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EDITORIAL

Psychological Theories in Mathematics Education

Introduction to the Special Issue

Andreas Obersteiner · Kristina Reiss · Aiso Heinze

The interaction between psychology and mathematics education, if well understood, is a rich source of new research topics, new concepts, and new theoretical views for both psychology and mathematics education. (Fischbein 1999, p. 58)

Mathematics education is a scientific discipline that has close connections to other disciplines. Psychology is one of these related disciplines, but the specific nature of the relationship between psychology and mathematics education is a matter of debate. This special issue aims at contributing to this debate, by presenting studies that rely on theories and models from (cognitive) psychology to address research questions relevant to mathematics education. In this introduction to the special issue, we briefly review commonalities and differences between the research disciplines of mathematics education and psychology, and then introduce the articles included in this special issue. We conclude with remarks on how mathematics education may benefit from integrating psychological theories.

1 Mathematics Education and Psychology—Closely Related Fields

Researchers from mathematic education and (cognitive) psychology share a common interest in understanding of how learning works. Aiming to improve mathematical learning in the classroom, researchers in mathematics education often take into ac-

A. Obersteiner (✉)
Freiburg University of Education, Kunzenweg 21, 79117 Freiburg, Germany
E-Mail: andreas.obersteiner@ph-freiburg.de

K. Reiss
TUM School of Education, Technical University of Munich, Munich, Germany

A. Heinze
IPN—Leibniz Institute for Science and Mathematics Education, Kiel, Germany

count the cognitive mechanisms of learning. Researchers in psychology, on the other hand, aiming at understanding general cognitive mechanisms of learning, need to study learning of certain topics, and one such topic, among others, may be mathematics. Thus, integrating research from both fields may occur naturally to some extent. In fact, there is a long tradition of this integration of both perspectives. Visible examples on the institutional level are the *International Group for the Psychology of Mathematics Education*, and the special interest group *Psychology and Mathematics Education* within the GDM, the society of mathematics educators with members primarily in countries of German language.

A research topic in which psychology has always had a particularly strong influence on research in mathematics education is learning of numbers. Mathematics education theories about learning of numbers have strongly been influenced by the work of Piaget (e. g., Piaget 1952). For example, the often-held assumption that children's ability to understand numbers is limited by their general cognitive maturation has certainly shaped educational approaches to teaching numbers in primary mathematics classrooms, and the way teaching of numbers was sequenced in mathematics curricula.

More recent research has pointed to children's very early intuitive sense for numerosity and numerical operations (Wynn 1998) and suggested that the human brain is well prepared for processing numerical information from early on (Dehaene 1998). Findings from these psychological and neuropsychological studies have generated novel perspectives on numerical development (Siegler and Lortie-Forgues 2014). This line of research is currently finding its way into the mathematics education literature (see, for example, the special issue on the integration of neuroscience and mathematics education in the journal *ZDM Mathematics Education*; De Smedt and Grabner 2016). However, there is controversy about the implications this recent research has for mathematics education (e. g., Verschaffel et al. 2016).

2 Challenges in Integrating Mathematics Education and Psychology

In spite of their shared interest in learning, there are also profound differences between the disciplines of psychology and mathematics education. For one, although studying the same topic, researchers from both fields do not always ask the same questions. Psychological research aims more strongly at understanding general mechanisms of learning and development in various domains. Mathematics educators, on the other hand, often consider the mathematical domain as a starting point, and aim at finding ways to support teaching and learning of that domain.

Different research questions go hand in hand with different theoretical frameworks and terminology. As Nolte (2015) pointed out, researchers from psychology and mathematics education do not always speak the same language. As an illustration, Nolte considers the term "number magnitude", which is commonly used in the psychological literature on numerical cognition. Mathematics educators—at least those in German speaking countries, where magnitude is often translated as "Größen"— may associate the term magnitude with the measurement aspect of numbers (e. g., 5 cm). Researchers in psychology, however, often use number magnitude to refer to the semantic meaning of numbers more general, including a variety of aspects such as the cardinality of collections of objects (e. g., Siegler 2016).

Another important difference between psychology and mathematics education concerns the research methods. Psychological research more often uses strictly controlled experiments to isolate potentially relevant factors. Mathematics education researchers, on the other hand, strive for understanding learning that occurs in more natural, realistic learning settings. When studying realistic settings, it is often not possible to use controlled designs, and doing so would threat external validity.

3 Questions Arising from Merging Perspectives

Considering the close relationship between mathematics education and psychology on the one hand, and the profound differences on the other, the question arises what the specific role of psychology for current mathematics education research is (Verschaffel et al. 2017). Addressing this issue bears on the very question of the nature of mathematics education as a discipline (Biehler et al. 1994). Do mathematics education researchers "borrow" theories of learning from psychology (an approach discouraged, for example, by Fischbein 1999). Do mathematics education researchers modify existing psychological theories so that they can fruitfully be used in mathematics education research? Or do they develop unique theories that emerge from merging mathematics education and psychological perspectives? This special issue provides insights into these questions.

In the sections above, we have used numerical learning as an example to illustrate the relation between psychology and mathematics education. However, there are many other areas in which psychology and mathematics education research have been integrated (e. g., Heinze and Reiss 2009, with respect to mathematical proof; or Seidel and Reiss 2014, with respect to general classroom instruction). The articles in this special issue extend this research and cover a wide range of topics.

4 Overview of the Contributions to the Special Issue

The aim of this special issue was to bring together empirical studies in which researchers include theories and models from psychology to address questions in mathematics education. The issue was open to researchers from both disciplines, and to studies of any mathematical domains and participant age groups. All authors were asked to state explicitly which psychological theory they adopted in their research. They were also encouraged to discuss how their research benefited from including the psychological perspective, and where the limitations were.

This special issue includes seven papers that cover a wide range of psychological theories, mathematical content areas, research methods, and participant age groups. The first two papers study arithmetic abilities in young children, and they include different psychological theories to address their specific questions. *Fritz, Ehlert, and Leutner* study the development of arithmetical abilities from a cognitive and developmental psychological perspective. In three longitudinal studies with children between pre-school and second grade, the authors aim at validating their theoretical model of arithmetic development (Fritz et al. 2013). The study by *Graß and Krammer* relies on cognitive and neurocognitive research such as the Triple-Code Model of number processing (Dehaene 1992) that suggests that processing of numbers may be related to spatial reasoning. Graß and Krammer hypothesize that spatial abilities are related to performance in mental arithmetic at the end of primary school, and that this relation is mediated by basic number processing.

Loibl and Leuders also study learning of numbers, although they focus on learning of fractions rather than natural numbers. They merge mathematics education and psychological perspectives on the instructional principles of productive failure and discovery learning (Kapur and Bielaczyc 2012). In their study, they explore whether fifth-graders can profit form the errors they make during an exploration phase.

Two studies focus on the cognitive processes involved in using specific external representations to solve mathematical problems. *Rolfes, Roth, and Schnotz* use theories about learning from text and pictures (Schnotz and Bannert 2003) to study sixth- and seventh-graders' performance differences in problems with different representations of functions. They link psychological mechanisms of processing information to specific features of representations of functions such as graphs and tables. *Böcherer-Linder, Eichler, and Vogel* also study why certain representations for the same problem may be more beneficial for understanding the problem situation than others. They focus on statistical reasoning problems that require the application of Bayes' theorem. There are different ways to verbally formulate and to visually represent these problems, such as tree diagrams and unit squares. Böcherer-Linder and colleagues use dual process theories (Kahneman 2011) to explain the specific challenges that university students face while solving problems in these representations.

There is another study that relies on theories of intuitive reasoning during problem solving. *Lehner and Reiss* present a study grounded on dual process theories to describe university students' strategies and their errors while solving problems represented with 2×2-tables. In addition to paper and pencil tests, they use eye tracking methodology, which is thought to tap closely into the cognitive mechanisms during problem solving (Mock et al. 2016).

Finally, *Rellensmann and Schukajlow* include a non-cognitive view on learning. The central psychological theory of their study is the control-value theories of achievement emotions (Pekrun 2006), which predicts that non-cognitive variables matter for learning. Rellensmann and Schukajlow address the often-held believe that mathematical problems are more interesting for students when they are embedded in real-life situations. In their study, ninth-graders rate their enjoyment and boredom in problems that have more or less strong connections to reality. They also study prospective teachers' predictions of students' responses.

5 Concluding Remarks

The articles included in this special issue cover a wide range of research topics. As such, they illustrate the breadth of the research field that integrates psychological theories and models to address research questions relevant to mathematics education.

However, the articles in this special issue do certainly not discuss the relationship between mathematics education and psychology in an exhaustive way. Rather, by documenting the current state of research, they may spark further discussion about ways to integrate both disciplines.

We notice that previous discussions about the relationship between mathematics education and psychology were centered around the primacy of one discipline over the other. We do not regard this a fruitful perspective. Rather, each perspective has its particular strengths, and merging perspectives requires being aware of the level of explanation that each perspective provides. We recommend developing a theoretical framework of how to integrate perspectives. Such a framework could help researchers communicating clearly which perspective they address, and how they relate various perspectives to one another (Obersteiner et al. in press).

We also notice that the field has made progress in merging perspectives, and that in some content areas integrating perspectives has become very common (e. g., in the area of learning of numbers, Norton and Alibali in press), so that it can even be difficult to make clear distinctions between the disciplines. In these areas, we may be on a path toward inherent integration of both perspectives (Star and Rittle-Johnson 2015), which may result in a truly integrated "psychology of mathematics education" (Fischbein 1999, p. 57). Such an integrated approach seems most promising for supporting teaching and learning in the best possible way.

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