *adjoint matrix* with the phrase *adjoint* (or *adjoint matrix*).

Page 194, line 8 from the bottom. Insert the word *as* after the word *and*.

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## 21

### Eigenvalues and Eigenvectors

Theorem 21.5.6 and its proof (pages 539–540). Make the following changes:

- (1) Take the equation number (5.2) to be the equation number for the inequalities that appear on line 3 of the theorem rather than for the result that appears on line 11 of the proof.
- (2) Replace the first sentence of paragraph 2 of the proof with the following sentence: Now, suppose that  $\mathbf{A}$  is symmetric, and take  $\mathbf{x}_1$  and  $\mathbf{x}_2$  to be any nonnull vectors for which the inequalities (5.2) hold for every nonnull vector  $\mathbf{x}$  in  $\mathcal{R}^n$ .

- (3) Modify line 6 of paragraph 2 of the proof by deleting the phrase [as is evident from result (5.2)].
- (4) (Optional) Dispensing with (3) above, replace the first sentence of paragraph 2 of the proof as in (2) above and replace the remainder of paragraph 2 with the following development: And observe that, for  $\mathbf{x} \neq \mathbf{0}$ ,

$$\frac{1}{\mathbf{x}'\mathbf{x}}\mathbf{x}'[\mathbf{A} - (\mathbf{x}'_1\mathbf{A}\mathbf{x}_1/\mathbf{x}'_1\mathbf{x}_1)\mathbf{I}_n]\mathbf{x} \geq 0$$

or, equivalently,  $\mathbf{x}'[\mathbf{A} - (\mathbf{x}'_1\mathbf{A}\mathbf{x}_1/\mathbf{x}'_1\mathbf{x}_1)\mathbf{I}_n]\mathbf{x} \geq 0$ . Thus,  $\mathbf{A} - (\mathbf{x}'_1\mathbf{A}\mathbf{x}_1/\mathbf{x}'_1\mathbf{x}_1)\mathbf{I}_n$  is a symmetric nonnegative definite matrix, and upon observing that

$$\mathbf{x}'_1[\mathbf{A} - (\mathbf{x}'_1\mathbf{A}\mathbf{x}_1/\mathbf{x}'_1\mathbf{x}_1)\mathbf{I}_n]\mathbf{x}_1 = 0,$$

it follows from Corollary 14.3.11 that

$$[\mathbf{A} - (\mathbf{x}'_1\mathbf{A}\mathbf{x}_1/\mathbf{x}'_1\mathbf{x}_1)\mathbf{I}_n]\mathbf{x}_1 = \mathbf{0}.$$

It is now clear that  $\mathbf{x}'_1\mathbf{A}\mathbf{x}_1/\mathbf{x}'_1\mathbf{x}_1$  is an eigenvalue of  $\mathbf{A}$ , that  $\mathbf{x}_1$  is an eigenvector of  $\mathbf{A}$  corresponding to  $\mathbf{x}'_1\mathbf{A}\mathbf{x}_1/\mathbf{x}'_1\mathbf{x}_1$ , and (since if  $\lambda$  is an eigenvalue of  $\mathbf{A}$ ,  $\lambda = \mathbf{x}'\mathbf{A}\mathbf{x}/\mathbf{x}'\mathbf{x}$  for some nonnull vector  $\mathbf{x}$ ) that  $\mathbf{x}'_1\mathbf{A}\mathbf{x}_1/\mathbf{x}'_1\mathbf{x}_1$  is the smallest eigenvalue of  $\mathbf{A}$ . It follows from a similar argument that  $\mathbf{x}'_2\mathbf{A}\mathbf{x}_2/\mathbf{x}'_2\mathbf{x}_2$  is an eigenvalue of  $\mathbf{A}$ , that  $\mathbf{x}_2$  is an eigenvector of  $\mathbf{A}$  corresponding to  $\mathbf{x}'_2\mathbf{A}\mathbf{x}_2/\mathbf{x}'_2\mathbf{x}_2$ , and that  $\mathbf{x}'_2\mathbf{A}\mathbf{x}_2/\mathbf{x}'_2\mathbf{x}_2$  is the largest eigenvalue of  $\mathbf{A}$ .