

Using multiple metaphors and multimodalities as a semiotic resource when teaching year 2 students computational strategies

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Abstract Recent research indicates that using multimodal learning experiences can be effective in teaching mathematics. Using a social semiotic lens within a participationist framework, this paper reports on a professional learning collaboration with a primary school teacher designed to explore the use of metaphors and modalities in mathematics instruction. This video case study was conducted in a year 2 classroom over two terms, with the focus on building children’s understanding of computational strategies. The findings revealed that the teacher was able to successfully plan both multimodal and multiple metaphor learning experiences that acted as semiotic resources to support the children’s understanding of abstract mathematics. The study also led to implications for teaching when using multiple metaphors and multimodalities.

Keywords Semiotic resources · Mental strategies · Primary mathematics · Video research

With the introduction of video technology, it is now possible to focus on the subtleties that take place within the teaching and learning process (Hackling et al. 2014). This study used video technology to explore the teaching and learning interactions when a teacher used semiotic resources, in particular metaphors and their modalities, to teach computational strategies to year 2 children (children aged 7–8). For two terms, the researchers worked collaboratively with the teacher, Elizabeth (pseudonym), focusing on her interactions with a focus group of six children.

Two important theories underpin this research. The first is social semiotic theory (Lemke 1990); the lens through which Elizabeth’s use of metaphors and multimodalities was explored, as she used these semiotic resources to help her children generate meaning. The second is sociocultural theory, and specifically a

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‘participationist’ perspective on teaching mathematics, which posits that novice learners need to be inducted into the practice of mathematics (Sfard 2008).

Semiotic resources: using multimodality and multiple metaphors

As mathematics is inherently abstract, the only way to access this subject is through the use of semiotic resources (Arzarello 2006; O’Halloran 2005; Radford 2003). Whilst using multiple representations has been suggested as an effective pedagogy for many years (Haylock 1984), the complexity of using signs to generate meaning has not been acknowledged (Kress 2010). Since Peirce’s seminal work on triadic models of representation (Peirce and Welby 1977), it has been apparent that the process of meaning making is dynamic, with the presented sign being interpreted before meaning is made. In the past mathematics teaching and learning concentrated on inert graphical representations (Lemke 2002; Sfard 2008) but recent research now recognises the importance of multimodality. Lemke states:

The point of these observations is that the total activity is an integrated whole with respect to meaning-making. Again and again it would not be possible to get a complete and correct meaning just from the verbal language in the activity, nor just from the mathematical expressions written and calculations performed, nor just from the visual diagrams, overheads, and chalkboard cues, nor just from the gestures and motor actions of the participants. It is only by cross-referring and integrating these thematically, by operating with them as if they were all component resources of a single semiotic system that meanings actually get effectively made and shared in real life (p.236).

This research paper focused on the combined use of modalities and metaphors acting as a semiotic resource within the teaching and learning process. Social semiotic theory proposes that various multimodal forms, such as verbal language, gestures, symbols, and concrete objects, act as semiotic resources from which students can generate meaning (Lemke 1990).

Meaning-making from all types of modalities is central to semiotic theory, and any particular community is central in creating relevant modalities for its members (Kress 2010). Researchers have begun to consider these different modalities and their impact on learning. For example, the framework created by McNeill (1992) outlined four types of gesture: pointing, iconic, metaphoric and beat. Alibali and Nathan (2007) noted that gesture may be used by teachers to support their own reasoning as well as to support students’ understanding, and Arzarello’s (2006) work established that the dynamic use of multimodalities in mathematics learning created a combined impact. He created the concept of the single semiotic set, which underpins this research study. The single semiotic set involves a set of modes that relate to each other as they have one underlying meaning structure. Arzarello also explored the grouping of semiotic resources and multimodal learning that occurs throughout a learning experience, coining the term ‘semiotic bundle’ (Arzarello 2006).

Another important component of a semiotic resource in mathematics teaching is that of *metaphor*. Sfard describes a metaphor as ‘a mental construction which plays a

constitutive role in structuring our experience' (1998, p. 9). Interacting with metaphors across a range of modes supports children's understanding of abstract concepts, therefore teachers need to understand the central nature of metaphors in the teaching and learning of mathematics.

Although various definitions of *conceptual metaphor* exist, this study uses Lakoff and Nunez' definition because of its clarity and simplicity. In their words: 'for the most part, human beings conceptualize abstract concepts in concrete terms, using modes or reasoning grounded in the sensory-motor system. The mechanism by which the abstract is comprehended in terms of the concrete is called conceptual metaphor' (2000, p.5). Lakoff and Nunez also state that metaphors are an effective learning tool as they 'allow us to ground our understanding of arithmetic in our prior understanding of extremely commonplace physical activities' (2000, p. 54).

Lakoff and Nunez specifically mention four different types of grounding metaphors that 'allow you to project from everyday experiences (like putting things into piles) onto abstract concepts (like addition)' p.53. The first of these grounding metaphors is the *object collection metaphor*, which is identified in discrete collections of concrete objects (the count). As counting discrete concrete objects is an activity often undertaken by young children before they begin school, it is an ideal way to begin to understand arithmetic. The second and third metaphors are *motion on a path* and a *measuring stick*, respectively, and these underpin the number line. This linear metaphor has become increasingly relevant as linear measurement is now a common activity in Australia and other westernised cultures. The final metaphor is *object construction*: the 'parts of a whole' that underpin rectangular representation. The object construction metaphor is also important in understanding that numbers are 'wholes made up of parts', and occur in this research where two rectangles are considered as part of one whole rectangle (2000, p.65).

As these everyday activities are part of the westernised, primary school context in which this study took place, use of Lakoff and Nunez's conceptualisation is appropriate, allowing the tasks described in this study to be categorised according to one of their four grounded metaphors. [This categorisation would not be appropriate in studies conducted in some other cultures: for example, Edmonds-Wathan (2012) found that the relevance of a number line is limited in some indigenous cultures.]. Figure 1 highlights how teachers may use every day concrete experiences that are familiar to students as metaphors to support their construction of new knowledge (Fig. 1).

Having outlined above the general importance of metaphors, the authors also assert that there is value in using multiple metaphors rather than just a single metaphor. A concern of using only one metaphor was noticed in previous research (Mildenhall 2015). In this research it was noted that mathematical tasks in textbooks often use a selected metaphor to teach a particular concept, after which the text moves on to

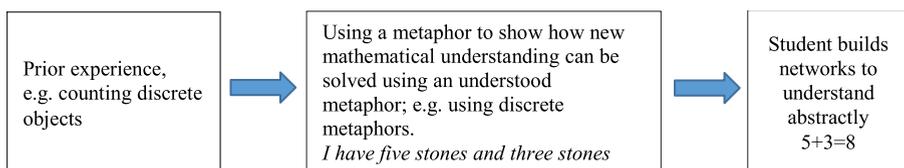


Fig. 1 Using metaphors to construct abstract mathematical understanding

another concept, rather than reinforcing the same concept using a different metaphor. For example, a text might contain questions about sharing matchsticks equally, to be followed by questions about fruit eaten on different days of the week. This may suggest to teachers and students that the metaphor is the concept rather than a conduit to the mathematical object.

Although models such as Peirce and Welby's (1977) focus on the particular understanding interpreted from a single sign or metaphor, using multiple, different signs or metaphors may be more effective in helping students build a rich understanding of the abstract mathematics (Griffin 2004). It has been suggested by researchers previously (for example, Lakoff and Johnson 1999) that a single metaphor may not be sufficient. Mowat and Davis (2010) assert that conceptual metaphors are central to understanding mathematical ideas, and that a number of different metaphors need to be accessed by the learner for a robust understanding of mathematics. Ernest concurs with Mowat and Davis, stating that 'mathematical knowledge, whether it be that of the learner, or of the culture of professional mathematics, is a growing richly connected network or set of networks with many metaphors and links' (Mowat and Davis 2010, p. 6). Children can, however, have problems learning from multi-representational environments and making connections between different metaphors (Ainsworth et al. 2009). It has also been found that some representations are more effective in representing some mathematics concepts than others (Ho and Lowrie 2014). Further research is needed so teachers are better equipped to select and combine metaphors effectively.

Teaching addition and subtraction computational strategies

This research was concerned with how semiotic resources including metaphors and modalities were used to teach addition and subtraction strategies. When considering how the semiotic resources were implemented, it is pertinent to consider the recent literature on teaching this area. Curriculum documents recommend a mental strategies approach to computation in the primary school (Australian Curriculum Assessment and Reporting Authority 2011; National Council of Mathematics Teachers 2006) and a focus on mental strategies was found to be highly effective in the seminal research of Askew et al. (1997).

Counting is the first step in children's mathematical development (Howell and Kemp 2005) and addition and subtraction develop from these counting experiences. As children progress, they need to possess a tool kit of mental strategies to draw upon to solve computational problems, rather than relying on procedural algorithms. It is important to teach students the structure of mathematics. Sarama and Clements (2004) state that the associative and commutative properties of addition are critical to understanding mental computational strategies. It is also important for young children's arithmetic that they focus on structural aspects, such as identifying that subtraction is the inverse of addition (Baroody 1984). Vilette (2002) also proposed that addition and subtraction be taught at the same time so that children realise that they can use their understanding of part-whole relationships.

Subtraction requires an understanding of both 'taking away' and 'finding the difference' (Torbeyns et al. 2009). Murdiyani et al. (2014), however, found that the focus in research on subtraction has often only been on taking away. Baroody (1984)

concur that it is important to use different strategies for subtraction. He also asserts that ‘counting down’ is challenging for children whereas ‘indirect addition’ or counting up reduces the cognitive load.

It is important that the latest research on how to teach computational strategies is used to underpin teacher’s pedagogy, and it is clear from the literature that a mental strategies focus is appropriate when working with young children. This research proposes that a mental strategies approach should be used (Sfard 2008), incorporating representations such as the number line and ten frames (Wright et al. 2014).

Conceptual framework

The conceptual framework for this study (Fig. 2) is underpinned by sociocultural theory (Vygotsky 1933). From this perspective learning is viewed as a social and cultural process and uses tools central to teaching and learning. This conceptual framework is informed by Sfard’s realisation tree (Sfard 2008) but incorporates the roles that the grounding metaphors (Lakoff and Nunez) and multimodality play in this process. The conceptual framework positions mathematical learning as taking place through discourse involving metaphors and modalities that act as semiotic resources. It represents how effective learning incorporates multimodality and can include verbal, iconic, object and gestural modalities (Sfard 2008) and specific grounding metaphors (Lakoff and Nunez 2000). The arrows in Fig. 2 demonstrate the process that children undertake as they begin to understand abstract mathematics. The framework includes Sfard’s term *realisation*, as it clearly articulates how students require perceptually accessible or tangible objects; i.e. a number line or counters, when learning about an abstract mathematical concept. An initial realisation task could be a child counting the fingers on one hand. As students then engage in multiple tasks or realisations using different grounding metaphors and modalities—for example, counting five frogs, identifying five on a number line, and subitising five on a die—they begin to identify the ‘fiveness’ in each of the multimodal tasks, and thus come to ‘realise’ or understand the abstract mathematical concept.

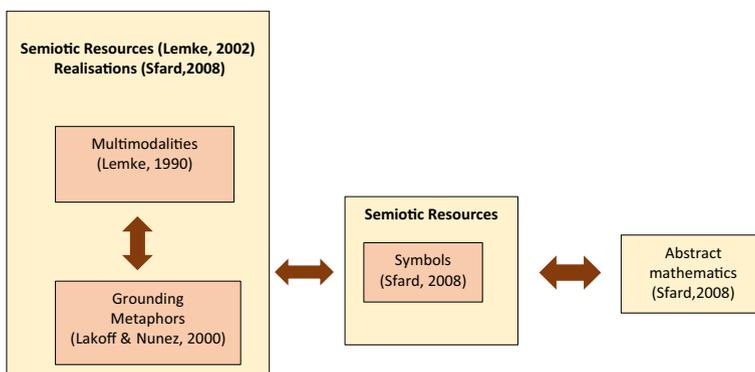


Fig. 2 Conceptual framework of the study

The conceptual framework shows that as children progress towards abstract mathematical understanding, they begin to use symbols with meaning. Symbols are the semiotic resource closest to the mathematical object, and connecting them to the metaphors in the multimodal task is important (Fig. 2). Notwithstanding previous research in this area, few studies have articulated how the combination of components of mathematical grounding metaphors and modalities act as semiotic resource to support the understanding of abstract mathematics. This research hopes to highlight the importance of these connections.

Purpose and research question

The purpose of this study was to explore how a year 2 teacher and her children used metaphors and multimodalities in lessons on computational strategies. This was done through fine-grained analysis of the lessons, which were captured on video. These findings have the potential to inform teachers and mathematics educators of how the interplay of different components within a classroom task develops children's mathematical understanding.

The research question for this study was:

How are metaphors and multimodalities utilised as a semiotic resource by a year 2 teacher and children when interacting in mathematical learning tasks designed to teach computational strategies?

Methodology

This research involved the teacher and two researchers working collaboratively in repeated cycles of inquiry rather than engaging in 'one shot' professional development. A case study was selected as it is ideally suited for collecting multiple sources of data in a rich context to answer 'how' questions regarding teaching and learning (Yin 2009). A case study is a bounded system that allows the researchers to gain an understanding of the people, situations, and events or programs involved (Yin 2009). The primary source of data was the video footage, but there were other data sources including student work samples, teacher planning documents and reflective notes.

The study was framed by a microethnographic approach to research as it explored a 'slice of a cultural experience'—how things are done in a particular place at a particular time—in order to understand how and why things are happening within that context (Garcez 1997). In this research, the focus was on one classroom's specific culture and how the teacher and children in that classroom conversed and interacted.

Participants

The case study participants were the classroom teacher, Elizabeth and a focus group of six year 2 children: Liam, Helen, Eric, John, Matilda and Christine. (Pseudonyms have

been used for all school-based participants.) Purposeful sampling was used to recruit a volunteer teacher.

Elizabeth was selected for several reasons, including her expertise and experience using a multimodal approach in her teaching, which was identified in a previous science study. She was also willing to work with the researchers during two professional learning sessions to develop and implement different metaphors and modalities into her teaching of computational strategies. Elizabeth had 17 years of experience as a generalist primary school teacher and, at the time of this study, had taught all grades in primary school both in Australia and in the UK. She had a particular interest in teaching mathematics and science subjects, and although she had explored how to use a multimodal approach when teaching science, using multiple metaphors and modalities in teaching mathematics was new to her. The children were selected by Elizabeth, included both genders, and represented the range of abilities in the class.

Data gathering techniques

This research was collaborative and involved two research cycles. Each cycle comprised the following elements: (1) a collaborative professional learning session where Elizabeth worked closely with the researchers in order to plan a multimodal and multimetaphoric approach to developing computational strategies, (2) Elizabeth's design and teaching of three mathematics lessons that incorporated metaphors and multimodal strategies and (3) the viewing (individually and collectively by Elizabeth and the researchers) of Elizabeth's practice and individual interviews with each child in the focus group. In total, six lessons were planned that focussed on different aspects of computation.

The teaching and learning activity took place in the children's classroom as part of their regular year 2 program. One of the researchers captured the interactions of Elizabeth working with the children using a *GoPro Hero 3* camera, which was positioned on the corner of the table where the learning activity was taking place. The video footage was central to the analysis, as it enabled the subtleties of the teaching interactions to be recorded, thus facilitating a deeper, more sophisticated understanding of how learning was taking place (O'Halloran 2005). The researcher also made notes whilst viewing the learning activities in situ. Work samples, teacher planning and reflections were also collected during the study.

Data analysis

The video footage was analysed using Erickson's (2006) iterative model of video analysis (2006). Video research is complex in that it involves the capture and analysis of a dynamic process. Flewitt (2006) asserts that although video research is 'messy' this inductive process is valuable as it allows extremely close scrutiny of the teaching process.

As this type of analysis does not involve a simple deductive coding process, it is important to outline in detail the data analysis that was undertaken.

Initially, the six lessons were watched in real time, giving the researchers a sense of the study as a whole. Four of the six lessons were then selected for closer analysis as they focused specifically on computational addition and subtraction strategies. It was also noted that in these lessons, Elizabeth combined different semiotic resources in her teaching of computational strategies although these were not closely analysed at that stage. Videos of the four lessons were reviewed and examined several times by the researchers both individually and together, in conjunction with lesson observation notes, Elizabeth's reflections and student work samples. The research question outlined above was used to guide the analysis. Three specific segments, during which semiotic resources were being used, were identified for detailed analysis.

When viewing the video, instances where Elizabeth used a rich combination of different metaphors and modalities were noted, and made into video clips for detailed microethnographic analysis. The researchers then watched each clip multiple times, often in slow motion, and both with and without sound. The discourse in these segments was fully transcribed and considered alongside the video clip in the analytic process (Erickson 2006).

Using a multimodal matrix, the researchers recorded the different modalities and metaphors that were used, along with the accompanying oral interactions. It was important at this stage of the analysis to include what was said and done independently, as well as how these modalities were combined (Fig. 3). It was also important to note significant interpretations of the data on the matrix, using techniques such as colour highlighting and bold text (Figs. 3 and 4). From this process, combined with the researcher's classroom observation notes, emerged thick descriptions and interpretations of the classroom instances (Flewitt 2006) that served to illuminate the complex way modalities and metaphors were used in that specific context.

Figure 4 provides an example of the analytic process. It reveals that the teacher had previously shown "adding 10" using the number line and was now showing "adding ten" in a very different way. As the teacher introduced the chart, the contrast of metaphors and modalities was recorded, and after considering why this was significant, these interpretations were noted on the matrix. This instance became part of the research findings. Through this analysis the researchers were not attempting to generalise, but to provide the reader with thick descriptions and interpretations (Flewitt 2006) that would enhance an understanding of the complex way modalities and metaphors were used in this specific context.

Time Stamp	Verbal	Gesture	Concrete object	Drawing	Comments
[00:03:28.06]	Eric: Um...				Elizabeth clarifies the word difference. Eric is looking very carefully at the line. Elizabeth scaffolds and states the word problem aloud.
[00:03:29.02]	Elizabeth: It's asking us what is the?	Elizabeth does a linear sweep to show that 8 pegs are at the tip pegs to show 21 pegs at the bottom.	Number line and the pegs. All the pegs are now on the number line.		Elizabeth uses sweeping gesture to show Eric where the 21 and 8 are.
[00:03:30.10]	Eric: Answer.				
[00:03:31.12]	Elizabeth: What is the difference? So there's 8 and there's 21.				

Fig. 3 Example of multimodal transcription matrix used to record multimodality

[00:00:55.16]	Teacher: Than doing it in 1's but what's good about adding 10. What's good about, what's easy about adding 10?	Elena picks up the 100 matrix card.	100 chart	Linking the number line and matrix (two metaphors) illustrating a similar mental strategy of jumping 10. Teacher introduces the 100 matrix card to reinforce the jumping 10 strategy. On the number line the jumping is a large jump. On the 100 matrix in contrast it is a small jump down to the next line. Ethan has an understanding of this on the 100 matrix.
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Fig. 4 Example of multimodal transcription matrix used to record interpretations of the text

Findings and discussion

Elizabeth encouraged the children to use different semiotic resources that incorporated concrete objects such as the number line, arrays and the 100 matrix to solve computational problems. The vignettes below have been selected to illustrate how the semiotic resources were used and from this analysis some possible pedagogical principles for teaching have emerged.

In lesson 2, the children were asked to represent word problems in different ways. They had been given a ‘think board’ to facilitate multi representational drawings. On their tables were counters, a number line with pegs and a rectangular representation. The children had worked in the previous week with the part-whole representation but the number line was new to them.

A number of metaphors were used by Elizabeth in this particular teaching interaction. Adding pegs to points on the number line exemplified the ‘measuring stick’ grounded metaphor of Lakoff and Nunez (2000). Elizabeth also included the object construction grounded metaphor (wholes made up of parts) rectangular representation (Fig. 5) to show part-part-whole (Ho and Lowrie 2014), a model used extensively in Singapore (Ho and Lowrie 2014).

In one interaction, Eric was given a word problem in which one person was 21 years old and another person was eight, and the task was to find the difference in their ages. Elizabeth started discussing with Eric and his partner how they could use the number line to calculate the solution to this problem. It became clear that initially Eric was unsure how to represent the mathematics on the number line. Elizabeth used the ‘think addition’ strategy as she scaffolded Eric’s learning. The number line had eight pegs attached above the numbers on the line and 21 pegs attached below the numbers on the line. Elizabeth pointed out that the number of pegs on both sides were the same up to 8, but after 8, there were only pegs on one side and that represented the difference. She explained:

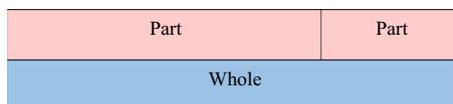


Fig. 5 Rectangular representation demonstrating part-part-whole strategy

Elizabeth: So there's 8 and 21. These, Eric, these match up together. See *up until* number 8, they match up together.

Elizabeth is using linear language such as 'up until' which is supporting the overall linear metaphor of the number line. As she was explaining the number line Elizabeth used a sweeping gesture (Fig. 6).

As Elizabeth also wanted Eric to identify that the pegs 'matched' up to eight, she used an up and down gesture to show the link between the eight pegs above the numbers and the eight pegs below the numbers on the number line. She also made some cutting gestures with her hand so that Eric could observe the parts on the number line. A summary of the use of the linear metaphor in vignette 1 is illustrated in Fig. 7. This description demonstrates that primary school teachers can incorporate multiple modalities into their teaching repertoire rather than simply focus on a tangible object, exemplifying the benefits of a multisensory learning experience as noted by Sfard (2008).

Eric's response to this instruction demonstrated that he was not able to use the number line to solve the problem. (Elizabeth is referred to as Teacher E in transcripts.)

Teacher E: So what is going to be the biggest number? Is the number going to be further than 21

Eric: Um maybe.

Teacher E: Maybe. Eric that's up to 8, he's 8 years old, Henry. Jim is 21—what is the difference? Can you point to the part that shows me the difference? These here are the same. Which—the bottom or the top?

Eric: The bottom.



Fig. 6 The sweeping gesture used by Elizabeth

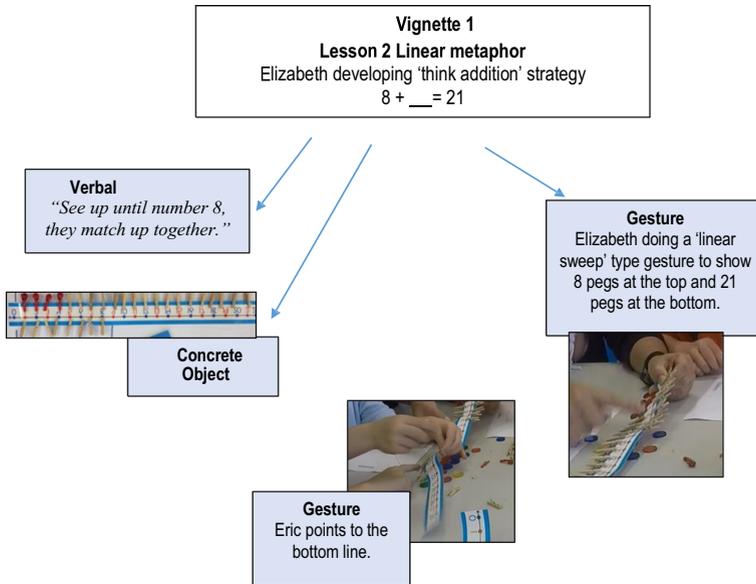


Fig. 7 Summary of the use of semiotic resources in vignette 1

Teacher E: These are the difference, so is the number going to get any bigger than 21?

Eric: Um yes

The teacher appeared to be asking questions to prompt Eric to consider what the 'whole' was, but as Eric could not extrapolate from this interaction that 21 was 'the whole', these prompting questions were unhelpful as he unsuccessfully navigated the number line (using the word *bigger* may have also been confusing with the focus on the linear metaphor).

In a previous lesson Elizabeth had used a 'parts of a whole' grounded metaphor (Fig. 5) so she decided to return to this familiar metaphor. Interestingly, she continued to use mostly linear gestures when showing Eric the rectangular representation. These included a linear sweep emphasising the length of the whole and parts of the rectangular representation. Only once did Elizabeth use a perimeter trace as a gesture to support the fact that the rectangular representation was made up of two rectangles and that the two rectangles combined to make the whole. The dissonance created by using different metaphors in the different modalities may have been confusing, particularly as Eric did not have a great deal of experience with the linear metaphor. Eric's struggle with the calculations suggests that Elizabeth should have focused on one particular metaphor using a number of modalities. In using the two different metaphors, Elizabeth was expecting Eric to make the cognitive shift required to identify the commonalities between the two, and identify the abstract mathematical concept common to both metaphors (Fig. 8). Unfortunately this did not happen.

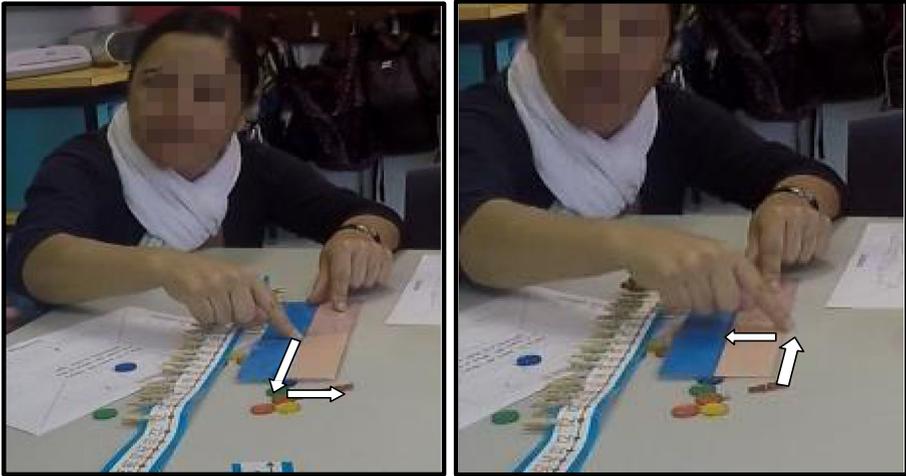


Fig. 8 Perimeter trace gesture to support the rectangular representation

As Elizabeth talked about ‘part’ and ‘whole’, it appeared that Eric was still working at the tangible level. He appeared to connect the words ‘part’ and ‘whole’ with the rectangular representation that he had been working with previously. Eric was still struggling to work on the tangible task that is shown on the conceptual framework as the beginning of mathematical understanding. This is supported by the fact that when Elizabeth asked Eric to point to the ‘part’ on the number line, he pointed to the ‘part’ on the rectangular representation instead:

Teacher E: What’s the other part? Point to the other part. Where’s the other part?

Eric: [Eric points to the 8 part on the rectangular representation.]

Teacher E: That’s already, this is this part [points to the rectangular representation]. Where’s this part on the number line? Can you point to the whole? This is one part on there.

Eric: And that’s the other part [points to the rectangular representation].

Teacher E: Point to it on this, on the number line.

Eric: [Eric then points to the part of the line with only one set of pegs on the line.]

Another example from the data that suggests that Eric’s understanding of the ‘think addition’ strategy and difference was connected only to the rectangular representation was his difficulty in stating which number represented ‘the whole’ on the number line. In this case, the length of the rectangular representation was physically a smaller representation in of the word problem than

the number line. Eric thought, when he visually placed the two representations alongside each other that ‘the whole’ on the rectangular representation was equivalent to the 16, not the 21 on the number line (where the rectangular representation ended).

Teacher E: See this is the whole, see that, that is the whole, this is that part. Can you point on the number line to what this part is? Does it stop at 16? It starts from here, absolutely right and your finger went to 16. Is that where it stops?

Eric: Yes

Teacher E: Are you sure?

Eric: Yes

Eric’s difficulties with the number line suggest that he was not working at an abstract level and had only understood the ‘think addition’ strategy in terms of manipulating the rectangular representation. He did not have sufficient experience working on the number line to identify the abstract mathematical connections between the two. This suggests that teachers need to plan the use of multiple metaphors strategically and ensure that children are comfortable working with the different metaphors before asking them to consider both. When both were introduced in this case, the concrete representations needed to be the same size.

After this lesson Elizabeth reflected that some of the children were not as engaged as she would have liked, and that this may have been due to the complexity of the multiple metaphor teaching approach as discussed earlier. Eric had not been able to solve the task using the number line, and it was made even more complicated when he was to make connections between the two. Elizabeth reflected that the children needed to develop their own understanding of what made sense to them, and participate in tasks to which they could more closely relate. This suggests that, in the initial stages of working with perceptually accessible objects, children need to spend considerable time using one metaphor in order to gain confidence before being able to work flexibly between metaphors. The transcript below highlights the teacher’s awareness of the children’s difficulties with the pedagogy she was using:

Confidence is something that Helen, Eric and Christine really need to develop in order to hold them in good stead for future learning. They need to be able to use these mental strategies well and with confidence in order to learn more complex maths. If they are still trying to remember their number bonds to 10 when the rest of the class can do this quite quickly, they will retreat and stop learning. I’ve already seen this in Eric and Christine whose first strategy is to look at their partner’s work for a starting point. They don’t engage in the thinking process on their own. This means they start all of their learning from a step behind their

peers, and this step seems to be overwhelming and therefore they stop, become disengaged and the gap gets bigger. (Elizabeth's reflection)

After reflecting on lesson 2, Elizabeth planned lesson 3, which would require the children to explain their strategies and how these made sense to them.

Teacher E: Today's learning goal is to explain what is happening in our minds—so that's your goal today.

This lesson was cognitively more straightforward than the previous one, as 'adding on' is less sophisticated than the 'think addition' strategy. The children were at that time more experienced with the number line so were able to manipulate it with ease.

Elizabeth asked the children to engage in a learning task that involved playing the game '99 or bust'. This was designed to develop addition computational strategies as the game involved reaching exactly 99 by adding on to the previous total. The game incorporated the linear metaphor as the children had to work their way up a number line.

Elizabeth modelled how different strategies could be used to add on to the previous number. She asked John how he added on 5 to 3. He used the 'turn around' strategy, adding 3 on to 5, and counting up.

Teacher E: So it's 20 something. 20 something. It's 27. I go all the way up to 27. Ahahaha... I'm winning but I don't want to bust. John, why don't you spin it. Oh 0 in the tens column. Roll your dice. 5. Ok this is where you need to start thinking. You're on 3. You need to add 5 to that.

John: 8.

Teacher E: How are you going to do that? You've said 8; you've said the answer really really quickly which is fantastic. Now you have to think about how you did that in your head. What did you do?

John: Um I knew that 5, you start with the lowest number and then I know from the, you start with the highest number and then you add on from the lowest.

Teacher E: So you started from 5

John: And then I added on 3

Teacher E: So did you just count up?

John: Yeah.

Elizabeth wanted the children to describe the images that were in their head. When doing the initial introduction to the lesson using the number line, Elizabeth asked the

children how they would work out $3 + 10$. Liam used linear verbal language, implying that he was visualising the number line.

Teacher E: Let's imagine that John spun a 1 in the tens column and rolled a 0 in the 1's column so he's on 3 and he rolled 10. Who can work that out?

Liam: 13.

Teacher E: How did you do that?

Liam: I just counted up 10 in my head.

As part of the introduction, Elizabeth had spontaneously decided to show what the 'adding 10' strategy would look like using a different metaphor. She introduced the parts of a whole metaphor as shown on the 100 matrix chart (hundreds chart), discussing 'adding 10' on the matrix. The modalities that Elizabeth used were in harmony: all supported the children in creating an image of 'adding ten' on a square divided up into parts (Fig. 9). In this specific instance, the children and teachers matched the metaphor used in the representation with the metaphors used in the other modalities. This appeared to be more successful than the approach used in vignette 1. Precisely, how to incorporate this harmonious use of metaphors needs further investigation.

In vignette 2, the verbal language related to moving the counter 'down' as the children moved the counter down in the game and this relates to what the learner physically does as ten is added on the matrix.

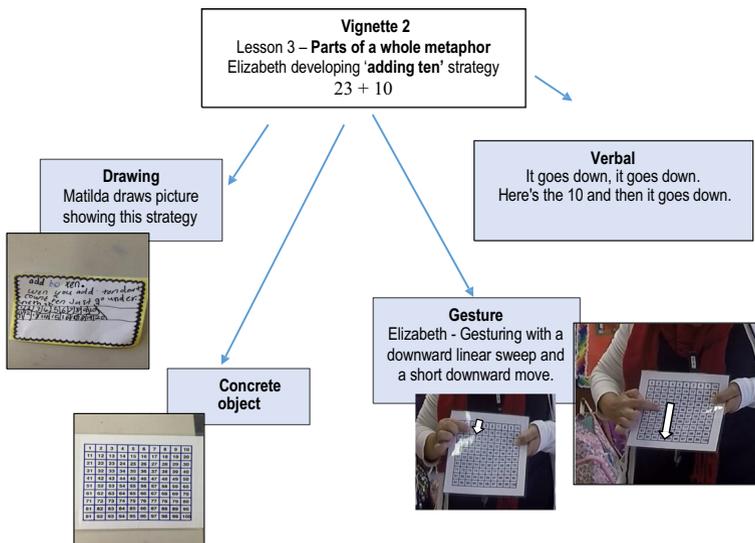


Fig. 9 Summary of the use of semiotic resources in vignette 2

Teacher E: Remember the 100's chart [Elizabeth picks up hundred's chart]. You need to think about what's happening in your brain. Here's the 100's chart. What happens to the number when we add 10? [Elizabeth points to the 3rd square]

Eric: It goes down, it goes down. [Elizabeth sweeps down with her finger]

Liam: Here's the 10 and then it goes down.

Teacher E: It goes straight down and which number stays the same all the way down that row?

Matilda: 3.

Teacher E: The 3. The 1's change, the 3's stay the same [Elizabeth keeps sweeping her hand up and down] so when I add 10 to a number, 3 stays the same and I add 1 to the tens column so it's 13. Add 10.

Children: 23.

Teacher E: Add 10. [Elizabeth does a beat to encourage a chorus reply from the children]

Children: 33.

Teacher E: 33. Add 10.

Children: 43.

Teacher E: 43. Add 10.

Children: 53.

After showing 'adding 10' on the matrix, Elizabeth returned to focus on the game that used a linear metaphor, demonstrating this calculation using linear metaphors in a multimodal way. This included using linear verbal language such as 'brings it up to' and linear gestures, such as, moving her thumb along the number line (thumb jump) and making a chopping gesture with her palm (vertical palm jump). These multimodal resources are shown in Fig. 10.

Elizabeth was playing the game with John to show the class how to play and use strategies to add on the numbers. When the game required John to add 22 to 8, he added the 20 first and then added the 8 and 2 to make 30. Elizabeth shared that her preferred way would be to add the 2 to the 8 to make 10 and then add the 20 to make 30.

John: Add the 20 and if you know your numbers adding up to 10 you can just add 8 and 2 so it's a 10.

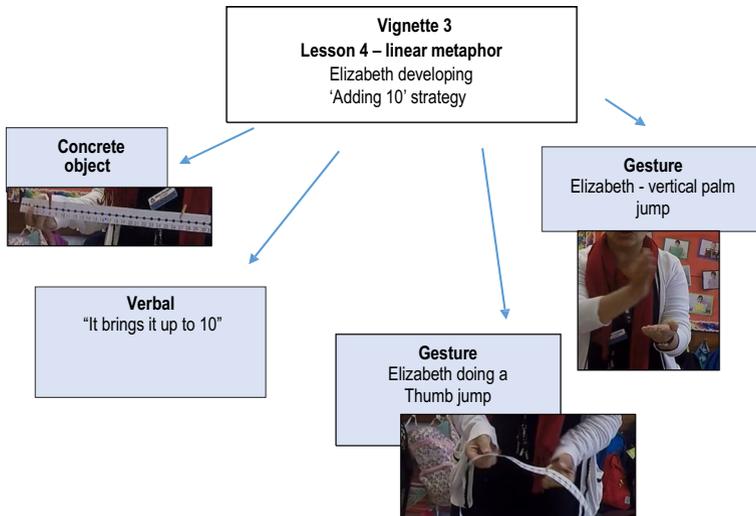


Fig. 10 Summary of the use of semiotic resources in vignette 3

Teacher E: So 22. You said add 8 and 2 and it's a 10 so why don't I just add the 2 to start with. Let's just add the 2 to start with because I know my tens. $8 + 2$ is 10. So that brings it up to 10. So I've added my 1's. What do I need to do now?

Student: Add 20.

Teacher E: Can we do that quite easily?

Student: Yes.

Teacher E: Um 10 plus 10?

Student: 20.

Teacher E: Is 20. Plus 10?

Student: 30.

Within this lesson, the children used both a linear and parts of a whole metaphor to dynamically and perceptually add 10. In vignette 1, 'adding 10' involved moving down in a multi-sensory way. Later, 'adding 10' involved jumping along a line. Towards the end of the session, Elizabeth focused on how the linear metaphor of 'adding 10' compared with the part of a whole metaphor of 'adding 10'. John thought that the matrix 'was easier than going across' and Elizabeth pointed out that the number line had some similarities.

She stated: 'It just gets to 10 and it goes to the next line. If I cut these out and put them in a row, it would be a number line'. The children then played the game again but used the matrix. The transcript below highlights the different thought processes of the focus group as they reflected on playing the game and made links between the two different concrete objects. With the 100 matrix, the children recognised that it was quite straightforward to simply look at the number directly below, with some children reflecting that visualising the '10 jump' was more challenging. The data in this specific instance suggest that using multiple grounding metaphor experiences may help children identify the abstract concept that is common to all of the tangible experiences. This is the goal of mathematics, as highlighted on the right of the conceptual framework.

Elizabeth: Talk about it with your group. Did it change the game and how did it change the game.

(Small group discussion)

Eric: It did change the game.

Liam: It changed the game a lot because if you got a number you could go straight down [*points his pencil showing movement down on the matrix*].

Matilda: You can't go down, you have to go sideways, you can't go down.
John: But I found the tens frame [*100 matrix*] too easy.

Liam: That's still horrible. Matilda: I know it is quite easy.

John: It was too easy because it was just so quick instead of actually moving [*gesturing along the number line*].

Eric: No this is much easier [*points to the 100 matrix*].

Matilda: This is better because, say I had 36 [*points to the same number on both the 100 matrix and the number line*] there's no 46 underneath I have to go sideways and count so it's slow so it's much easier and better.

Helen: Yeah and it's like easier for the little kids

Matilda: Yeah this [*picks up the 100 matrix*] would help them learn like 5 year olds.

Eric: Yeah.

John: Yeah that's what I find, that's too easy

(Whole class discussion)

Teacher E: 5, 4, 3, 2, 1. OK the question was ‘how did the 100 square change the game?’ So which person is speaking at this group?

Helen: Matilda.

Teacher E: Matilda, how did the 100 square change the game?

Matilda: Because it was easier, because in the number line [*holding the number line*] we don't have—say I was on 70 [*points to 70 on the line*] I wouldn't have 80 underneath—I would have to count 10 more on [*gesturing moving up the line*] but here you could just go down for 10 [*moves down on the 100 matrix*].

Teacher E: So it made adding 10 a lot easier. Did it change anything else? Did it change anything else? I'll come back....

John: Um I found the 100 square too easy because all you did is go down and it was over too quickly.

Teacher E: Did it change the speed of the game?

John: Yes.

Teacher E: Did it make the game quicker?

John: Yup.

Teacher E: Is that because people were calculating quicker?

John: Mmhh [*affirming*].

Teacher E: OK

John: I liked the one going across because it was like a long game.

Liam: I don't.

John: because it was like a long game.

Matilda: So that's why we said...

Teacher E: And it required you to think differently, didn't it John?

John: Yeah.

Having these discussions, making links between metaphors and comparing the thought processes that occurred in the two different learning experiences

should lead to a rich generalised and abstract understanding of mathematical concepts. Certainly the children tackled the tasks far more confidently in Vignette 2.

In the following transcript, it is clear that Eric is engaging in the learning activity in a way that he was unable to in the earlier vignette (1). Elizabeth asked the children to consider how to add $48 + 15$. Eric independently offers the answer to part of the solution $48 + 10 = 58$; and when further scaffolded, he was able to split the 5 into 3 and 2, add the 2 to 58 to make 60 and then the 3 to make 63. This reflected his increased engagement and confidence.

Teacher E: Great. Eric's on 48 he has to add 15. How are we going to do that in our head? Don't move it yet—I want you to try and think about ... Eric the challenge here is to try and think about working it out in your head before you move the peg. So 48 plus 15—what could you do? That's quite a tricky one.

Liam: Add 10 first.

Teacher E: He could add 10 first. Do you want to add 10 first? So if you've got 48, don't move it, if you've got 48 and you add 10 what do you end up with? 48 plus 10?

Student: 58.

Eric: 58.

Teacher E: Have a look here. 48 plus 10 is?

Eric: 58.

Teacher E: What do you think you might do Eric? You've got 58 and you need to add 5. What might you do? You could count up.

Liam: You could count up 3 and then add the 2.

Teacher E: What could you do? What if you thought about your 5?

Eric: 3 and 2.

Teacher E: Absolutely. So let's use the 2 to get us to the next lot of 10. And then you just have to add your 3 so you're up to?

Eric: 63.

As the project came to a close, Elizabeth recognised the need for extensive time to be spent on multimodal experiences to develop computational strategies, and that this

needed to be considered when planning teaching sequences. This was necessary so that the children could grasp the concepts for themselves. In her reflections she wrote:

We have to get better at providing our children with learning difficulties more hands on manipulatives with multimodal learning experiences, but the time that I spent with Eric needs to be spent with at least six more boys.

Implications for teaching

Analysis of the interactions in these brief teaching sequences has implications for teaching computational strategies that may be useful for teachers in other contexts.

1. Mathematics teaching and learning experiences need to include all modalities rather than use of concrete objects alone

The teaching in this study used multimodal approaches including gesture, drawings, concrete objects and verbal language. In this context it was possible, through the use of video, to identify how valuable these multimodalities were in creating a rich learning experience. In all three vignettes the teacher used specific gestures such as linear sweeps to convey particular metaphors. The teacher incorporated these with her use of verbal language and concrete objects. The finding that more than one modality can be used to convey a particular meaning has also been expressed in previous research. The awareness of connecting modalities that have one underlying meaning structure has been explored by Arzarello (2006) when he defined the term semiotic set and semiotic bundle.

2. Multimodalities need to be strategically planned to create an effective resource

In this study, Elizabeth used different modalities to teach a particular grounding metaphor. In vignette 2, the ‘parts of a whole’ grounding metaphor (Lakoff and Nunez 2000) was taught using a concrete representation of the 100 matrix, gesturing down and using drawing to show the important equal boxes. Whilst in the past, the introduction of concrete objects has involved extensive planning, in this study, the initial planning considerations included modalities such as gesture and drawings. By using a combination of semiotic resources to convey the same grounding metaphor, concepts were reinforced and confusion minimised. This research therefore suggests that when using grounding metaphors in Western primary classrooms, teachers need to carefully plan how different modalities can support learning.

3. The use of more than one metaphor to teach the same concept supports understanding of different aspects of abstract mathematics

Incorporating more than one grounding metaphor into teaching is important to support children’s understanding of abstract mathematical concepts but this process

must be planned and implemented carefully so that it does not confuse the learner. Ensuring that children can confidently complete mathematical tasks using one grounding metaphor is important before introducing others. This study showed that once children were able to work confidently with one grounding metaphor, they could then be taught the same mathematical concept using another grounding metaphor.

4. Metaphors need to be linked to support the formulation of abstract concepts

Creating links between grounding metaphors that address the same abstract mathematical concept enhances students' understanding of abstract mathematical concepts. In this study this was implemented through working on the mathematical concept of 'adding 10' using both the number line and the 100 matrix. This process appeared to engage children and increase their confidence when discussing abstract mathematics.

Conclusion

Multiple metaphor and multimodal tasks have the potential to support abstract reasoning and enhance children's understanding of mathematical concepts. The use of these resources, however, demands intentional planning if confusion is to be avoided and children's understandings are to be enhanced.

This study has built on existing research (Arzarello 2006, Lemke 2002) into the use of the social semiotic lens to support learning, and revealed how multiple metaphors and multimodalities can be utilised as a semiotic resource within a purposeful process. A combination of verbal, drawing, concrete and gestural modalities enabled year 2 children to solve abstract mathematical problems.

Few studies have articulated the semiotic resource components of mathematical metaphors and modalities and this study's findings suggest that this is a fruitful area for further research with larger samples, and investigating a wider range of metaphors. Detailing the processes involved in using multimodal resources has great potential to support teachers as they strive to deepen the mathematical understanding of their students.

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