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ORIGINAL ARTICLE

The influence of the mass on the finish time in skeleton and luge competitions, and the fairness of rules and regulations

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Abstract

In gravity-powered sports, more mass at the same drag area results in a shorter finish time. Based on the body mass data and the finish times of the Skeleton and Luge competitions at the 2018 Winter Olympics, this study investigated the fairness of rules, by establishing trends between finish time and body mass or simulated system mass. A glide model served for the assessment of the sensitivity of mass, drag area and coefficient of friction, and for determining how much mass were required to tie with the next higher rank of the first four athletes of each competition. The rules of Skeleton and Luge competitions permit the use of ballast mass as a function of the athletes' body mass, but the reference mass was up to 10 kg too low. When correlating the finish time with the body mass, all trends were significant ($p \leq 0.032$; $\alpha = 0.05$) which indicated that the finish times were mass dependent. Correlating the finish time with the simulated system mass reveals the same result except for the men's Luge competition. The sensitivity analysis showed that 15% change of an input parameter resulted in about 1% change of the finish time. Despite the low sensitivity, the masses required to tie with the next highest rank ranged from 0.02 to 20 kg, with a median of 0.7 kg. The skeleton rules were improved in 2020 which now provide equal conditions across a wide range of body masses; however, the current Luge rules still disadvantage lighter athletes even when using the permitted ballast mass.

Keywords Luge · Skeleton · 2018 Winter Olympics · Rules · Body Mass · System Mass · Finish Time · Glide Model · Fairness

1 Introduction

In gravity-powered sports, the system mass of the athlete plus equipment is directly related to the speed and thus to the finish time. The anthropometric code number [1], derived from the terminal velocity (gravity-powered system with zero acceleration), equals $\sqrt{(BW/Ad)}$, where BW is the body weight (N) and Ad is the drag area (m^2; frontal area times drag coefficient). Therefore, the heavier the athlete and the smaller the product of frontal area and drag coefficient, the greater is the terminal velocity and the faster is the athlete. Heavier athletes are therefore advantaged. This is, for example, the case in skiing, particularly in alpine skiing and in ski cross [2].

In luge and skeleton competitions, because of the use of heavier equipment (sled), the mass is regulated [3, 4], and ballast mass is permitted, to allow athletes with a low combined body and equipment mass (system mass) to reach a mass limit suggested by the rules. Ballast mass is supposed to make competitions fairer by reducing, if not removing entirely, the advantage of a heavier body mass. Nevertheless, the influence of the mass is downplayed, by claiming that "*…the athlete with the best driving skills, the ability to relax on the sled, a fast start, the best preparation, and a good work ethic, will win.*" [5].

The finish time, however, depends on several factors, in addition to the mass (body, equipment, or ballast): aerodynamic drag, start time, equipment (coefficient of friction; ease of steering), track conditions (coefficient of friction; temperature; changes of track conditions throughout the competition), psychological factors (stress; mental strength;

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feel of the equipment), and a factor commonly referred to as 'skill'.

The influence of the start order is taken care of by a start sequence based of the reversed ranking order of the previous run or race heat [3, 4]. In bobsledding, however, the start order, together with the temperature and the start time, explained about half of the variance in the performance [6] estimated from the data (including turn times of 14 turns) of the 1988 Winter Olympic Games in Calgary. The authors of this study recommended that the then "*existing rule concerning the start order in a heat be modified to guarantee a fair competition*" [6]. Based on data of the 1994 Lillehammer Winter Olympic Games, the start times and accelerations of luge and bobsled competitions correlated significantly with the finish time [7]. Interestingly, both studies [6, 7] did not assess the influence of mass (neither body mass not system mass).

As far as the geometry of track is concerned, Hubbard et al. [8] developed a mathematical model in silico for three-dimensional bobsled turning that allows evaluating the influence of track design. Zhang et al. [9] simulated the 1994 Lillehammer Winter Olympic Track and calculated the travel times for free traveling and optimal control by including the steering forces. Gong et al. [10] simulated the Igls ice track (1976 Innsbruck Winter Olympic Games) to explore the control strategies of steering. Braghin et al. [11] simulated the Cesana Pariol Olympic track (2006 Torino Winter Olympic Games) and obtained a good agreement between calculated and measured speed along the entire track.

In contrast to previous studies, this one concentrates on the influence of mass (body, and simulated system) on the finish time in luge and skeleton competitions. Fortunately, the International Olympic Committee (IOC) [12] published both the finish times and the body mass of the athletes for the luge and skeleton competitions at the 2018 Winter Olympic Games. This study does not evaluate the relationship between finish time and any other factors, due to the lacking availability of data, such as track temperature and details of each segment of the track (e.g., slope angle, as well as radius and bank angle of each turn at the Alpensia Sliding Centre near Pyeongchang, South Korea).

The available data of mass and finish time allow for testing of the following hypotheses, based on the fairness principle, expressed as null hypotheses:

(a) The body mass of the athletes is ***not*** correlated to their finish times; and
(b) A simulated system mass (athlete + equipment + maximum ballast mass permitted) is ***not*** correlated to the finish times either.

The basic aim of this study was to reject the null hypotheses – for two reasons:

(1) From first principles, heavier athletes are advantaged in gravity-powered sports;
(2) The more the correlation of finish time vs mass worsens (increasing regression p-value) when considering the permitted ballast mass, the fairer are the rules.

The extended aims of this study were to (1) thoroughly examine the luge and skeleton rules regarding mass regulations; (2) to analyse data from competitions to establish or refute evidence of the statistical influence of mass on the finish time; and to (3) apply a glide model to analyse the effect of this influence. The bigger picture of this study was to understand whether the rules are fair and offer equal conditions and opportunities across the body mass ranges.

2 Methods

2.1 Rules and regulations

The rules of Luge [3] and Skeleton [4] in force during the 2018 Winter Olympic competitions were analysed to understand the principles of adding ballast mass to the athletes' and the sleds' masses, and how the different mass components are regulated.

2.2 Data of the PyeongChang 2018 Winter Olympics

The IOC [12] provided the anthropometric data (including body mass) of the athletes participating in the men's and women's Luge and Skeleton competitions, and the finish time data of six training and four final runs of the four competitions. Although the athletes plus sled may be weighed before or after each race [3, 4], the IOC has unfortunately not published these data. As such, the masses of sled, gear and ballast were unknown.

In this study, the two best finish times of each athlete were correlated with their body mass to estimate its influence on the finish time. In general, the influence on the finish time is multifactorial and it depends on physical and skill parameters. The physical parameters that vary between the athletes and between the equipment are the mass, the aerodynamic drag area, and the coefficient of friction. In terms of the mass, only the body mass of the athletes was known, but not the other masses such as sled, apparel, and ballast. When correlating the finish time with the body mass by means of a regression model, then the coefficient of determination R^2 informs of the influence of the body mass on the finish time (as a percentage: $100R^2\%$), and the p-value informs whether the apparent trend of the regression slope (increasing or decreasing) is real or just an illusion. We expect a negative slope in the first place, as a greater mass is associated with a shorter finish time. Therefore, the null hypothesis and the alternative hypothesis in a *decreasing* trend are as follows:

- H0: the effect is greater than or equal to zero.
- H1: the effect is smaller than zero (negative slope).

As these tests cannot distinguish between zero and an effect in a particular direction (thus: *greater than or equal to zero*), these tests are inherently one-sided. Accordingly, the one-sided p-value of the regression was calculated for rejecting ($p < 0.05$), or failing to reject, the null hypothesis. If $p < 0.05$, then 100 $(1–R^2)$% corresponds to the unexplained influence, due to masses other than the body's, as well as due to drag, friction, or skills.

When correlating the finish time (t) to the athlete's body mass (m) in this study (Fig. 1), then the R^2 and p-values were reported together with the linear regression function f ($t = fm$) and the regression scatter plot was presented. From the regression function the residuals were calculated and tested for normality with the Shapiro–Wilk test. If the calculated Shapiro–Wilk statistic W is greater than critical value of W (at 5% significance level), and therefore the Shapiro–Wilk p-value is greater than 0.05 (one-tailed p), then the residuals are normally distributed about the regression fit line. If the residuals were not normally distributed, and as the central limit theorem did not apply because of small numbers of data, a new method was developed to achieve normality. This method is inherently connected to finish times in the sporting context. When exploring the distribution of the residuals (Fig. 2), it is less likely to find outliers at the tail of the short finish times. Outliers at the 'short' tail at the left side of the distribution occur only if athletes are advantaged by a revolutionary technology (e.g., aerodynamics) that gives them the edge over the rest of the competitors. It is more likely to find outliers at the tail of the longer finish times, due to general factors such as human errors, poor strategy, low skill level, insufficient equipment, equipment failure, fatigue, etc. The hypothesis here is that these outliers spoil the normal distribution, and outlier removal is expected to restore normality. Accordingly, the residuals were plotted as a box-whisker diagram (Fig. 2) and the outliers were identified with the Tukey's fences test from the interquartile range (IQR; data more than 1.5 IQR below the first quartile or more than 1.5 IQR above the third quartile are considered outliers). This method was repeated until normality was achieved (Figs. 1 and 2). If the residuals of the regression were still not normally distributed after all outliers were removed, further data were excluded starting with the most distant residual on the right side of the distribution until normality was achieved.

In summary, both methods, (1) correlating the original finish time data to the mass (body or system) and (2) correlating the finish time data with normally distributed residuals to the mass, explore the influence of the mass on the finish time, where $100(1–R^2)$% corresponds to the unexplained influence (masses other than the body's [only if the body mass is used], or drag, friction, skills). However, in the 2nd method, the unexplained influence does no longer include skill errors. Therefore, the 2nd method serves three purposes: (1) achieving normality of the residuals' distribution, (2) reducing the influence of skill that is confounded with the influence of the body mass, and (3)

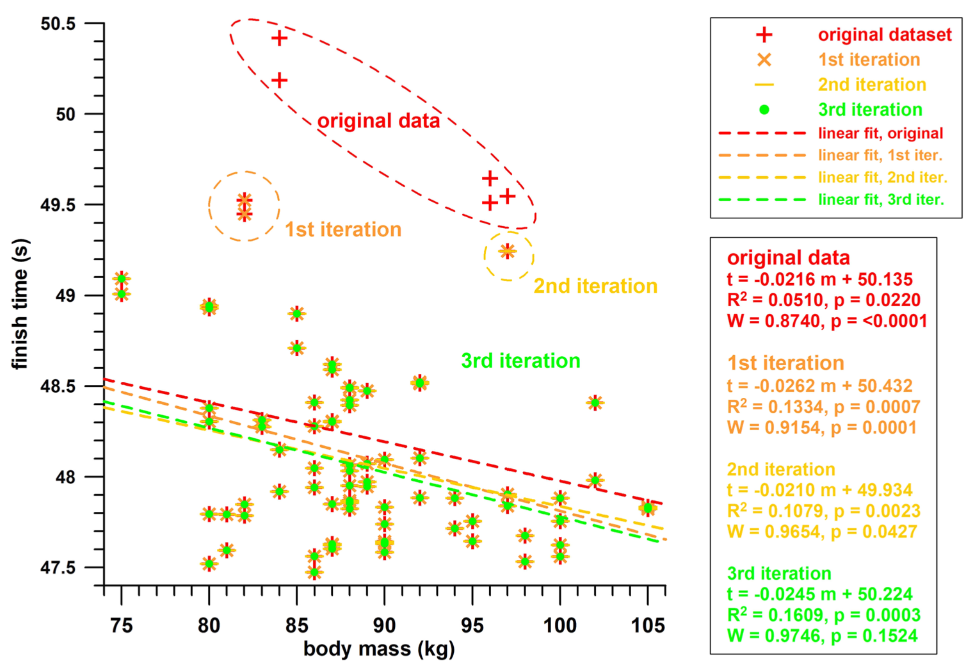

Fig. 1 method for outlier removal, required for assuring normality of the residuals; Luge men competition, PyeongChang 2018 Winter Olympics, two best finish times of each athlete against the athletes' body mass; the original data underwent three iterations of outlier removal; 5 outliers were removed (dashed red ellipse) from the original dataset, 2 outliers from the 1st iteration (dashed orange ellipse), and 1 from the 2nd iteration (dashed yellow ellipse); the 3rd iteration showed a Shapiro–Wilk (W) p-value of 0.1524 (> 0.05), which proved the normality of the residuals; note that both the regression p-value and the Shapiro–Wilk p-value are one-tailed (α = 0.05) (color figure online)

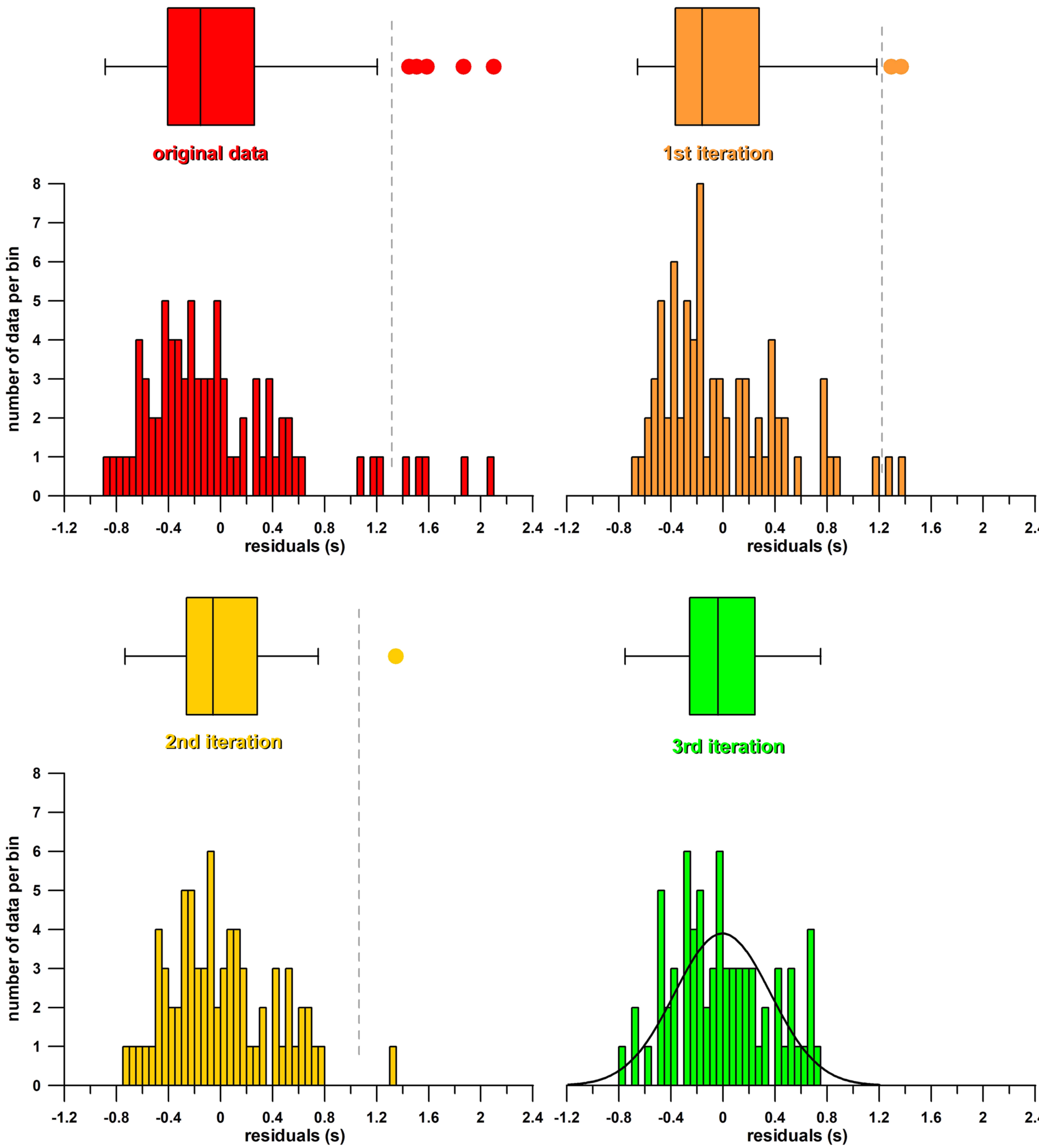

Fig. 2 distribution of the regression residuals of Fig. 1 (exemplified by the data of the men's Luge competition, PyeongChang 2018 Winter Olympics) when removing the outliers; the data and the iterations are the same as in Fig. 1; the abscissas apply to both the histograms and the box-whisker plots; the width of the histogram bins is 0.05 s; the dashed grey line separates the outliers (●) and the associated bins form the rest of the data (color figure online)

testing whether the result of the 1st method is still valid, e.g., negative slope with $p < 0.05$. As outliers can only be removed because of *unusual conditions* [13] among other reasons, the unusual conditions are here suspected skill errors, and do not serve to improve the p-value for achieving significant results.

In addition to evaluating the influence of the mass on the finish time, the influence of the start time was investigated too, as the IOC published the start time together with the finish time.

2.3 Glide model

The glide model developed by Fuss [2] for skiing is applicable to skeleton and luge too and is briefly explained subsequently. Full details are given by Fuss [2].

According to the Free Body Diagram of Fig. 3, the following forces are in equilibrium:

$$F_{Gy} = F_R \tag{1}$$

$$F_{Gx} = F_I + F_F + F_D \tag{2}$$

where F_{Gx} and F_{Gy} are the 2 components of the gravitational force F_G; F_R is the ground reaction force; F_I is the inertial force; F_F is the friction force, and F_D is the aerodynamic drag force. Note that the lift force is not considered in Eq. (1) as no data are available in the literature. As $F_F = \mu F_{Gy}$; $F_I = m\,a$; and $F_D = 0.5\,\rho\,Ad\,v^2$, where μ is the kinetic friction coefficient, m is the system mass, a is the system acceleration parallel to the track surface, ρ is the air density, Ad is the drag area, and v is the system velocity parallel to the track surface, Eq. (2) is modified as follows:

$$F_{Gx} = m\,a + \mu\,F_{Gy} + 0.5\,\rho\,Ad\,v^2 \tag{3}$$

Considering that $F_{Gx} = m\,g\,\cos\theta$, and $F_{Gy} = m\,g\,\sin\theta$, where g is the gravitational acceleration and θ is the track inclination angle, rearranging Eq. (3) yields

$$a = g\cos\theta + \mu\,g\sin\theta + 0.5\,\rho\,Ad\;m^{-1}\,v^2 \tag{4}$$

The resulting differential equation is

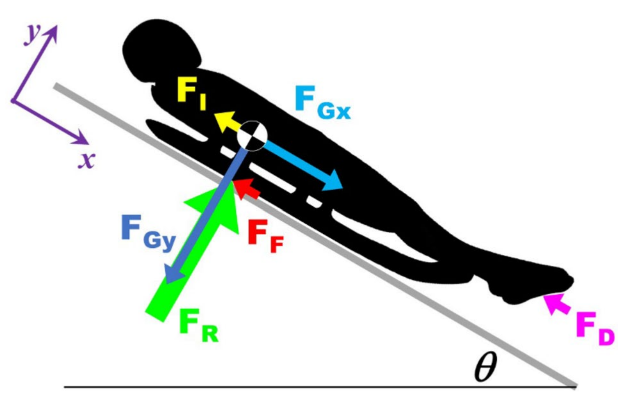

Fig. 3 Free Body Diagram of a Luge athlete; $F_{Gx,y}$ = components of the gravitational force; F_I = inertial force; F_D = drag force; F_F = friction force; F_R = ice reaction force; x, y = coordinate system; θ = track slope angle (intentionally exaggerated to accommodate for a reasonable display of force vectors) (color figure online)

$$\frac{dv}{dt} = c_1 - c_2\,v^2 \tag{5}$$

where $c_1 = g\cos\theta + \mu\,g\sin\theta$; and $c_2 = 0.5\,\rho\,Ad\,m^{-1}$.

Solving Eq. (5), considering that $t_0 = 0$, yields

$$v_t = \sqrt{\frac{c_1}{c_2}}\tanh\left[\tanh^{-1}\left(v_0\sqrt{\frac{c_2}{c_1}}\right) + t\sqrt{c_1 c_2}\right] \tag{6}$$

Simplifying Eq. (6) by defining three further constants, $c_3 = \tanh^{-1}\left(v_0\sqrt{\frac{c_2}{c_1}}\right)$, $c_4 = \sqrt{c_1 c_2}$, and $c_5 = \sqrt{\frac{c_1}{c_2}}$, yields

$$v_t = c_5\tanh(c_3 + c_4 t) \tag{7}$$

Integrating Eq. (7) for calculating the displacement x on the track, for initial conditions of $t_0 = 0$ and $x_0 = 0$, yields

$$x_t = \frac{c_5}{c_4}\left\{\log\left[\cosh(c_3 + c_4 t)\right] - \log\left[\cosh(c_3)\right]\right\} \tag{8}$$

where log denotes the natural logarithm. Solving Eq. (8) for t yields

$$t_x = \frac{\cosh^{-1}\left\{e^{\frac{c_4}{c_5}x + \log\left[\cosh(c_3)\right]}\right\} - c_3}{c_4} \tag{9}$$

i.e., the time t as a function of the track distance x.

The glide model served for three different purposes.

First, a sensitivity analysis was conducted to identify the individual influence of the different model parameters on the finish time, namely the influence of the drag area Ad, the mass m of the system, and the kinetic coefficient of friction μ. The input data were the mass data obtained from the rules and regulations [3, 4]; the mean drag area data of luge (0.047 m^2) and skeleton (0.056 m^2) calculated from Brownlie's [14] data of frontal area and drag coefficient; and the kinetic friction coefficient of steel on ice (0.01 [15]) at a normal force of 0.5 N [15] corresponding to a contact pressure of 0.07 MPa. This pressure was selected to prevent ice fracture [15]. While ice exhibits a velocity-weakening effect (decreasing μ as the sliding velocity increases [15, 16]), there is evidence for a force-strengthening effect [17] which could lead to greater μ as the normal force increases. Thus, μ might be underestimated in the glide model. Further input data were the track length (1376 m) and drop (116 m) of the 2018 Olympic Sliding Centre, as well as its mean altitude (870 m), temperature (− 5 °C) and humidity (66%) in February, resulting in an air density of 1.185 kg/m^3. The three parameters (Ad, m and μ) were changed by ± 5%, 10% and 15%, and the corresponding relative changes of the finish time were calculated and visualized with a Spider plot [18]. Comparable sensitivity analyses were performed in skiing [1] and in wheelchair racing [19].

Second, the body mass data [12] of the 2018 Winter Olympic athletes were converted to the permitted system mass data [3, 4]. Although the actual system mass data are unknown, the ideal system mass data provide a valid estimate of the fairness of the rules [3, 4] when correlated to the finish times [12]. Under ideal conditions, the slope of a linear regression should be insignificantly different from zero ($p > 0.05$). However, to test whether a linear regression is justified, the glide model is required for providing an alternative fit for a non-linear regression. When considering that the initial velocity on the track is zero, then the constants c_3 and $\log[\cosh(c_3)]$ become 0 and Eq. (9) reduces to

$$t_x = \frac{\cosh^{-1}\left(e^{\frac{c_4}{c_5}x}\right)}{c_4} = \frac{\cosh^{-1}\left(e^{c_2 x}\right)}{\sqrt{c_2}\sqrt{c_1}} = \frac{\sqrt{m}}{\sqrt{0.5\,\rho A d}}\,\frac{\cosh^{-1}\left(e^{\frac{0.5\rho A d x}{m}}\right)}{\sqrt{c_1}} \tag{10}$$

Exchanging independent variable x and constant m, the finish time as a function of the mass m is

$$t_m = \frac{\sqrt{m}}{Z_2}\,\frac{\cosh^{-1}\left(e^{\frac{Z_2^2 x}{m}}\right)}{Z_1} = A\sqrt{m}\cosh^{-1}\left(e^{\frac{B}{m}}\right) \tag{11}$$

where A and B are the coefficients of the fit function. Equation (11) was used to fit the finish time vs. the ideal system mass data with Matlab 2021a (MathWorks, Natick, MA, USA), by calculating the R^2 and its p-value and comparing them to their counterparts of the linear fit.

Third, the differences in finish times between the four highest ranks were calculated for the four competitions. Subsequently, by means of the glide model, mass was added to the athlete of the lower rank to match the finish time of the next higher (medal) rank, by assuming ideal mass conditions of the athlete of the lower rank, based on the maximum mass permitted by the rules and regulations [3, 4] and the athlete's body mass [12]. This analysis served to understand the difference even small mass additions could make, under realistic conditions.

In addition to evaluating the sensitivity of the mass, the sensitivity of the start time was addressed as well, by varying v_0, and the percentage of in/decrease of v_0.

3 Results

3.1 Rules and regulations

3.1.1 Mass components and their terminology

Both the Skeleton and the Luge Rules [3, 4] use the term 'weight' instead of 'mass' although the unit is indicated in kilograms. To match the physical quantity and the unit, the term 'mass' shall be used throughout this paper. The term 'system mass' shall correspond to the combined mass of all four mass components of the athlete-equipment system:

(1) Body mass (of the athlete);
(2) Apparel mass (helmet, shoes, gloves, and clothing / race suit; referred to as 'complete race equipment' in the Skeleton rules, and 'race clothing' in the Luge rules);
(3) Sled mass;
(4) Ballast mass (attached to the sled in Skeleton competitions; attached to the athlete's body in Luge competitions).

3.1.2 Skeleton rules

Ballast mass attached to the sled to reach maximum permitted mass limit is considered an inherent construction element of the sled [4]. In contrast to the Luge Rules, ballast mass on the athlete's body is prohibited.

The combined system mass of the sled (with ballast mass) and the athlete (with complete race equipment) '*may not*' exceed 115 or 92 kg in men's or women's competitions, respectively. The mass of the sled alone '*may not*' exceed 43 or 35 kg in men's or women's competitions, respectively. If the combined mass of the sled and the athlete exceeds 115 or 92 kg, then the mass of the sled alone '*may not*' exceed 33 or 29 kg in men's or women's competitions, respectively.

The implications of the latter rule are accepting the violation of the permitted limit (115 or 92 kg) by keeping the maximum sled mass at 33 or 29 kg if the body mass (plus gear) is greater than 82 or 63 kg, respectively. Without taking this option, the sled mass must decrease to 15 or 12 kg (Fig. 4), respectively, if the body mass (plus race gear) approaches 100 or 80 kg in men's or women's competitions, respectively, to meet the permitted limit (115 or 92 kg).

The system mass and the sled mass plus ballast as a function of the body mass (plus race gear) is shown in Fig. 4a.

3.1.3 Luge rules

The mass of singles sled (including attached accessories [i.e., not the ballast mass]) is minimally 21 kg and maximally 25 kg [3]. The mass calculation is based on 23 kg, whereby missing or increased mass of the sled (i.e., ± 2 kg)

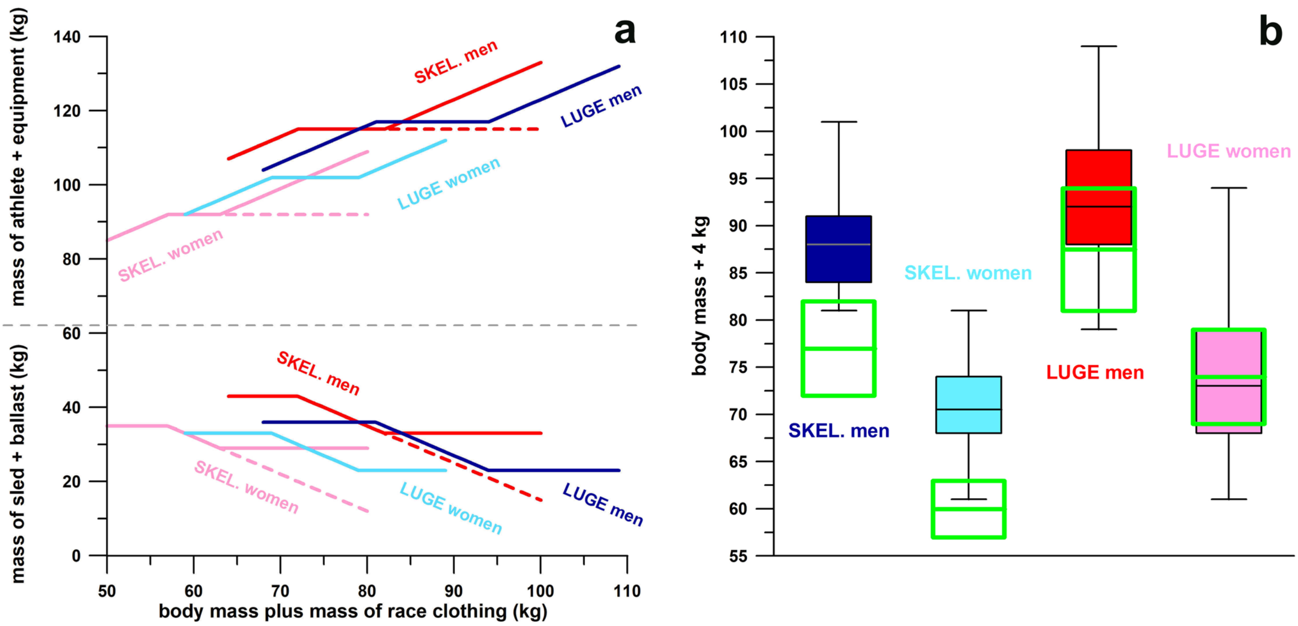

Fig. 4 (**a**) rules in force during the 2018 Winter Olympic Games; different masses (athlete + equipment, and sled + ballast) as a function of the body mass (+ race clothing); the dashed lines refer to the masses when disregarding the rule of the smaller sled mass (maximally 33 kg or 29 kg in men's or women's Skeleton competitions); note that the ballast mass has to be attached to the athlete's body in luge competitions, whereas in skeleton competitions it is an integral part of the sled; (**b**) body mass distribution of skeleton and luge athletes during the 2018 Winter Olympic Games; the green boxes denote the equal system mass window, SKEL = Skeleton (color figure online)

can be adjusted through the additional mass (ballast mass) and the mass of the gear (apparel mass).

Men or women may use an additional mass (ballast mass) amounting to all of the difference between body mass and a reference mass of 90 or 75 kg, respectively [3], whereby this additional mass must not exceed 13 or 10 kg in men or women, respectively [3]. In contrast to the skeleton rules, the ballast mass must be carried on the body of the athlete but not on the sled [3]. The mass of the race clothing may amount to 4 kg [3].

The implications of these rules are that the ballast mass reduces to zero when the body mass equals the reference mass of 90 or 75 kg. This in turn means that the system mass must increase if the body mass exceeds the reference mass, as the system mass is not specifically regulated by the rules. Consequently, if the body mass exceeds the reference mass, then the system mass is the sum of body mass, 4 kg mass of race clothing, and 23 kg mass of the sled, as the ballast mass cannot be less than zero.

The system mass and the sled mass plus ballast as a function of the body mass (plus race gear) is shown in Fig. 4a.

3.1.4 Analysis of rules

Within two body mass boundaries, there is an 'equal system mass' window (Fig. 4a), where the system mass is the same for all athletes, independent of their body mass, if, and only if, the athletes maximise their system mass within the mass rules and regulations. This window is not applicable to extreme body mass, i.e., too light, or too heavy, resulting

Table 1 Mass components and ranges according to the rules in force during the 2018 Winter Olympic Games, and actual mass ranges of the athletes participating in the 2018 Winter Olympic Games

Discipline, gender	Equal system mass (kg)	Range of body mass plus apparel mass (kg) across the equal system mass window	Mean ± half range (kg)	Actual range of body mass (plus 4 kg apparel mass) at the 2018 Winter Olympic Games	Mean ± half the actual range (kg)
Skeleton, men	115	10 (72–82)	77 ± 5	20 (81–101)	91 ± 10
Skeleton, women	92	6 (57–63)	60 ± 3	20 (61–81)	71 ± 10
Luge, men	117	13 (81–94)	87.5 ± 6.5	30 (79–109)	94 ± 15
Luge, women	102	10 (69–79)	74 ± 5	33 (61–94)	77.5 ± 16.5

in a system mass lighter or heavier, respectively, than the equal system mass.

Figure 4a shows this equal system mass windows for each competition as a function of the body mass. The boundaries of this window, in terms of body mass plus race apparel, are shown in Table 1.

Before or after these boundaries, the system mass is smaller or greater, respectively, than the equal system mass (Fig. 4a). This means that lighter or heavier athletes are disadvantaged or advantaged, respectively.

The mean of the equal system mass in Skeleton competitions is substantially smaller than the one in Luge competitions (Table 1). While the range of body mass across the equal system mass window (green box in Fig. 4b) is within the range of the actual athletes' body mass in Luge competitions at the 2018 Winter Olympics, the equal system mass range (green box in Fig. 4b) is outside the range of the actual athletes' body mass in Skeleton competitions. Indeed, the equal system mass range in Skeleton favours only the extremely light athletes and disadvantages the rest. The two interquartile ranges (body mass across the equal system mass range shown as a green box in Fig. 4b; and the actual body mass range shown as blue rectangles in Fig. 4b) overlap only by 1–2 kg (Fig. 4b; Table 1). In contrast to this, the interquartile range of the body mass in female Luge athletes is almost identical to the equal system mass range (interquartile), which is therefore well placed. In male Luge athletes, the two interquartile ranges (body mass and equal system mass range) overlap only by half the interquartile range.

Throughout the windows of equal system mass, the mass of sled plus ballast decreases (Fig. 4a) to compensate for the increasing body mass (plus apparel mass).

3.2 2018 Winter Olympics data – mass

The 2018 Winter Olympics Data are shown in Figs. 5 and 6 by correlating the finish time to the body mass (Fig. 5) and system mass (Fig. 6). In both cases the data are displayed as original data and as data with normally distributed residuals after outlier removal. The latter is summarised in Table 2, by showing the iterations involved in the outlier removal process with the associated statistical data. The data of the women's Skeleton competitions were normally distributed in the first place (Table 2).

Figure 5 shows the relationship between the finish time and the body mass. The linear regression has a negative gradient, expected from the principle 'the heavier, the faster'. The trends of all linear regression functions shown in Fig. 5a–d are significant ($p < 0.05$).

After outlier removal, required to achieve the normal distribution of the regression residuals, the R^2 values of Skeleton competitions remain about the same, whereas in men's Luge competitions, the R^2 increases threefold, and in women's Luge competitions decreases by one third.

From descriptive statistics, the R^2 values explain the influence of the body mass on the finish time (if $p < 0.05$). After outlier removal, in men's Skeleton competitions, merely 6% of the finish time is explained from the body mass (Fig. 5c), whereas in women's Skeleton competitions, 29% of the finish time was explained from the body mass (Fig. 5d). These results reject the null hypothesis, that the body mass of the athletes is ***not*** correlated to their finish times.

Figure 6 shows the relationship between the finish time and the system mass, by simulating that the system mass is at its permissible maximum, according to the rules (Fig. 4a). The results are comparable to Fig. 5 when comparing the original data, with one exception: the trend seen in the Luge men's competition, although with an expected negative gradient, is no longer significant ($p = 0.13$), indicating that the system mass would not influence the finish time. After outlier removal, all trends are significant.

In Skeleton competitions, when considering both the original data and the data after outlier removal, the R^2 values remain basically the same (Figs. 5c, d and 6c, d), suggesting that the mass regulations do not make any difference. This effect is reflected in Fig. 4, indicating that the 'equal system mass' window is almost entirely outside the actual body mass range of the 2018 Winter Olympics male and female Skeleton athletes.

In Luge competitions, when considering original data only, the R^2 values drop (Fig. 5a&b compared to Fig. 6a and b), suggesting that the mass regulations would make a difference. This effect is equally reflected in Fig. 4, indicating that the 'equal system mass' window overlaps with the actual body mass range (entirely in female, and partially in male athletes) of the 2018 Winter Olympics Luge athletes. However, when considering the data after outlier removal, then the R^2 values of male athletes drop to one third (Fig. 5a compared to Fig. 6b), whereas the R^2 values of female athletes increase sixfold (Fig. 5b compared to Fig. 6b), suggesting that an unequivocal effect of mass regulations is missing.

After outlier removal, in men's competitions, merely 5.1–5.6% of the finish time was explained from the body mass (Fig. 6a and c), whereas in women's competitions, 29.8% (Fig. 6d) to 52.3% (Fig. 6b) of the finish time is explained from the body mass. These results reject the null hypothesis, that a simulated ideal system mass (maximum ballast mass permitted) is ***not*** correlated to the finish times.

The data were fitted with linear and non-linear regression functions, the latter according to Eq. (11). There is a good match between the two functions in Fig. 6, with comparable R^2 values, so that a simpler linear regression model is justified (Fig. 5). The difference between the two corresponding R^2 values (linear and non-linear regression) was not significant ($p > 0.8$) in any of the correlations (Fig. 6).

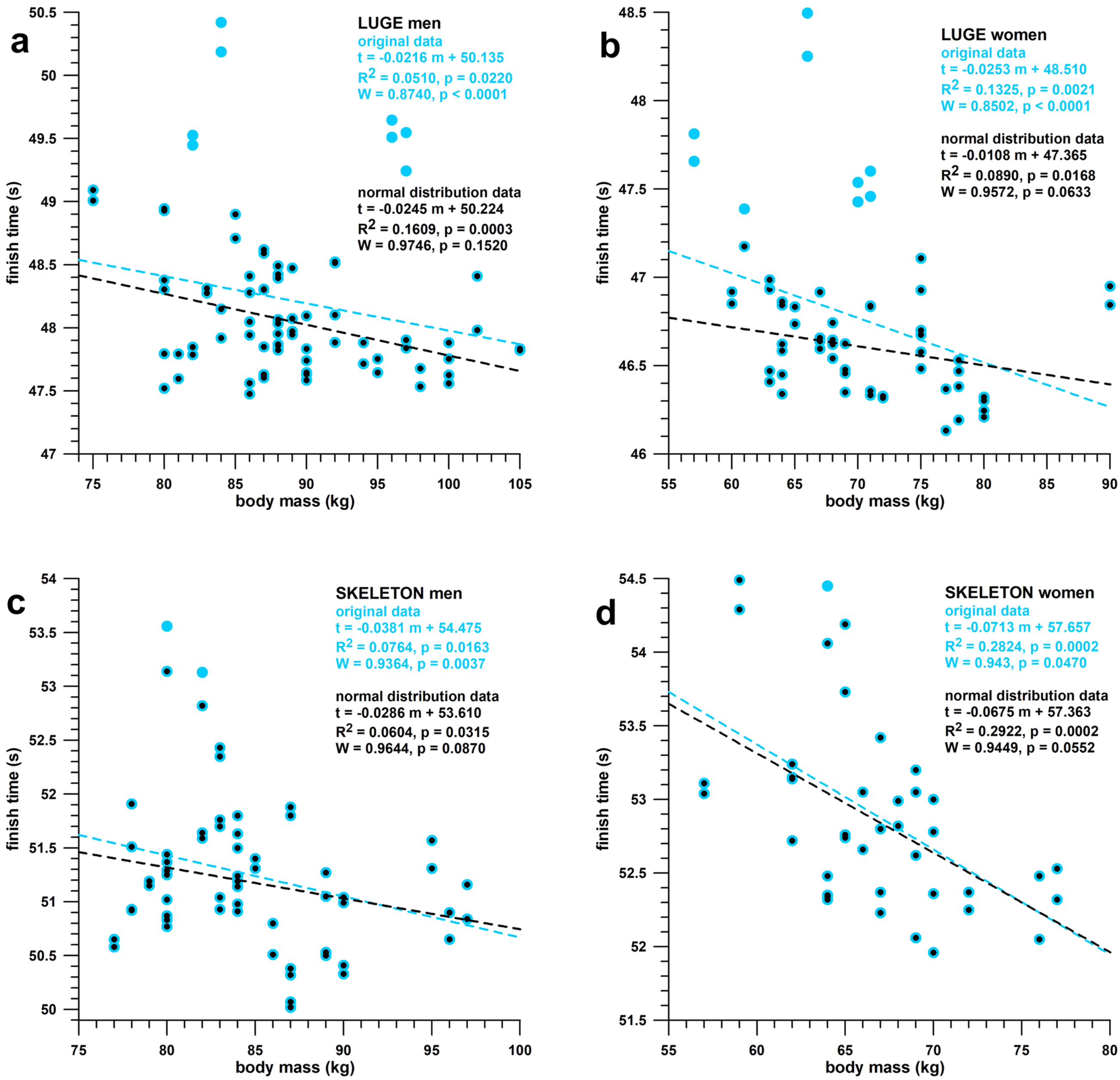

Fig. 5 Finish time against the athletes' body mass at the 2018 Winter Olympic Games; the original data are shown in blue, and the data with normally distributed residuals in black; the equation of the linear regression function (same color code) is shown for t (finish time) as a function of m (body mass); W = Shapiro–Wilk statistic; the original data of these graphs are copyrighted by Purdue University, and were reproduced with permission from: Purdue University, Purdue e-Pubs, ISEA2022 – International Sports Engineering Association Conference, Symposium Contributions [22] (color figure online)

3.3 2018 Winter Olympics data – start time

When correlating the start time to the finish time with the same method applied to the mass, three out of four competitions showed a positive correlation. In the men's Skeleton competition, $R^2 = 0.4235$ ($p < 0.0001$); in the men's Luge competition, $R^2 = 0.5887$ ($p < 0.0001$), and in the women's Luge competition, $R^2 = 0.4456$ ($p < 0.0001$). The exception was the women's Skeleton competition, with an R^2 of 0.0004 ($p = 0.5615$). While the magnitude of the finish time was explained in 6% to 29% from the body mass, it

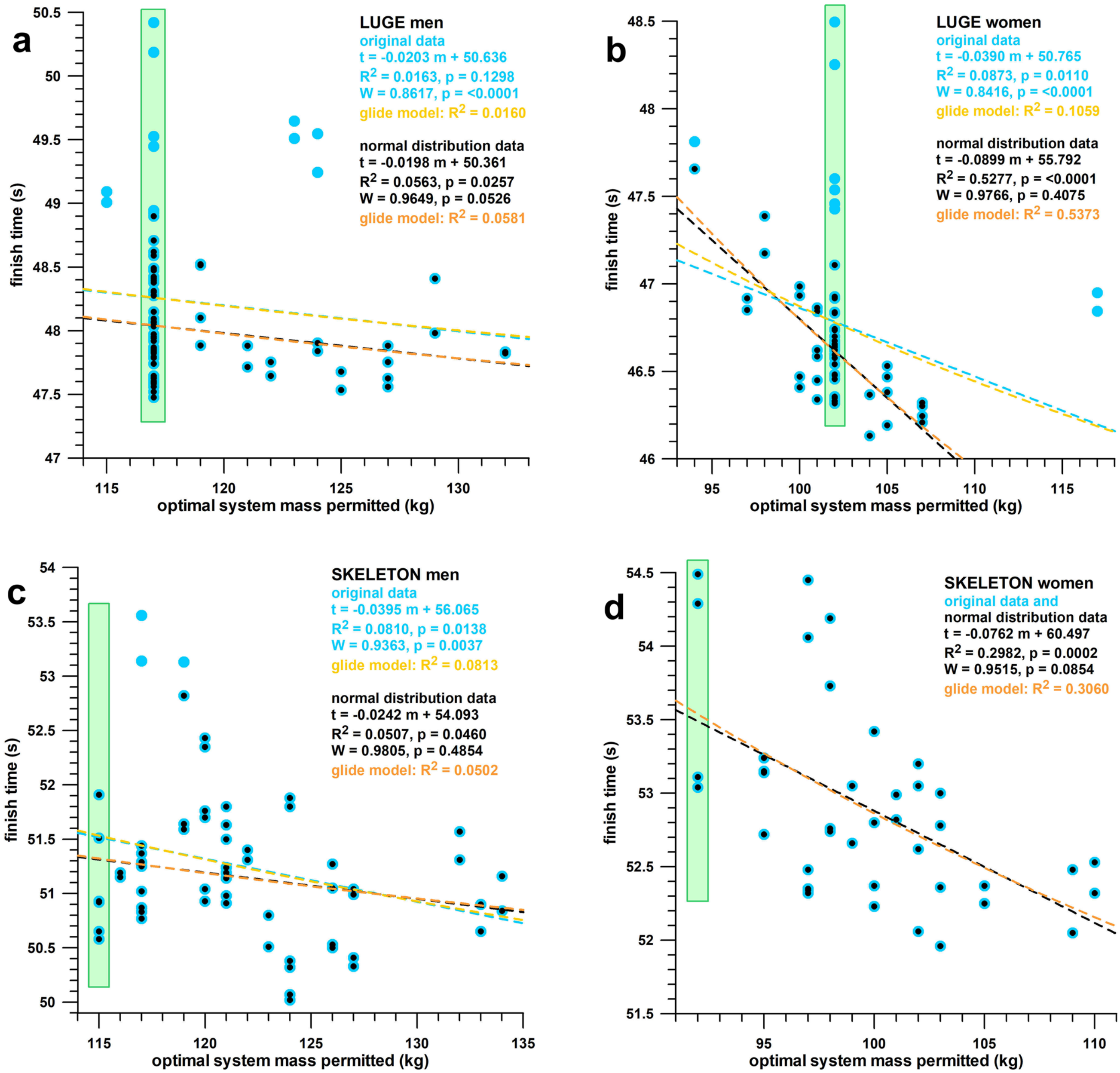

Fig. 6 Finish time against the optimal system mass (when abiding by the rules and using the maximal ballast mass permitted) at the 2018 Winter Olympic Games; the original data are shown in blue, and the data with normally distributed residuals in black; the linear regression fit is shown as a dashed blue line (original data) or dashed black line (normally distributed data), whereas the nonlinear glide model fit according to Eq. (11) is indicated by a dashed yellow line (original data) or dashed orange line (normally distributed data); W = Shapiro–Wilk statistic; the green boxes refer to the equal system mass window (color figure online)

was explained in 0% to 59% from the start time. The start time had therefore a greater influence on the finish time.

3.4 Simulation – mass

The sensitivity analysis (Fig. 7) expectedly indicates that increasing the mass of the system shortens the finish time, whereas increasing the aerodynamic area or the coefficient of friction lengthens the finish time. All three parameters, mass, drag area and friction coefficient exhibited a low sensitivity effect on the finish time, namely, as a rule of the thumb, a change of 15% of each parameter results in a change of about only 1% of the finish time. As far as the mass and the drag area are concerned, the athlete-skeleton system, because of its greater drag area, has a slightly greater effect on the finish time than the athlete-luge

Table 2 removal of finish time outliers for achieving a normal distribution of residuals when correlating the finish time to the body mass or system mass with a linear regression function

Reference mass	Discipline, Gender	Iteration number	n data	n out-liers	R^2	p (R^2)	W	W_{crit}	p (W)
body mass	Luge, men	0	80	5	0.0510	**0.0220**	0.8740	0.9691	< 0.0001
		1	75	2	0.1334	**0.0007**	0.9154	0.9674	0.0001
		2	73	1	0.1079	**0.0023**	0.9654	0.9666	0.0427
		3	72	0	0.1609	**0.0003**	0.9746	0.9662	**0.1524**
	Luge, women	0	60	6	0.1325	**0.0021**	0.8502	0.9605	< 0.0001
		1	54	2	0.1531	**0.0017**	0.9170	0.9569	0.0012
		2	52	***1***	0.1183	**0.0063**	0.9474	0.9555	0.0227
		3	51	0	0.0890	**0.0168**	0.9572	0.9548	**0.0633**
	Skeleton, men	0	60	2	0.0764	**0.0163**	0.9364	0.9605	0.0037
		1	58	2	0.0604	**0.0315**	0.9644	0.9594	**0.0870**
	Skeleton, women	0	40	***1***	0.2824	**0.0002**	0.9439	0.9447	0.0470
		1	39	0	0.2922	**0.0002**	0.9449	0.9436	**0.0552**
optimal system mass per-mitted	Luge, men	0	80	6	0.0163	0.1298	0.8617	0.9691	< 0.0001
		1	74	2	0.0560	**0.0211**	0.9350	0.9670	0.0009
		2	72	***4***	0.0881	**0.0057**	0.9557	0.9662	0.0128
		3	68	0	0.0563	**0.0257**	0.9649	0.9645	**0.0526**
	Luge, women	0	60	3	0.0873	**0.0110**	0.8416	0.9605	< 0.0001
		1	57	4	0.1253	**0.0020**	0.8917	0.9588	0.0001
		2	53	2	0.1988	**0.0003**	0.9200	0.9562	0.0017
		3	51	0	0.5277	**< 0.0001**	0.9766	0.9548	**0.4075**
	Skeleton, men	0	60	1	0.0810	**0.0138**	0.9363	0.9605	0.0037
		1	59	2	0.0681	**0.0230**	0.9492	0.9600	0.0155
		2	57	1	0.0507	**0.0460**	0.9805	0.9588	**0.4854**
	Skeleton, women	0	40	0	0.2982	**0.0002**	0.9515	0.9447	**0.0854**

A zero iteration number indicates the original data before outlier removal, n data = number of data at each iteration, n outlier = number of outliers that were identified for removal (data shown in italic and bold font were not outliers per se but residuals with the greatest value, required for exclusion to achieve normally distributed residuals), R^2 = coefficient of determination of each correlation (data in bold font if $p < 0.05$), p (R^2) = one-tailed p-value of the regression slope, W = Shapiro–Wilk statistic; W_{crit} = critical W-value; p (W) = p-value of the Shapiro–Wilk statistic W, indicating a statistically significant difference from a normal distribution (data in bold font if normally distributed, i.e., if $p > 0.05$)

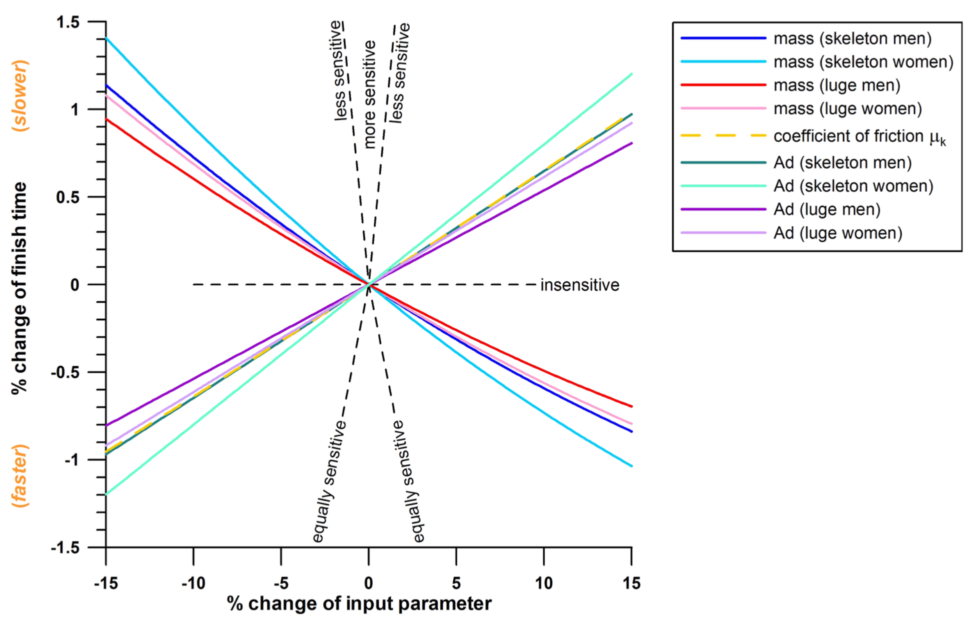

Fig. 7 Spider plot of the model sensitivity analysis, describing how much the model output value (finish time) is affected by changes in the model input values; 'equally sensitive' means that input and output changes are identical; note: curves of the coefficient of friction (not influenced by other factors) and of the *Ad* (skeleton men) coincidentally share equal sensitivity (color figure online)

Table 3 Finish time differences of the athletes participating in the 2018 Winter Olympic Games, and the additional mass (kg) required to tie with the next higher rank

Discipline, gender	Ranking	Time difference (four races) between rankings (s)	Time difference (single race mean) between rankings (s)	Percentage time difference relative to the mean finish time of two ranks	Additional mass (kg) required to tie with the next higher rank
Skeleton men	1–2	1.63	0.4075	0.809	20.05
	2–3	0.02	0.005	0.010	0.22
	3–4	0.11	0.0275	0.054	0.98
Skeleton women	1–2	0.45	0.1125	0.217	3.67
	2–3	0.17	0.0425	0.082	1.19
	3–4	0.02	0.005	0.010	0.12
Luge men	1–2	0.026	0.0065	0.014	0.34
	2–3	0.204	0.051	0.107	2.38
	3–4	0.002	0.0005	0.001	0.02
Luge women	1–2	0.367	0.09175	0.198	3.59
	2–3	0.045	0.01125	0.024	0.40
	3–4	0.069	0.01725	0.037	0.68

system; and female athletes, because of their smaller mass mean, have slightly greater effect on the finish time than male athletes (Fig. 7).

These influences, specifically the influence ratio of $^1/_{15}$, seem to be small, however, from a practical point of view, the differences in winning time at the medal ranks are often so minute, that even a small additional mass can make a difference (Table 3).

In the men's Luge competition at the 2018 Winter Olympics, the time difference between rank 3 (bronze medal) and 4 was merely 2 ms across four races, with a single race mean of 0.5 ms (Table 3). Under the assumption that the athletes fully exploit the ballast mass according to the rules, solely 20 g of additional mass would have sufficed to tie with the bronze medalist. In contrast to this, in the men's Skeleton competition, the time difference between rank 1 (gold medal) and 2 was 1.63 s across four races, with a single race mean of 0.41 s. 20 kg of additional mass would have been required to tie with the gold medalist (Table 3). Out of the 12 additional masses listed in Table 3, seven are smaller than 1 kg, and one thereof (0.34 kg) were required to tie with a gold medalist.

3.5 Simulation – start time

The sensitivity of the start time v_0 depended heavily on the reference start time itself. The percentage of the finish time $\%t_x$ variation can be expressed by the following equation:

$$\%t_x \approx \; k \; v_0 \; \%v_0 \tag{12}$$

where $k = -0.00553$, and $\%v_0$ is the percentage of the variation of v_0. For example, if $v_0 = 20$ kph and $\%v_0 = +40\%$, then t_x changes by –4.43%. The approximation sign accounts for the negligible effect when varying the mass or the drag coefficient. The influence of the start time is therefore substantially greater than the influence of mass (Fig. 7).

4 Discussion

This research exemplifies that there is empirical and theoretical evidence that the mass of the athlete-equipment system influences the winning time. Although both the Skeleton and the Luge Rules [3, 4], in force during the 2018 Winter Olympic competitions, aim at enabling an equal system mass across all athletes, independent of their body mass, the 'equal system mass' window has neither the right width nor the right position within the range of the actual athlete's body mass distribution (at least for the body mass distribution during the 2018 Winter Olympic Games; Fig. 4b). Heavier athletes were still advantaged, and lighter ones disadvantaged. In the Skeleton competitions, the position of the 'equal system mass' window was about 10 kg too low with respect to the actual body mass distribution, and in the men's Luge competitions about 5 kg too low. The first parameter to be adjusted is the position of the 'equal system mass' window, ideally centered at the mean of the athletes' mass distribution (to be done individually for all four competitions), to guarantee that most of the athletes fall into the equal system mass window. The second parameter to be adjusted is the width of the equal system mass window. From the 2018 Winter Olympics data, for having 70% or 80% of the participating athletes inside an equal system mass window, the required window width is: Skeleton men – 10 or 12 kg, respectively; Skeleton women – 9 or 11 kg; Luge men – 15 or 18 kg; Luge women – 14 or 17 kg. The actual window widths according to the rules were 10 kg and 6 kg in men's and women's Skeleton competitions, and 13 kg and 10 kg in men's and women's Luge competitions, respectively (Table 1). Based on these data, an equal mass window width of 15 kg seems feasible, for having at least 70% of the athletes inside the equal system mass window.

From a psychological point of view, maximizing the system mass within the rules might not be the best choice for every athlete. We know these issues from muscle-powered sports, where reducing the system mass is the ultimately goal [19]. Wheelchair racers, for example, do not prefer wheelchairs that are too light as they feel unstable and lack inertia (personal communication, Coleman R, 2011). If the equipment does not feel right, the performance of athletes might be affected. For Luge and Skeleton athletes, lighter sleds may accelerate better at the start and steer more easily in a turn, allowing them to follow a different track.

New editions of the Skeleton and the Luge Rules in force during the 2018 Winter Olympic competitions [3, 4] were published in 2020 [20, 21], but only the Skeleton rules [4] were revised and the following changes were implemented:

Skeleton 2020 rules [21]:

The combined system mass '*may not*' exceed 120 kg (*formerly 115 kg*) in men's competitions and 102 kg (*formerly 92 kg*) in women's competitions. The mass of the sled alone '*may not*' exceed 45 kg (*formerly 43 kg*) or 38 kg (*formerly 35 kg*) in men's or women's competitions, respectively.

The special case '*if the combined mass … exceeds 115 kg or 92 kg*' was removed from the new rules. The removal of this special case results in the following implications. The regression slope is *no longer* significantly different from zero when plotting the 2018 Winter Olympic finish times against the total mass according to the 2020 rules in both men's and women's competitions. The sled mass of the heaviest 2018 athletes (body mass of 97 or 77 kg for men or women, respectively) would have been 19 or 21 kg in men or women, respectively. The width of the equal system mass window would have been 45 or 38 kg in theory, essentially the maximally permitted mass of the sled, with an effective zero mass of the sled if the athlete's body mass (with complete race equipment) reaches 120 or 102 kg. To reduce the width of the equal system mass window to practical values, the absolute minimum mass of the sled must be known, at which the sled is still functionally operating.

The *Luge* 2020 rules [20] are the same as the *Luge* 2014 rules [3].

Therefore, the preliminary recommendations resulting from this study, subject to further research into body mass distribution in men's and women's Luge competitions, are:

- The system mass should be equal across all athletes within a 15 kg window,
- The centre of which should be placed at the mean body mass BM_{avg}, with respect to two different BM distributions (Luge men and women);
- Accordingly, the ballast mass at the window centre is 7.5 kg;
- THE ballast at, or smaller than, BM_{avg} minus 7.5 kg equals 15 kg;
- The ballast at, or greater than, BM_{avg} plus 7.5 kg equals 0 kg.

The limitations of this study are as follows.

(1) The dataset used in this study is confined to the 2018 Winter Olympic Games only, as the body mass of the participating athletes was neither reported before these games, nor was it published at the recent 2022 Winter Olympic Games. Nevertheless, the limited data provide evidence that the body mass influences the finish time.

(2) The sensitivity analysis was performed at a constant slope, but at the track length (1376 m) and drop (116 m) of the 2018 (Alpensia) Olympic Sliding Centre. A constant slope was necessary as the exact track profiles were not available. Furthermore, changing the slope angle does not affect the sensitivity, i.e., the relative influence of the model parameters (mass, drag area, coefficient of friction) on the finish time. This means that at any slope angle larger than the critical angle (transition from deceleration to acceleration [2]) and at any velocity smaller than the terminal one (zero acceleration [2]), the relative influence of the model parameters on the finish time remains the same.

5 Conclusions

The purpose of this study was the understanding of fairness of Luge and Skeleton competitions in terms of using the ballast mass within the rules and regulations at the time of the 2018 Winter Olympic competitions. The term fairness refers to reducing the parameters influencing the finish time solely to the athlete's skills. The data of the 2018 Winter Olympic Games provide sufficient evidence for rejecting the principle of fairness at the time of the 2018 Winter Olympic Games, however, offer the opportunity to revise the current Luge rules by recommending alternative ones towards greater fairness for Luge competitions. The Skeleton rules were already revised in 2020, however, the minimum operational sled mass needs to be known to reassess the fairness of the new rules.

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Declarations

Conflict of interest The author declares that he has no conflict of interest.

References

1. Luethi SM, Denoth J (1987) The influence of aerodynamic and anthropometric factors on speed in skiing. Intl J Sport Biomech 3:345–352
2. Fuss FK (2018) Slipstreaming in gravity powered sports: application to racing strategy in ski cross. Front Physiol 9(article1032):1–8. https://doi.org/10.3389/fphys.2018.01032
3. FIL (International Luge Federation) (2014) International Luge Regulations. Salzburg Austria. Available: https://www.ibsf.org/en/inside-ibsf/downloads. Accessed: 17 February 2018
4. IBSF (International Bobsleigh & Skeleton Federation) (2015) International Skeleton Rules. Lausanne, Switzerland. Available: https://www.fil-luge.org/en/rules/rules-artificial-track. Accessed: 17 February 2018
5. USA Luge (2010) What Makes a Successful Luge Athlete? Available: https://www.teamusa.org/-/media/USA_Luge/Documents/15GoodAthlete.pdf?la=en&hash=4D43279A81E87AA5B7DFA0F52E3F5B2F364B6CA3. Accessed: 31 January 2022
6. Morlock MM, Zatsiorsky VM (1989) Factors Influencing performance in bobsledding: influences of the bobsled crew and the environment. J Appl Biomech 5(2):208–221. https://doi.org/10.1123/ijsb.5.2.208
7. Brüggemann G-P, Morlock M, Zatsiorsky VM (1997) Analysis of the Bobsled and Men's luge events at the XVII olympic winter games in lillehammer. J Appl Biomech 13(1):98–108. https://doi.org/10.1123/jab.13.1.98
8. Hubbard M, Kallay M, Rowhani P (1989) Three-dimensional bobsled turning dynamics. J Appl Biomech 5(2):222–237. https://doi.org/10.1123/ijsb.5.2.222
9. Zhang YL, Hubbard M, Huffman RK (1995) Optimum control of bobsled steering. J Optim Theory Appl 85:1–19. https://doi.org/10.1007/BF02192297
10. Gong C, Phillips CWG, Rogers E, Turnock SR (2016) Analysis of performance indices for simulated skeleton descents. Procedia Engineering 147:712–717. https://doi.org/10.1016/j.proeng.2016.06.253
11. Braghin F, Cheli F, Donzelli M, Melzi S, Sabbioni E (2011) Multi-body model of a bobsleigh: comparison with experimental data. Multibody Syst Dyn 25:185–201. https://doi.org/10.1007/s11044-010-9218-7
12. IOC (International Olympic Committee) (2018) Data of Luge and Skeleton athletes (name, date of birth, body height, body mass) and final results of the Olympic Sliding Centre. Available: https://olympics.com/en/olympic-games/pyeongchang-2018. Accessed 9–28 February 2018
13. Frost J (2020) Hypothesis Testing. Jim Publishing, State College, PA, USA, pp 254–273
14. Brownlie L (2021) Aerodynamic drag reduction in winter sports: The quest for "free speed." Proc IMechE Part P: J Sports Eng Technol 235(4):365–404. https://doi.org/10.1177/1754337120921091
15. Scherge M, Böttcher R, Spagni A, Marchetto D (2018) High-speed measurements of steel–ice friction: experiment vs. calculation. Lubricants 6:1–8. https://doi.org/10.3390/lubricants6010026. ((**article26**))
16. Tikanmäki M, Sainio P (2020) Experiments on friction of dry and wet ice. Cold Reg Sci Technol. https://doi.org/10.1016/j.coldregions.2020.102990
17. Liefferink RW, Hsia FC, Weber B, Bonn D (2021) Friction on ice: how temperature, pressure, and speed control the slipperiness of ice. Phys Rev X. https://doi.org/10.1103/PhysRevX.11.011025
18. Eschenbach TG (1992) Spider plots versus tornado diagrams for sensitivity analysis. Interfaces / INFORMS J Appl Analyt 22(6):40–46
19. Fuss FK (2009) Influence of mass on the speed of wheelchair racing. Sports Eng 12:41–53. https://doi.org/10.1007/s12283-009-0027-2
20. FIL (International Luge Federation) (2020) International Luge Regulations. Salzburg Austria. Available: https://www.ibsf.org/en/inside-ibsf/downloads. Accessed: 8 March 2022
21. IBSF (International Bobsleigh & Skeleton Federation) (2020) International Skeleton Rules. Lausanne, Switzerland. Available: https://www.fil-luge.org/en/rules/rules-artificial-track. Accessed: 8 March 2022
22. Fuss FK (2022) Does the body mass influence the winning time in skeleton and luge competitions? The Engineering of Sport 14 – Proceedings of the 14th Conference of the International Sports Engineering Association (ISEA 2022,), Purdue University, West Lafayette, Indiana, USA, 6–10 June 2022