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Quasicrystals and art: interesting new facts

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Abstract

Quasiperiodic ornamental patterns represent only a small percentage of patterns when compared to the entire body of periodic patterns. Decagonal pattern is known since twelfth century Iran and fourteenth century western Islam (Andalusia and Morocco). A rich spectrum of octagonal patterns exists at the latter localities (fourteenth century and later), whereas a sole example of a dodecagonal pattern comes from Morocco. Later copies exist in all these regions. My most recent studies were concentrated upon the Andalusian and Moroccan regions, in which the fourteenth century (and later) wall mosaics occur as uninterrupted coatings of entire walls so that the motif of individual panels had to be adjusted to secure continuity of their underlying bar-and-band structure. In Andalusia, the tetragonal structure of the panels and their complexes were locally adjusted to become octagonal quasiperiodic. Only two geometric types of such octagrids were derived in Andalusia, in agreement with the rarity of quasiperiodic ornaments in general. In Morocco, before the panel substructure became heavily masked by an overflow of rosettes of several sizes, the mosaic panel was based on an octagonal quasiperiodic grid and ornamental rosettes were placed in it, disposed in the form of concentric octagons. As a prominent example, the octagonal motif of the Nejjarine Fountain and its plaster encasement will be discussed.

Keywords Quasiperiodic patterns · Decagonal patterns · Octagonal patterns · Alcázar in Seville · Alhambra · Meknes in Morocco

1 Introduction: how much is known?

Quasiperiodic ornamental patterns represent only a small percentage of Islamic ornamental patterns if compared with the entire body of periodic patterns (Bonner 2017; Broug 2013, among others). The currently known beginnings of the quasiperiodic geometric art date back to twelfth century. The key construction, the Blue Tomb (Gunbad-i-Qabud), was built in the years A.D. 1196–97 in the town of Maragha in western Iran. Decagonal quasiperiodic panels (Makovicky 1992) adorn sides of a ten-sided tomb tower (Fig. 1) that was built for an unknown Seljuk personality. This ornament, together with the following examples, marks the eastern Islamic tradition of quasiperiodic patterns, which were constructed by composing Kond-style tiles (Makovicky 1992, 2007, 2008; Lu and Steinhardt 2007).

The design heritage of the quasiperiodic Maragha pattern ranges from simple copies to fanciful modifications; most prominent among them are: the much discussed tympanum of the Imamzada Darb-e-Imam in Esfahan (A.D. 1453/age recently questioned, see Makovicky (2015) for the voluminous references), Karatay Medresa in Konya, Turkey (A.D. 1251–52, Rigby 2005), and portals of the Masjid-e-Hakim (A.D. 1656–1662, Makovicky 2015) in Esfahan. Less strict approximations can be found in the Jameh Mosque in Yazd and among several panels and tympana from Esfahan.

Each wall panel of the Maragha pattern contains ¼—cartwheel of the quasiperiodic pattern, with some artistic embellishments. The tiles used in the Maragha pattern can be classified either as *basic tiles* (Kond tiles; a pentagon, butterfly and marked lozenge, Fig. 2) or as *composite tiles* (aggregates composed of basic tiles and subject to artistic modifications) (Makovicky 2008).

It is worth of note that the original artists already realized that the composite tiles with their contents can be rotated by

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Fig. 1 The Blue Tomb (Gunbad-i-Qabud), built in A.D. 1196–97 in Maragha, western Iran. Photograph for Figure was taken by M. Ghari (Teheran)

Fig. 2 Kond-type tiling of the Blue Tomb: basic tile types (dark shading) versus the two kinds of composite tiles and their rotation derivatives

$n \times 72^o$ without disturbing the continuity of the pattern, and used this opportunity in their mosaic art. Moreover, the lozenges of the basic set actually are sites of dual possibilities, which can be materialized by positioning the small pentagons in two slightly different positions (i.e., these choices are related by flipping the small pentagons over a line parallel to the long lozenge axis, followed by rearrangement of some of the adjacent tiles), and they also mark partial overlap of the large *composite* pentagons.

The empty circular 'sun' element, used by later versions (e.g., Darb-e-Imam), actually is one of the composite tiles in Fig. 2, but *emptied* of its contents. In principle, the Maragha version hosts concurrently all the pattern variations just described, i.e., it is a *dynamic pattern* akin to the de-Boissieu's model (2022) of dynamic disorder in 2D-quasiperiodic quasicrystals.

The Kond tiles in the Maragha interpretation (Makovicky 1992, 2017) are related to the pentagon-based version of Penrose tiles in a simple way. First, we inscribe a smaller pentagon in each Penrose pentagon by connecting the midpoints of all sides of the latter. Then, the other kinds of Maragha/Kond tiles will *automatically* appear. Penrose's stars and half-stars in the pattern will become sites of tile-flipping choices of the above described kinds, a phenomenon which is more natural for the decagonal tiling, than the fixed star shapes themselves.

In Western Islamic tradition (Andalusia and Morocco), the decagonal quasiperiodic tiling was based on a different concept, that of Ammann quasilattice (Grünbaum and Shephard 1987; Senechal 2004) constructed according to the artists' own definition (Makovicky et al. 1998). The known examples come from the period about A.D.1325–1350, and the principal examples are found in The Alhambra Museum (Granada) (Fig. 3) and in the Mosque Al Attarin (Fez) (Fig. 4); later copies were being created up to modern times.

Quasilattice intervals (i.e., the line spacing) are 1 ('S bars') and $(1+\sqrt{5})/2$ ('L bars'). As already the artists themselves knew, in this quasiperiodic bar sequence occasional paired L bars occur, but not pairs of S bars. Transcript into

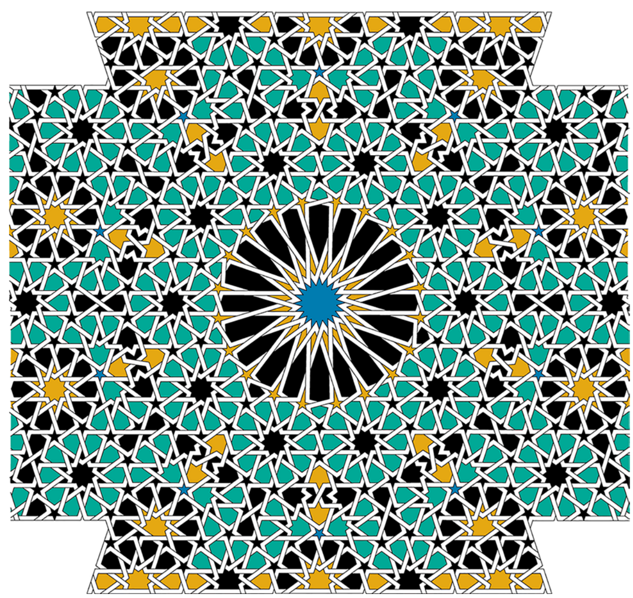


Fig. 3 Decagonal quasiperiodic pattern from the Alhambra museum (Granada, Spain). Quasiperiodic sequences of S and L bars in five orientations. Colors simulate the original hues

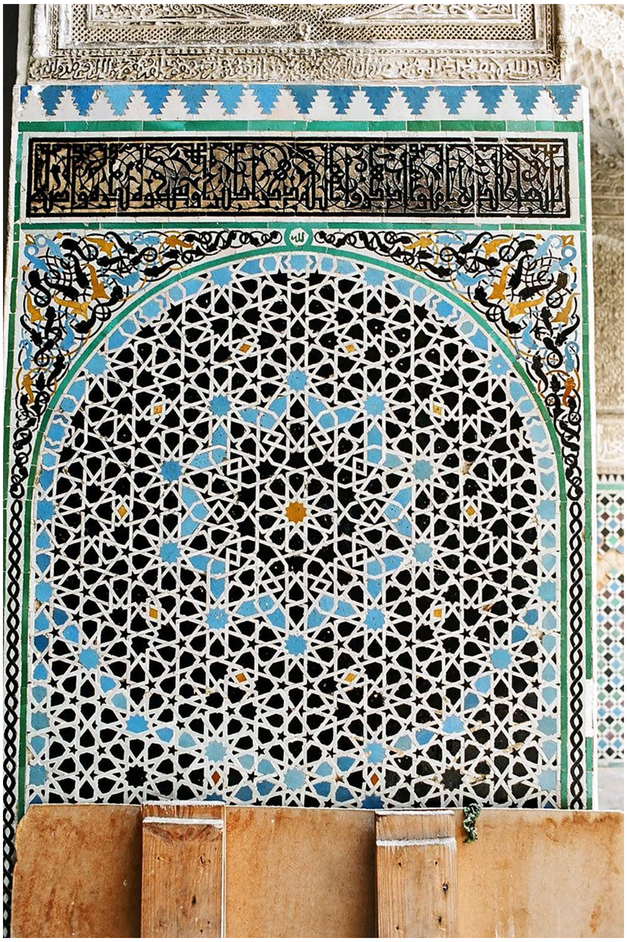


Fig. 4 Decagonal quasiperiodic pattern from the Madrasat-al-Attarin in Fez, Morocco. Slight artistic modification of the motif

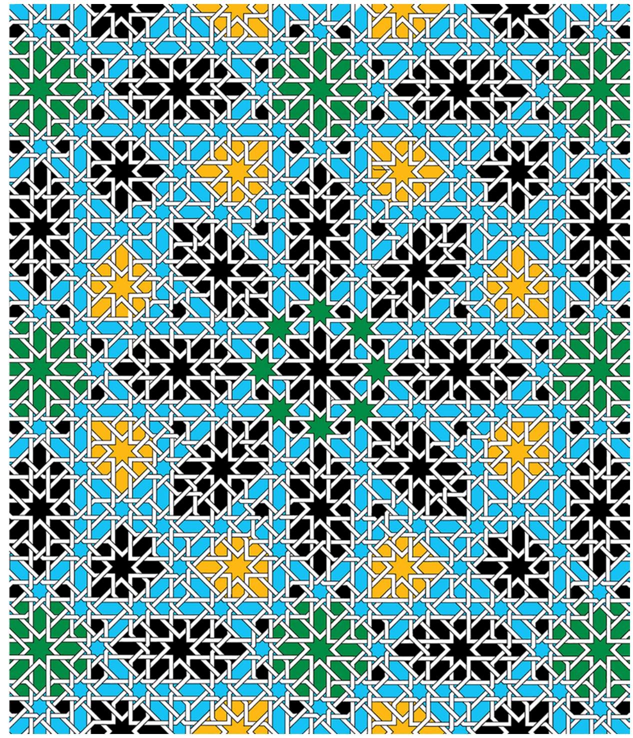


Fig. 5 Octagonal quasiperiodic pattern from the Patio de las Doncellas, Reales Alcazares, Sevilla, Spain. Quasiperiodic sequence of S and L bars of two widths; octagons outlined by ornamental stars are superimposed on the quasiperiodic pattern. Type A panel of Gonzalez Ramírez (1995)

a tile pattern (e.g., Penrose tiling) is always possible (Grünbaum and Shephard 1987) although in the originals it is absent.

The decagonal patterns from both localities contain abundant flips of bar pairs (SL → LS, i.e., quasilattice phasons, Figs. 3 and 4). In their geometry and often also in the 'intensity' distribution of conspicuous rosette adornments these 'multigrids' resemble the *reciprocal lattices* of real crystals.

Modern constructions of the octagonal quasiperiodic tiling were performed by tile-composition (Ammann in: Senechal 2004, Castéra 1996, among others). In their common form, tiles are squares and 45° rhombs, with edge and corner markings on tiles and a fixed ratio of tile frequencies in the pattern. This approach is applicable to Moroccan rosette-saturated mosaics.

The Andalusian and Moroccan compositions were produced in a different way, by the artists' version of the *Islamic octagrid* (actually *Ammann quasilattice,* Grünbaum and Shephard 1987) method (Makovicky and Fenoll 1996). The *quasiperiodic sequence* consists of alternating unit bars (denoted as S bars in the following text, with their width equal to 1) and L bars (width equal to $\sqrt{2}$); occasional pairs of unit bars occur in the quasiperiodic array. The quasiperiodic arrangement generates identity of sequences on axes and on the 45° diagonals. We shall see, however, that the developed Andalusian and Moroccan varieties differ in their underlying geometry.

In a typical *quasiperiodic octagonal mosaic* from the Patio de las Doncellas, Reales Alcazares, Sevilla (Fig. 5), the quasiperiodic region extends somewhat beyond the large inscribed circle of conspicuously colored tiles but it does not extend to the square corners. Boundaries to adjacent panels consist of several S and L bars in an inconspicuous but fixed combination (Fig. 5).

Dodecagonal quasiperiodic tiling has been located as a single pattern at the Zaouia Moulay Ismail in Fez, Morocco (Makovicky and Makovicky 2011). It is a part of historical ornamentation of one of the gates; its age is uncertain. Bar sequences in the Amman-type quasilattice of this pattern can be matched as unity (S bars, width of 1) and $(1 + \sqrt{3})/2$ (L bars). Similar to the previous patterns, this pattern could be matched with the electron diffraction pattern of Ta telluride by Conrad et al. (1998). It was reproduced with small alterations by multigrid method (Aboufadil et al. 2013).

2 Practical problems

The fundamental questions concerning these quasiperiodic patterns are: The artists probably *did not understand the notion of* infinitely quasiperiodic—they wanted to construct *cart-wheel* (centered circular) *ornaments which could be fitted* to regularly repeating wall panels, starting each time from the panel center (the site of rotation axis). We know very little about their ideas. In spite of large Islamic geometric literature concerning properties and construction of single polygons (e.g., a pentagon), *literature dealing with patterns based on these polygons* is scarce and often it was destroyed by religious zealots as, e.g., in Granada.

What appears well established is that, once the prescription has been found by an ingenious master ornamentalist, it was *copied to the last detail* and applied widely, often over vast geographic areas. Both the exact and the creatively modified copies exist, as we know, e.g., for the decagonal patterns (Makovicky 2008).

What is very interesting is that, except for the Kond tilings of the decagonal patterns from the eastern Islamic regions, the accent was not on defining individual tiles which, when composed together according to definite rules, would yield the quasiperiodic pattern. Accent was on a quasilattice which could be decorated with prominent ornaments (rosettes, stars..) placed on suitable bar intersections (preferably multiple S-bar intersections). When, in more peripheral regions of the pattern, there was lack of these, rearrangement of broken S bars was called upon to increase and amend the ornamental possibilities.

Andalusian and Moroccan artists also faced another problem: their dados were supposed *to cover entire long walls without break,* eventually enveloping windows or door frames. At that time period, the mosaics which they created were not supposed to be a repetition of the same motif, panel after panel. Those times required variability in the overall unity or overall unity in individual variability. Moreover, especially the breaks between panels were frowned upon. All these requirements resulted in *a new approach* to the construction of classical octagonal ornaments in Andalusia and Morocco.

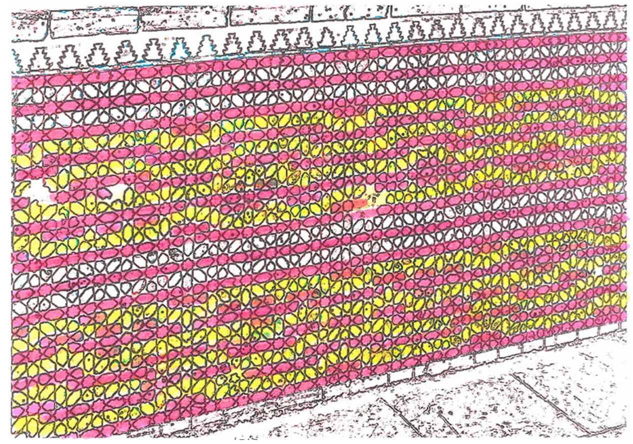


Fig. 6 Bar-and-band scheme (horizontal bars) of the tetragonal wall mosaic in the Mezquita Aljama de Cordoba (Spain)

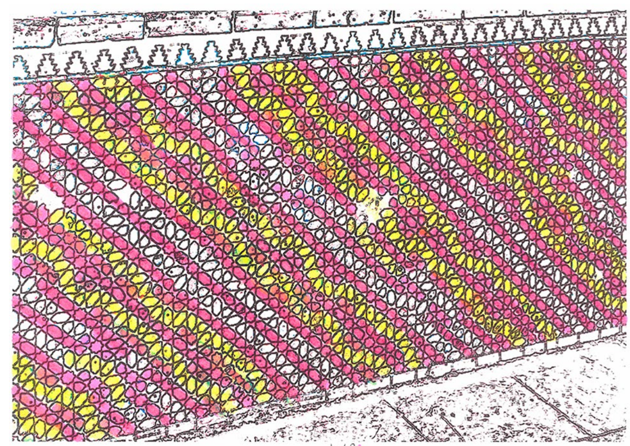


Fig. 7 Bar-and-band scheme (diagonal bars) of the tetragonal wall mosaic in the Mezquita Aljama de Cordoba (Spain)

3 Tetragonal patterns and 'octagonalization'

First, we have to observe that at least a full half of panels encircling the Patio de las Doncellas (Sevilla) are tetragonal panels and all the evidence described below suggests that they were the starting point for 'octagonalization'. As already mentioned, all panels of one wall are united into an uninterrupted sequence by containing through-going bars.

The reasons for the original tetragonality assumption are illustrated by the tetragonal bar-and-band schemes of the wall panels. In the Mezquita Aljama de Cordoba, the wall panel displays complicated (5S + 4L) *bands* with frequent bar-flipping (i.e., with a phason-rich) character (these are marked in Fig. 6 by yellow L tiles) which alternate with undisturbed (3S + 2L) *bands* (marked by white L bars) (Fig. 6). The S + L ↔ L + S flips in the former bands are

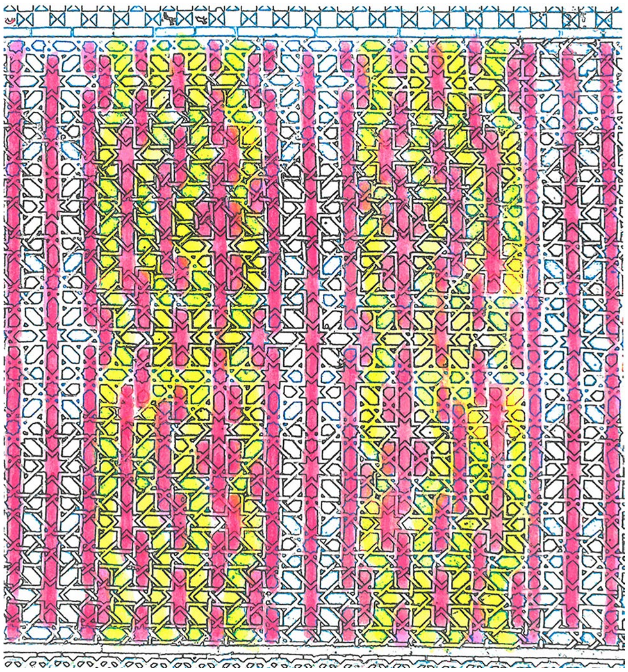


Fig. 8 The vertical band scheme of the A-type octagonal rearrangement of bars during octagonalization of the motif. Note the 'narrowing' and bar breaking in the waist portions of the panel

necessary for accommodating the rosettes and stars of four-fold, and in the following text also of eight-fold disposition schemes, where-ever they would clash locally with the

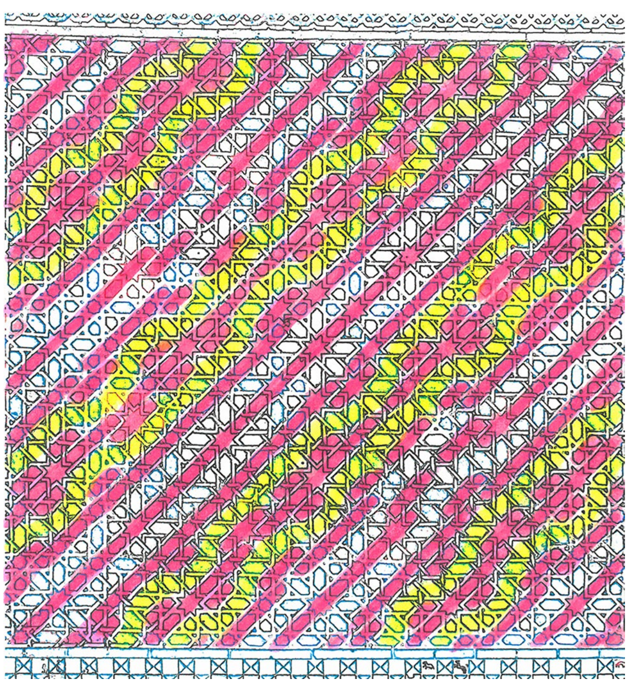


Fig. 9 The diagonal band scheme of the A-type rearrangement of bars during octagonalization

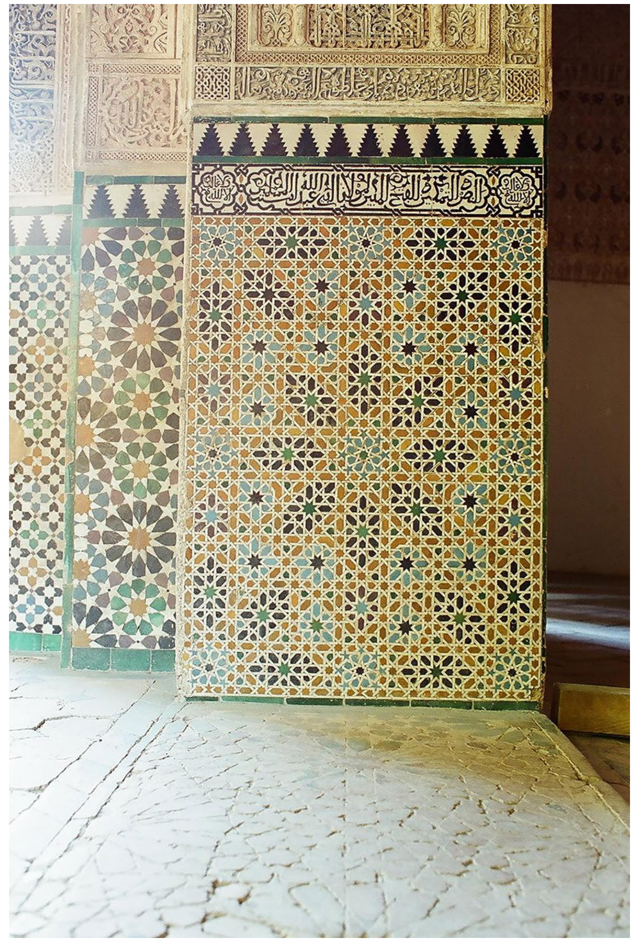


Fig. 10 The quasicrystalline mosaic panel in the Mirador de Lindaraja complex of the Alhambra Palace (Granada, Spain). Modifications of the A-type bar rearrangement for this panel are described in the text of the paper and in Fig. 11

regular (even quasiperiodic) S and L scheme of unbroken bars. These constructions were the practical adjustments (on the level of the times) for application of zero-dimensional patterns onto the two-dimensional quasiperiodic background.

Figures 6 and 7 illustrate a tetragonal pattern in which the horizontal bar-and-band sequences are identical with the vertical sequences (the latter not illustrated) but the sequences on diagonals differ by a factor of $\sqrt{2}$. Therefore, I decided to demonstrate separately the axial and diagonal band schemes *of the same pattern* in these two figures*, and in those of the following patterns.* In the illustrated case, diagonal (3S + 2L) bands with flipping character alternate with (3S + 2L) bands of undisturbed bars (white).

Another tetragonal (the C-type of Gonzalez Ramírez 1995) panel, from the Patio de las Doncellas, Reales Alcazares, Sevilla, shows the horizontal SLSLS bands (L tiles white) alternating with flipping (1S + 1L) bands (L tiles yellow). These are the thinnest bands possible. On the diagonals

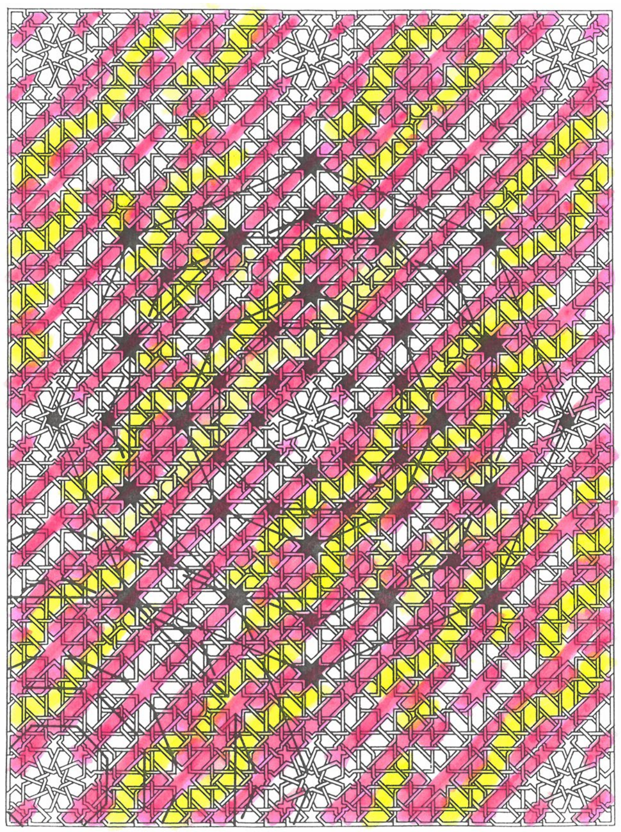


Fig. 11 The diagonal bar-and-band scheme of the Mirador de Lindaraja panel, with star adornment added in black (one can see the use of bars and bar fragments to properly accommodate the rings of ornamental stars in the panel). The uppermost and lowermost panel portions were adjusted to fit the elongated panel by means of 'pattern-grafting'

of this pattern, the bar-flipping (3S + 2L) bar scheme alternates with steady (3S + 2L) (white) bands. These two cases indicate the hierarchy of band widths when the collective width of the band scheme increases. Viewing such figures, we should remember that the ends of horizontal bars in them already belong to adjacent panels. *Importantly, two diagonal, and one vertical plus one horizontal, bar sequences can be traced in every mosaic panel.*

In all her detailed drawings of panel patterns, Gonzalez Ramírez (1995) found only two types of cart-wheel patterns, which I now reclassify as octagonal patterns. The rest are patterns, which I describe as tetragonal in character. One of the two octagonal patterns occurs in a less- and in a more-ornamented form (her D and E types).

Creation of these octagonal patterns was performed by applying a *strictly organized breaking and rearrangement of the bar schemes which were initially observed in the tetragonal patterns.* When compared, however, the two outstanding categories, A and D, of octagonal patterns have remarkably different rearrangements of bar fragments.

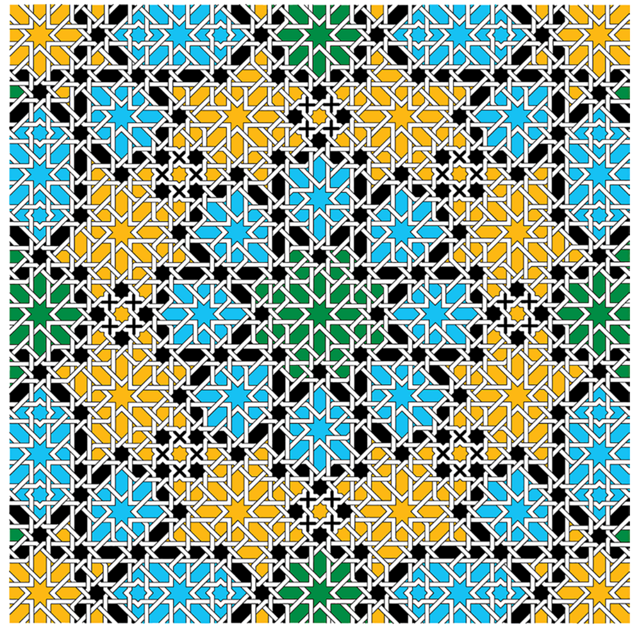


Fig. 12 The D-type panel from the Patio de las Doncellas in the Reales Alcazares (Sevilla, Spain). Coloring expresses the eight-fold point group symmetry of the cart-wheel pattern. Band-and-bar sequences are illustrated in Fig. 13

Reader is reminded that a given tile can be a portion of the S bar or of the L bar, depending upon the way how the tile and the bar are oriented towards one another (in our tetragonal/octagonal case, it can be oriented in parallel with the bar, or at 45° to it). Equally important is that every tile participates at the same time in a horizontal bar, in a vertical bar, and in two diagonal bars. Thus, in bars of different orientation the same tile will play a different role. As illustrated in the following examples, these complex rearrangements are directly connected with, and required by, the distribution of point-group ornaments (stars and rosettes) over the pattern/panel.

4 Octagonal patterns in Andalusia

The 'A' type of eight-fold grid. The representative figure (Fig. 5) copies a tiling from the Patio de las Doncellas in its original coloring. The white spacers between adjacent tiles and tile bars demonstrate additional interlacing of the spacer skeleton. The quasiperiodic disc covers most of the figure (it exceeds the circle of orange patches) but the corners and SLSLS spacers between panels are not quasiperiodic; they are just conveniently finishing it off (Makovicky 2021).

For the same pattern, the vertical band scheme of the A-type octagonal rearrangement of bars (Fig. 8) shows only rather subtle changes of the original tetragonal scheme, with the 'narrowing' and bar breaking in the waist portion of the

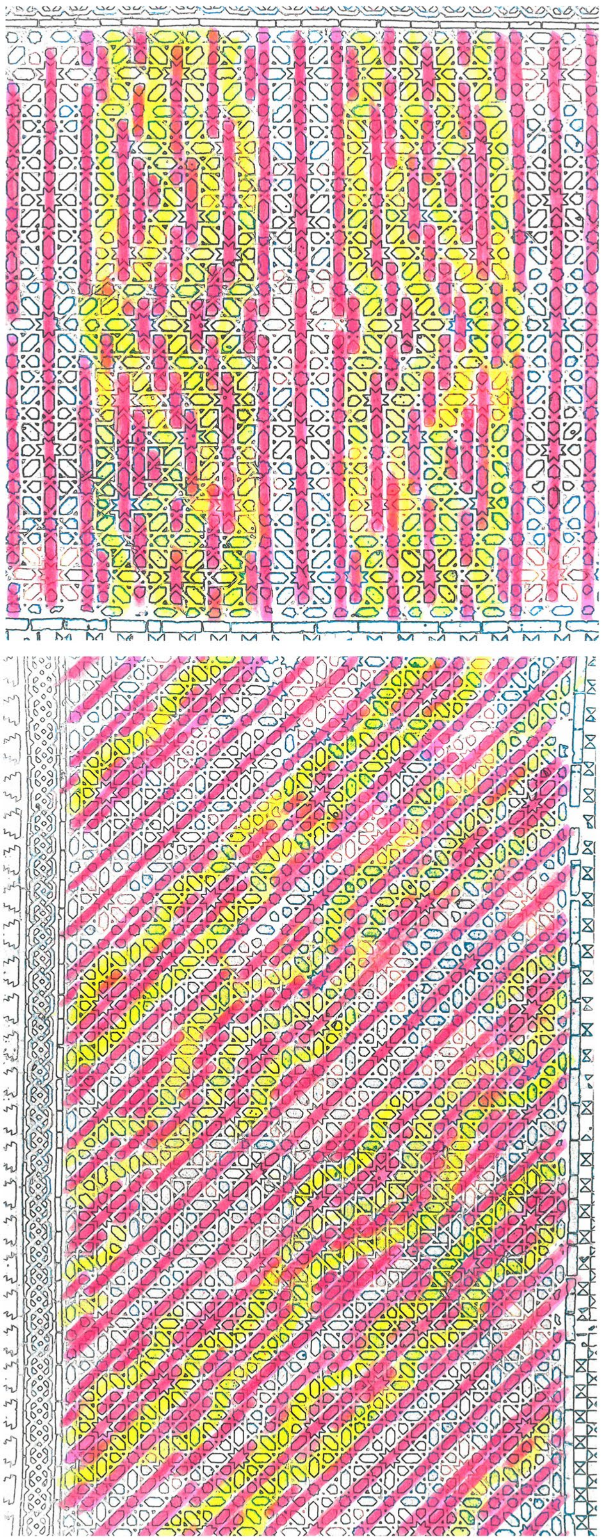
a b


Fig. 13 The vertical and the diagonal bar-and-band schemes for the D-type panel from the Patio de las Doncellas. 'Ascending' and 'descending' sequences of S-bar fragments are sandwiched between 'sinusoidal' sequences of L tiles. The SLSLS band (white L tiles) runs through the origin and the panel boundaries

Fig. 14 Two conjoint octagonal panels from the Mausoleum of Sultan Moulay Ismail in Meknes, Morocco. The S bars of the two panels cross freely their boundary. In spite of different color schemes, the underlying geometry of the two panels is almost identical

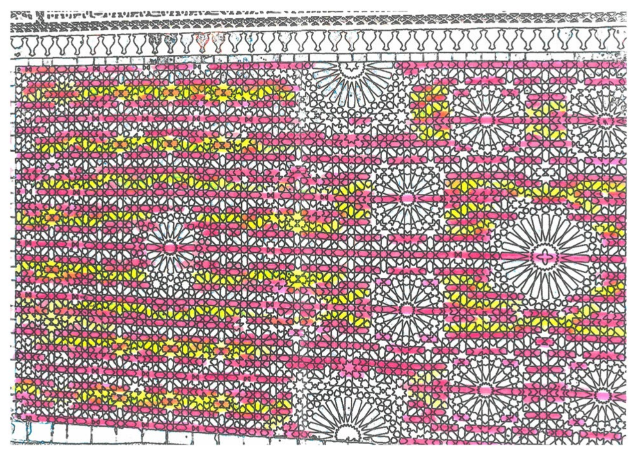


Fig. 15 The horizontally running bar-and-band schemes of the panels from Fig. 14. While the band schemes (indicated in white and yellow, for the bars non-flipping and the bars flipping schemes, respectively) differ, the bar schemes (S bars) are through-going and nearly identical

panel. The octagonal rearrangement for the diagonal band sequence of the original tetragonal bar scheme is illustrated in Fig. 9. Observe again the 'inward' displacements of broken bars in the waist portions. Tile arrangements in both 45° directions are identical, so that each of the two band schemes described here is valid in two mutually perpendicular directions.

The Alhambra contains the most sublime mosaic panel of this type: a very fine 'eightfold' geometric mosaic of fine tiles and white spacers (Fig. 10) in the Mirador de Lindaraja (Makovicky and Fenoll Hach-Alí 1996) is akin

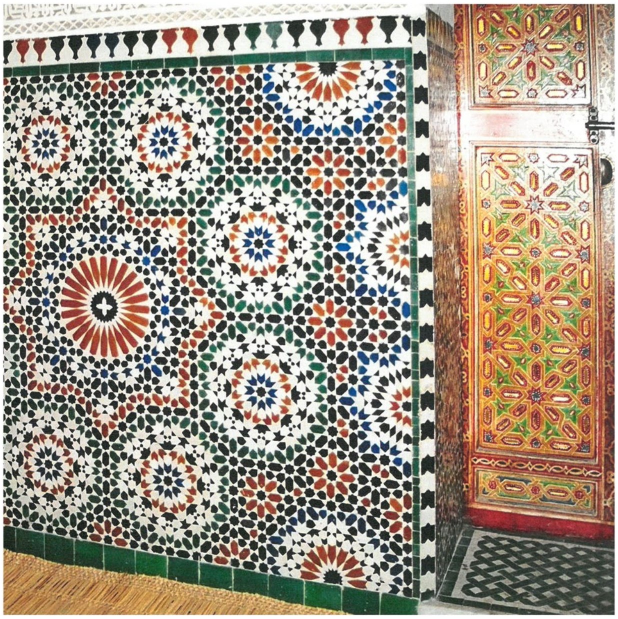


Fig. 16 An isolated mosaic panel from the Mausoleum of Moulay Ismail, Meknes. Octagonal underlying S and L scheme with superimposed octagons of rosettes

to the octagonal mosaics from the Patio de las Doncellas, Sevilla, reminding us of friendship between the Granada dynasty and Pedro the Cruel of Sevilla. However, there are substantial differences!

The relatively narrow rectangular pattern from Mirador de Lindaraja (Fig. 10), is of the A type (the diagonal bars and bands are illustrated in Fig. 11) but the upper and lower *quarters* of the panel were displaced laterally against the large central field. The purpose of this geometric exercise was that their white central SLSLS bands should lead 'properly' towards the corners of the panel. Comparing the two figures, the original A-type pattern and the Alhambra pattern, we see that the S and L bars accommodate easily this kind of exercise, which I can only characterize as 'pattern-grafting' upon the central pattern.

As mentioned before, there are two coloring/tiling varieties of the D-type bar scheme, which are present among the wall panels of the Alcazares patio (Fig. 12). They are based on much more extensive reorganization of the bar scheme. The vertical and diagonal bar-and-band schemes for the D-type panel from the Patio de las Doncellas (Fig. 13) show that the 'ascending' and 'descending' sequences of S-bar fragments (red) are sandwiched by 'wavy' sequences of L-bar tiles (yellow). 'Piling up' of S-bar fragments in these two schemes is similar but not identical either. The SLSLS band runs through the center, the next such band is only on the lateral panel boundaries. One should remember that in Fig. 13 the D-panel is the central one, the wings already belong to other panels.

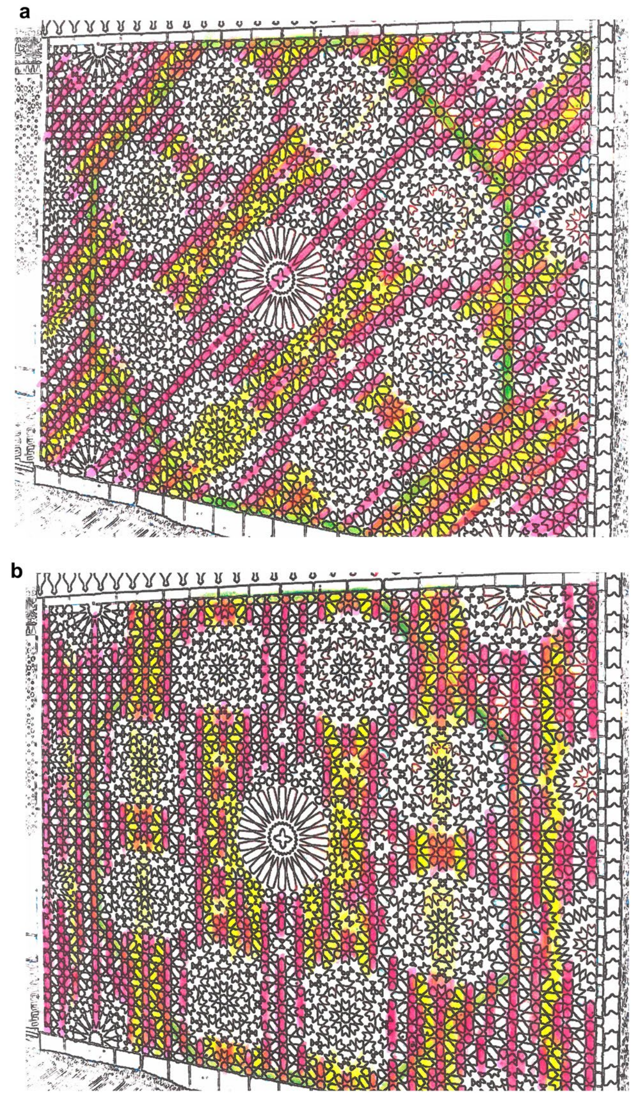


Fig. 17 The diagonal and the vertical band schemes for the panel from Fig. 16 are identical, demonstrating its octagonal quasiperiodic character (within green boundaries). Both band types display several different widths, to satisfy the positional requirements of ornamental rosettes

5 Octagonal patterns in Morocco

Development in Morocco started first in a way similar to Andalusia but it developed a much more decorative, 'flowery' aspect with time. I shall illustrate features of this development primarily using the dados from the Mausoleum of Sultan Moulay Ismail in Meknes. Serious building activity in this mausoleum started after about 1700.

Figure 14 shows two octagonal panels based on octagonal bar schemes. What is important and universally observed, the diagonal bar transition along the lateral contact of the two octagonal panels/bar schemes is smooth and uninterrupted.

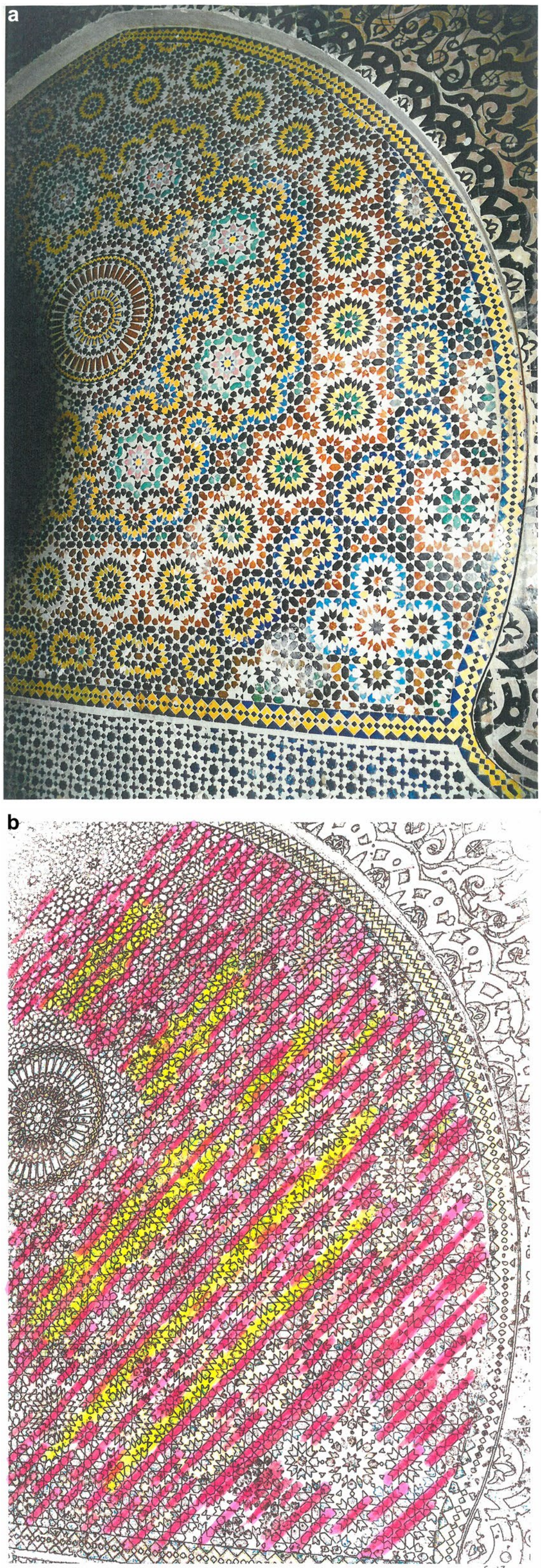


Fig. 18 A large octagonal *zellij* wall panel from the old city of Fez (undated). It is an accumulation of rosette types with different diameters, and the underlying bars are visible only at the margins. 'Shower' of short broken S-bar fragments indicates areas entirely composed of small rosettes

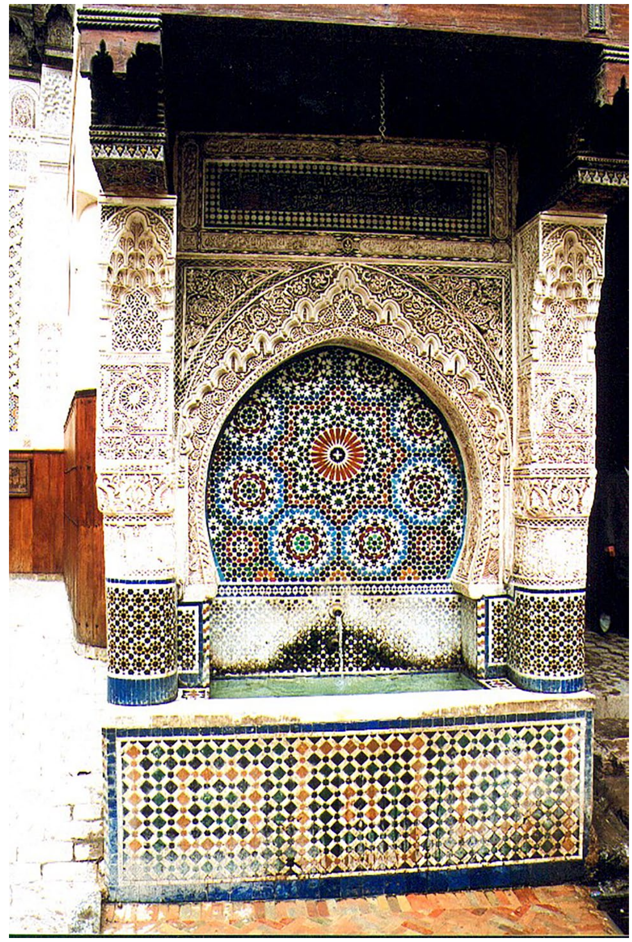


Fig. 19 The Fountain of Nejjarine in the old city of Fez (Morocco). The octagonal mosaic is surrounded and partly covered by sumptuous plaster work. Age is controversial; Broug (2013) suggests A.D.1711, other sources consider it younger. I think that mosaic is of Meknes type and age, whereas plaster might be younger

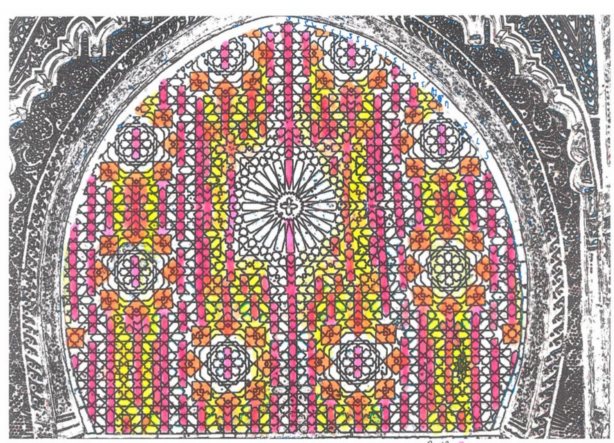


Fig. 20 The band and bar sequence of the Nejjarine fountain. The sequence is truly octagonal, with axial and diagonal orientations fully identical, and in all aspects it is of Meknes type

A horizontal band-and-bar sequence over these adjacent octagonal panels is continuous as well (Fig. 15). Still, the two schemes are differently configured, independent of one another in details, and each of them has its own system of fixed-bar and bar-flipping bands. In spite of this, the S bars and the L bars continue from one panel into another.

Another mosaic panel from the Moulay Ismail Mausoleum in Meknes is in Fig. 16. It is easy to trace in the explanatory Fig. 17, that the diagonal and the horizontal/vertical band-and-bar schemes are identical. Boundary of the quasiperiodic octagonal field is outlined in green; the outside area is just a continuation.

Large octagonal *zellij* wall panel in the old city of Fez, Morocco, undated, illustrates further development of the Moroccan concept (Fig. 18). It is a geometrically organized *accumulation of rosettes* of several sizes with overall eightfold symmetry. In the bar scheme, the areas of the 'shower of short S-bar fragments' indicate fields with aggregations of smallest rosettes. The octagonally oriented bar patterns, which form the background, become visible only at the margins of the panel.

The mosaic panel of the famous Fountain of Nejjarine (old city of Fez) (Fig. 19) corresponds in style to those of the Meknes type. Note also the discordance between the geometry of the *zellij* mosaic and that of the rich plaster arc enclosing it (Fig. 19). When we analyze the band-and-bar sequence for the mosaic in the Fountain of Nejjarine (Fig. 20), the vertical, horizontal and diagonal band-and-bar sequences are identical. They resemble closely the mosaic panels from the Meknes Mausoleum, as also is the coloring of the tiles. According to the majority of literary sources, the fountain should be younger than the Meknes period, although Broug (2013) suggests the year 1711. As a tentative conclusion, I suggest that it is only the heavy plaster adornment (Fig. 20), which is considerably younger, but not the 'Meknes-like' *zellij* mosaic, which existed for an important time span without that encasement.

6 Conclusions

At all known localities, the spectrum of truly different quasiperiodic patterns is severely limited (they were created by exceptional masters!) although later ornamental variations can and do occur.

The low number of construction-wise 'perfect' decagonal quasilattices and a single case of dodecagrid do not offer enough background for 'genetic' considerations about their development and origins.

Individual octagonal and tetragonal panels in the Andalusian mosaics are parts of an uninterrupted sequence of panels and motifs, which are interconnected both in bar-and-band schemes.

Octagonal quasiperiodic panels in Andalusia were created from tetragonal panels by rearrangement of fragments of S and L bars. This explains a much more complicated S and L distribution than should result from theoretical S–L multigrids.

Developed Moroccan octagonal panels have octagonal bar sequences. Adjacent sequences have been adjusted to cross the panel boundary unbroken or with minor change. They increasingly became means of accommodating circular/polygonal octagonal arrangements of rosettes without regard to quasiperiodicity.

The geometric role of bands with fixed bar sequences and of those with flipping bar sequences in Moroccan patterns can differ from the Andalusian examples.

The principal conclusion, however, is that the western Islamic artists created an ingenious and flexible combination of bars and bands as a practical solution for composing tetragonal and (especially) octagonal mosaic patterns on the quasilattice background.

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Availability of data and materials All buildings/objects studied or examined are publicly accessible without prohibitions on photography

Declarations

Conflict of interest No competing interests of financial or personal nature were found.

Ethical approval Not applicable.

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