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*Erratum*

# Erratum to: A Higher Frobenius–Schur Indicator Formula for Group-Theoretical Fusion Categories

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Group-theoretical fusion categories are defined in terms of a group $G$, a subgroup $H$, a three-cocycle $\omega\colon G \times G \times G \to \mathbb{C}^\times$, and a two-cochain $\theta\colon H \times H \to \mathbb{C}^\times$ such that $\omega|_{H^3}$ is the coboundary of $\theta$. The quadruple $(G, H, \omega, \theta)$ is called group-theoretical data.

The paper gives first a formula for the Frobenius–Schur indicators in group-theoretical fusion categories defined by special group-theoretical data, namely for the case where $\theta$ is trivial, thus the restriction $\omega|_{H^3} = 1$, but even more strictly $\omega|_{G\times G\times H} = 1$; we call such a three-cocycle $\omega$ *adapted*. Natale has shown that, essentially, this is not a restriction, since every group-theoretical data is equivalent to one in which $\omega$ is adapted and $\theta = 1$.

From the formula for adapted cocycles, the paper deduces formulas for the general case, using Natale's recipe to pass from general group-theoretical data to an adapted cocycle; as an application, a formula for twisted Drinfeld doubles is given. Unfortunately, there is a mistake in the passage from the special to the general case, which also affects the special case of doubles. Embarrassingly enough, the error only came up thanks to computer calculations: In joint work (in progress) with Michael Mignard the indicator formulas were implemented in GAP [1] with the homological algebra package HAP; the uncorrected formula, however, produced many values that were not algebraic integers and thus incompatible with the basic theory of higher Frobenius–Schur indicators.

The calculation in the displayed formula above Corollary 2 is hopelessly incorrect. First, our attempt to follow the proof of Proposition 4.2 in [2] suffered an error from changing sides. We repeat the calculation in detail to amend this. Thus, we choose a

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section $Q$ of the right $H$-cosets in $G$. For $p, q \in Q$ and $h, h', h'' \in H$ let

$$\eta_1(ph, qh') := \omega(p, h, h'),$$
$$\omega_0 := \omega(d\eta_1).$$

Then clearly $\eta_1|_{H\times G} = 1$, $\eta_1|_{Q\times G} = 1$ and

$$\begin{aligned}\omega_0(ph, h', h'') &= \omega(ph, h', h'')\eta_1(h', h'')\eta_1^{-1}(phh', h'')\eta_1(ph, h'h'')\eta_1^{-1}(ph, h')\\ &= \omega(ph, h', h'')\omega^{-1}(p, hh', h'')\omega(p, h, h'h'')\omega^{-1}(p, h, h')\\ &= \omega(h, h', h'') = 1,\end{aligned}$$

that is $\omega_0|_{G\times H\times H} = 1$.

Next, let

$$\eta_2(ph, qh') := \omega_0^{-1}(ph, q, h')\omega_0(p, h, q),$$
$$\omega_1 := \omega_0(d\eta_2).$$

Then we obviously have $\eta_2|_{G\times H} = 1$ and $\eta_2|_{H\times Q} = 1$, and

$$\begin{aligned}&\omega_1(ph, qh', h'')\\ &\quad= \omega_0(ph, qh', h'')\eta_2(qh', h'')\eta_2^{-1}(phqh', h'')\eta_2(ph, qh'h'')\eta_2^{-1}(ph, qh')\\ &\quad= \omega_0(ph, qh', h'')\eta_2(ph, qh'h'')\eta_2^{-1}(ph, qh')\\ &\quad= \omega_0(ph, qh', h'')\omega_0^{-1}(ph, q, h'h'')\omega_0(p, h, q)\omega_0(ph, q, h')\omega_0^{-1}(p, h, q)\\ &\quad= \omega_0(ph, qh', h'')\omega_0^{-1}(ph, q, h'h'')\omega_0(ph, q, h')\\ &\quad= \omega_0(q, h, h'')\omega_0^{-1}(phq, h', h'')\omega_0(ph, qh', h'')\omega_0^{-1}(ph, q, h'h'')\omega_0(ph, q, h')\\ &\quad= 1\end{aligned}$$

so that $\omega_1 = \omega(d\eta)$ is adapted for $\eta := \eta_1\eta_2$.

The correct $\eta$ for "adapting" $\omega$ found, we can correct the calculation of $\tilde{\pi}_m$ in the situation of Corollary 2 using the formula (5.6) from Theorem 2.

Thus, let $g \in G$ and $h \in H$, assume that $s = gh$ satisfies $s^m \in S = \mathrm{Stab}_H(gH) \subset H$, and that we have chosen $Q$ so that $g \in Q$. Then

$$\begin{aligned}\eta(s, s^m) &= \eta_1(s, s^m)\\ &= \omega(g, h, s^m),\\ (d\eta_1)(s^m, g, h) &= \eta_1(g, h)\eta_1^{-1}(s^m g, h)\eta_1(s^m, gh)\eta_1^{-1}(s^m, g)\\ &= \eta_1^{-1}(s^m g, h)\\ &= \eta_1^{-1}(g(g^{-1} \triangleright s^m), h)\\ &= \omega^{-1}(g, g^{-1} \triangleright s^m, h),\end{aligned}$$

and

$$\begin{aligned}\eta(s^m, s) &= \eta_2(s^m, s)\\ &= \omega_0^{-1}(s^m, g, h)\\ &= \omega^{-1}(s^m, g, h)(d\eta_1^{-1})(s^m, g, h)\\ &= \omega^{-1}(s^m, g, h)\omega(g, g^{-1} \triangleright s^m, h),\end{aligned}$$

thus

$$\begin{aligned}\eta(s,s^m)\eta^{-1}(s^m,s) &= \omega(g,h,s^m)\omega^{-1}(g,g^{-1}\triangleright s^m,h)\omega(s^m,g,h)\\ &= \omega(g,h,s^m)\omega^{-1}(g,h\triangleright s^m,h)\omega(s^m,g,h)\\ &= \alpha_{s^m}(g,h).\end{aligned}$$

Further

$$\eta(s^m,g)=1,$$

and

$$\begin{aligned}\eta(s^m g,g^{-1}\triangleright s^{-m}) &= \eta_1(s^m g,g^{-1}\triangleright s^{-m})\\ &= \eta_1(g(g^{-1}\triangleright s^m),g^{-1}\triangleright s^{-m})\\ &= \omega(g,g^{-1}\triangleright s^m,g^{-1}\triangleright s^{-m}),\end{aligned}$$

and hence, using (5.6):

$$\begin{aligned}\tilde{\pi}_m(s) &= \pi_m(s)\alpha_{s^m}(g,h)\omega(g,g^{-1}\triangleright s^m,g^{-1}\triangleright s^{-m})\\ &= \pi_m(s)\alpha_{s^m}(g,g^{-1}s)\omega(g,g^{-1}\triangleright s^m,g^{-1}\triangleright s^{-m})\\ &= \pi_m(gh)\alpha_{(gh)^m}(g,h)\omega(g,(hg)^m,(hg)^{-m}).\end{aligned}$$

Thus, Corollary 2 needs an additional factor in the indicator formula, which should read, correctly

$$\begin{aligned}\nu_m(M) &= \frac{1}{|S|}\sum_{\substack{r\in gH\\ r^m\in S}}\pi_{-m}(r)\alpha_{r^{-m}}(g,g^{-1}r)\omega(g,g^{-1}\triangleright r^{-m},g^{-1}\triangleright r^m)\chi(r^{-m})\\ &= \frac{1}{|S|}\sum_{\substack{h\in H\\ (gh)^m\in S}}\pi_{-m}(gh)\alpha_{(gh)^{-m}}(g,h)\omega(g,(hg)^{-m},(hg)^m)\chi(r^{-m}).\end{aligned}$$

In the proof of Theorem 4, the old (second) correction factor to $\pi_m$ obediently disappears as previously stated, but the new factor does not. The analog of $\alpha_{((g,1)(x,x))^m}((g,1),(x,x))$ for $\varpi$ is $\alpha_{(gx)^m}(g,x)$. Thus the correct indicator formula in Theorem 3 ought to have been

$$\nu_m(M)=\frac{1}{|C_G(g)|}\sum_{\substack{x\in G\\ (gx)^m=x^m}}\frac{\pi_m(gx)}{\pi_m(x)}\alpha_{x^m}(g,x)\chi(x^m).$$

The conclusions of Proposition 4 hold true without changes. To see this observe that the analog of $\alpha_{(gx)^m}(g,x)$ is trivial if $g\in N$, but also if $g\notin N$, for then either $x\in N$ or $gx\in N$.

## References

1. GAP—Groups, Algorithms, and Programming, Version 4.7.4. The GAP Group (2014)
2. Natale, S.: Frobenius–Schur indicators for a class of fusion categories. Pac. J. Math. **221**(2), 353–377 (2005). ISSN:0030-8730

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