Discussion/Reply

Authors' Reply to Comments on "A New Analytical Approach to Predict Spacing Selection in Lamellar and Rod Eutectic Systems"

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The authors, Song *et al.*, of the article[1] claim that a "fatal mistake" was found in our paper[2] that discusses the spacing selection in binary eutectic systems. The issue that Song *et al.* bring into question is the correctness of Eq. [9a] of Reference 2, which is also presented in an explicit form as Eq. [4] in Reference 1 and reproduced here for clarity from our original paper:

$$B_o = \left[\left(f_\alpha v_\alpha + f_\beta v_\beta\right) - \left(f_\alpha v_\alpha k_\alpha + f_\beta v_\beta k_\beta\right)\right] \cdot \left(C_E + B_o\right) + \left[v_\alpha(1-k_\alpha) - v_\beta\left(1-k_\beta\right)\right] \cdot \sum_{n=1}^{\infty} B_n \frac{\sin(n\pi f_\alpha)}{n\pi}. \quad [9a]$$

At this point, we would like to invite the authors of Reference 1 to carefully consider the boundary condition given by Eq. [3] of their paper, which is identical to Eq. [8] of our paper, and integrate it within the appropriate limits, also by accounting that $C_I(x)$ is the liquid concentration at the solid/liquid interface as given by Eq. [1] of Reference 1 (*i.e.*, for $z = 0$). We trust that they will obtain the result shown in Eq. [4] of Reference 1. Certainly, this is different from Eq. [10] of Song *et al.* The reason for these different results resides in the fact that Song *et al.* substituted $C_I(x)$ for the eutectic concentration, C_E, in the boundary condition (see Eqs. [7-1] and [7-2] of Reference 1). This substitution is a simplification proposed in the original treatment of Jackson and Hunt (JH)[3] which, as explained by JH in their note, can be used "with quite good accuracy for most cases." Therefore, when setting $v_\alpha = 1$ and $v_\beta = 1$, the result obtained by Song *et al.* for the B_o term (Eq. [10] of Reference 1) becomes identical to that of JH, except for being expressed in a different form. Consequently, the results of Song *et al.* are also correct, as they are exactly the JH solution.

The reason for the simplification proposed in the JH treatment was to make the mathematics tractable when calculating the Fourier coefficients of the general solution for liquid concentration. This was used by JH when calculating the B_o term as well as the coefficients B_n. Catalina *et al.*[2] also used this simplification when calculating B_n, but, in an attempt to improving the solution accuracy, chose to use the actual $C_I(x)$ for the calculation of B_o. Unfortunately, this detail was missed by Song *et al.*, although it is clearly stated when the JH approximation was used in Reference 2. It must be pointed out that there is still an ongoing effort aimed to refining the solution of eutectic growth and extend it from binary to multicomponent systems. Examples of such effort can be found in References 4 through 6, just to name a few.

Most of the time, the solidification processes can be described mathematically as boundary value problems, with the solid/liquid interface boundary being also the solution of the problem. Consequently, many different solutions can be obtained for the same process, depending on the assumptions used in the mathematical formulation. It is exactly the case discussed in this article. Therefore, the statement of Song *et al.* that a "fatal mistake" made its way in our formulation has no validity as it is the consequence of their failure to recognize the difference in the boundary conditions used in our approach compared to the JH treatment.

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