



## Lithospheric Stress Tensor from Gravity and Lithospheric Structure Models

MEHDI ESHAGH<sup>1</sup> and ROBERT TENZER<sup>2</sup>

**Abstract**—In this study we investigate the lithospheric stresses computed from the gravity and lithospheric structure models. The functional relation between the lithospheric stress tensor and the gravity field parameters is formulated based on solving the boundary-value problem of elasticity in order to determine the propagation of stresses inside the lithosphere, while assuming the horizontal shear stress components (computed at the base of the lithosphere) as lower boundary values for solving this problem. We further suppress the signature of global mantle flow in the stress spectrum by subtracting the long-wavelength harmonics (below the degree of 13). This numerical scheme is applied to compute the normal and shear stress tensor components globally at the Moho interface. The results reveal that most of the lithospheric stresses are accumulated along active convergent tectonic margins of oceanic subductions and along continent-to-continent tectonic plate collisions. These results indicate that, aside from a frictional drag caused by mantle convection, the largest stresses within the lithosphere are induced by subduction slab pull forces on the side of subducted lithosphere, which are coupled by slightly less pronounced stresses (on the side of overriding lithospheric plate) possibly attributed to trench suction. Our results also show the presence of (intra-plate) lithospheric loading stresses along Hawaii islands. The signature of ridge push (along divergent tectonic margins) and basal shear traction resistive forces is not clearly manifested at the investigated stress spectrum (between the degrees from 13 to 180).

**Key words:** Gravity, lithospheric stress, Moho, tectonics.

### 1. Introduction

Several different theories have been proposed to explain driving forces of plate tectonics. In a pioneering study, Runcorn (1962) was reasoning that the continental drift is a consequence of convection flow in the mantle (see also Runcorn 1980). After a better

understanding of tectonic processes as well as the Earth's inner structure based on the analysis of various geophysical and geodetic data, different hypotheses have been proposed to explain mechanisms of lithospheric plate motions. Some authors suggested that, rather than global mantle flow (e.g., Ricard et al. 1984; Bai et al. 1992), the lithospheric plate boundary and body forces are responsible for the plate motion. They include ridge push (e.g., McKenzie 1968, 1969; Richardson 1992; Ziegler 1992, 1993; Bott 1991a, b, 1993), slab pull (e.g., Forsyth and Uyeda 1975; Chapple and Tullis 1977), trench suction (e.g., Wilson 1993), collisional resistance (e.g., Forsyth and Uyeda 1975), and basal drag (e.g., Wortel and Vlaar 1976; Jacoby 1980; Fleitout 1991; Richardson 1992). These tectonic forces as well as the lithospheric load, volcanism, elasticity of the lithosphere, viscosity of the asthenosphere, and other rheological parameters and geodynamic/geological processes contribute to the overall stress state of the lithosphere. Some authors suggested that the origin of large-scale lithospheric stresses is mainly aligned to a frictional drag due to global mantle flow (e.g., Hager and O'Connell 1981; Steinberger et al. 2001), while others argue that the lithospheric stresses are attributed mainly to the lithospheric plate boundary and body forces (Ricard et al. 1984; Bai et al. 1992; Jurdy and Stefanick 1991).

The first low-degree global gravity models in the 1960s determined from the orbital parameters of early satellite missions, were used in studies of the Earth's inner structure and processes. In context of stress studies, Kaula (1963) developed a method based on minimizing the strain energy and using the low-degree gravitational and topographic harmonics to estimate the minimum stresses in an elastic Earth. Runcorn (1964, 1967) formulated a functional relation between the stress and the gravity based on solving the Navier–Stokes' equations for modelling the horizontal shear

<sup>1</sup> Department of Engineering Science, University West, Trollhättan, Sweden. E-mail: Mehdi.eshagh@hv.se

<sup>2</sup> New Technologies for the Information Society (NTIS), University of West Bohemia, Plzen, Czech Republic. E-mail: rtenzer@sgg.wbu.edu.cn

stresses below the crust, while considering a two-layered model for the Earth so that the upper layer (crust) is floating on the viscous layer (mantle) below. He then used the low-degree spherical harmonics of the Earth's gravity field to deduce the global horizontal stress pattern, and found a correlation between the convergent and divergent sites established by the plate theory. Liu (1977, 1978, 1979) applied Runcorn's theory to construct maps of the convection-generated stresses driving the movements of tectonic plates. Liu (1980) also studied the relation between the mantle-convection generated stresses and the intra-plate volcanism. Huang and Fu (1982) and Fu and Huang (1983) extended Runcorn's definition for the full stress tensor represented by six independent components, while assuming a constant depth of the lithosphere. They solved the boundary-value problem of elasticity according to Love (1944) to model the stress field inside the lithosphere, and used Runcorn's formulae to define the boundary conditions to solve this problem.

Major theoretical deficiencies of these definitions are related to disregarding mantle viscosity variations and crustal deformations, because lithospheric stresses reflect the rheology and thermal state of the lithosphere as well as driving forces of plate motions. After a rapid expansion of global seismic networks and a development of space techniques for a precise determination of plate motions (such as VLBI, GPS), seismic tomography data and global tectonic plate motion models become preferably used to model tectonic stresses. Among existing studies we could mention earlier works by Richardson et al. (1979), Richardson (1992), and Bai et al. (1992), or more recent studies by Steinberger et al. (2001), Lithgow-Bertelloni and Gynn (2004), and Naliboff et al. (2009, 2012). Furthermore, bore-hole breakouts, hydraulic fracturing, volcanic alignment, seismic focal mechanisms, heat flow on faults, transform fault azimuths, and other in situ stress measurements were also used to investigate and predict the global stress pattern (Zoback and Zoback 1991; Sperner et al. 2003; Heidbach et al. 2007, 2010) and to compile the World Stress Map Database (Zoback 1992; Heidbach et al. 2016).

Even though global seismic networks and locations covered by in situ stress measurements increased steadily over the last few decades, most of them are

concentrated in seismically active regions, while large parts of the world are still not yet sufficiently covered by these data. Global gravity models (which have almost global and homogenous coverage) could therefore be used as an alternative source of information to detect the lithospheric stresses. Following this concept, the lithospheric stress determination from gravity data has recently been addressed by Eshagh (2014a, b), Eshagh and Tenzer (2015), Tenzer and Eshagh (2015), Tenzer et al. (2017), and Eshagh (2017). Moreover, gravimetric methods could also be used to investigate the lithospheric stresses of planetary bodies (cf. Tenzer et al. (2015)). In aforementioned studies, the horizontal shear stresses computed according to Runcorn's (1967) theory. Moreover, the two-layered model (used by Runcorn for the crust and the mantle) should correctly be assumed for the solid lithosphere and the viscous asthenosphere, because information about the lithospheric thickness is now available (e.g., Conrad and Lithgow-Bertelloni 2006). In this study, we apply the method developed by Huang and Fu (1982) and Fu and Huang (1983) to evaluate the lithospheric stress tensor globally from the gravity and lithospheric structure models, while assuming a variable lithospheric thickness. We further subtract the long-wavelength signature of global mantle flow in order to enhance the medium-to-higher frequency spectrum of the lithospheric stresses. We then interpret results with respect to tectonic and loading forces. The comparison of the gravimetrically-determined lithospheric stresses with results from seismic tomography and in situ stress measurements is out of the scope of this study.

## 2. Method

In this section we give a brief summary of the boundary-value problem of elasticity and its solution for finding the functional relation between the lithospheric stress tensor and the gravity field.

### 2.1. Lithospheric Stress Tensor

To begin with, let us define the stress tensor in its generic form (e.g., Turcotte and Schubert 1982; Stuewe 2007)

$$\mathbf{S} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}, \quad (1)$$

where  $\{\sigma_{ij}; i, j = x, y, z\}$  are the elements of stress tensor. For an infinitesimal element of the spherical shell, the diagonal elements represent the normal stress and the off-diagonal elements define the shear stress.

To obtain the lithospheric stress tensor, we solve the partial differential equation of elasticity in the spherical domain for a thin spherical shell (cf. Fu and Huang 1983)

$$(\chi + \mu) \nabla(\nabla \cdot \mathbf{S}) + \mu \nabla^2 \mathbf{S} = 0, \quad (2)$$

where  $\mathbf{S}$  denotes the displacement field,  $\nabla^2$  is the Laplace operator,  $\nabla$  is the gradient operator,  $\nabla \cdot \mathbf{S}$  is the divergence of tensor  $\mathbf{S}$ ,  $\chi = 0.71 \times 10^{12}$  and  $\mu = 0.56 \times 10^{12}$  are the elasticity parameters of the Earth's lithosphere (cf. Fu and Huang 1990).

To solve the differential equation in Eq. (2), we define the following boundary conditions (cf. Runcorn 1967)

$$F_r = 0, F_\theta = 0, F_\lambda = 0, \quad \text{for } r = R, \quad (3)$$

$$F_r = 0, F_\theta = \sigma_{xz}, F_\lambda = \sigma_{yz}, \quad \text{for } r = R - D, \quad (4)$$

where  $F$  denotes the force,  $D$  is the lithospheric depth, and  $R$  is the Earth's mean radius. The 3-D position is defined by the geocentric spherical coordinates, with the radius  $r$ , co-latitude  $\theta$ , and longitude  $\lambda$ . The boundary conditions, given in Eqs. (3) and (4), specify that forces in the radial, northward, and eastward directions equal zero at the upper boundary of the lithosphere. At the base of the lithosphere (i.e., the lithosphere-asthenosphere boundary), the radial force is zero, while global mantle flow induces the shear stresses defined by the components  $\sigma_{xz}$  and  $\sigma_{yz}$ . According to this formulation, viscosity variations of the asthenosphere are completely disregarded and boundary conditions are defined for an undeformed surface. The gravitational field is then directly related to the horizontal shear stress component, under a number of assumptions which might not realistically represent the actual rheology and crustal deformations (cf. Runcorn 1967; Phillips and Ivins 1979). This over-simplistic model thus assumes that the lithospheric stresses could be detected solely from the

gravity data without using any constraining information such as the global lithospheric plate velocity model and the rheological model of lithosphere/asthenosphere (detected from seismic tomography). Nevertheless, this model might be applicable to study the terrestrial stress pattern over regions with an insufficient coverage of seismic data and in situ stress measurements, and stresses of planetary bodies (due to a limited knowledge about the rheology of planetary interiors).

## 2.2. Shear Stress Components

Runcorn (1967) formulated the expression for computing the shear stress components  $\sigma_{xz}$  and  $\sigma_{yz}$  as follows

$$\begin{pmatrix} \sigma_{xz} \\ \sigma_{yz} \end{pmatrix} = \frac{Mg}{4\pi(R-D)^2} \sum_{n=2}^{\infty} s^{n+1} \frac{2n+1}{n+1} \begin{pmatrix} \frac{\partial T_n}{\partial \theta} \\ \frac{\partial T_n}{\sin \theta \partial \lambda} \end{pmatrix}, \quad (5)$$

where  $s = R/(R-D)$ ,  $M$  is the Earth's total mass (including the atmosphere), and  $g$  is the Earth's (surface) mean gravity. The surface spherical harmonics  $T_n$  of the disturbing potential  $T$  (i.e., the difference between the actual and normal gravity potentials) in Eq. (5) are given by

$$T_n = \sum_{m=-n}^n T_{nm} Y_{nm}(\theta, \lambda), \quad (6)$$

where  $Y_{nm}(\theta, \lambda)$  are the (fully-normalized) spherical harmonic functions of degree  $n$  and order  $m$ , and the (fully-normalised) coefficients  $T_{nm}$  of the disturbing potential are obtained from the coefficients of a global gravitational model after subtracting the spherical harmonic coefficients of the normal gravity field.

## 2.3. Stress Tensor in Terms of Gravity

By analogy with Runcorn's (1967) solution for the shear stress components in terms of gravity (in Eq. 5), we could find an equivalent solution for all elements of the stress tensor in Eq. (1).

The solution of the differential formula in Eq. (2) with the boundary conditions from Eqs. (3) and (4) yields the displacement field for a spherical shell (cf. Beuthe 2008). Using this displacement field and the

elastic theory, the final stress equations are found in terms of the spherical harmonics  $T_n$  and their partial derivatives. From Fu and Huang (1983), the normal stress components are defined by

$$\sigma_{xx} = \sum_{n=2}^{\infty} \left\{ \left[ \left( \frac{\chi}{r'} \right) K_n^1 + \left( \frac{2\mu}{r'} \right) K_n^3 \right] T_n + \left( \frac{2\mu}{r'} \right) K_n^5 \frac{\partial^2 T_n}{\partial \theta^2} \right\}, \quad (7)$$

$$\sigma_{yy} = \sum_{n=2}^{\infty} \left\{ \left[ \left( \frac{\chi}{r'} \right) K_n^1 + \left( \frac{2\mu}{r'} \right) K_n^3 \right] T_n + \left( \frac{2\mu}{r'} \right) K_n^5 \left[ \frac{1}{\sin^2 \theta} \frac{\partial^2 T_n}{\partial \lambda^2} + \cot \theta \frac{\partial T_n}{\partial \theta} \right] \right\}, \quad (8)$$

$$\sigma_{zz} = \sum_{n=2}^{\infty} \left[ \left( \frac{\chi}{r'} \right) K_n^1 + \left( \frac{2\mu}{r'} \right) K_n^3 \right] T_n, \quad (9)$$

and the shear stress components read

$$\sigma_{xy} = \sum_{n=2}^{\infty} \left\{ \left( \frac{\mu}{r' \sin \theta} \right) K_n^5 \left[ \frac{\partial^2 T_n}{\partial \theta \partial \lambda} - \cot \theta \frac{\partial T_n}{\partial \lambda} \right] \right\}, \quad (10)$$

$$\sigma_{xz} = \sum_{n=2}^{\infty} \left\{ \left( \frac{\mu}{r'} \right) (K_n^4 - K_n^5 + K_n^3) \frac{\partial T_n}{\partial \theta} \right\}, \quad (11)$$

$$\sigma_{yz} = \sum_{n=2}^{\infty} \left\{ \left( \frac{\mu}{r' \sin \theta} \right) (K_n^4 - K_n^5 + K_n^3) \frac{\partial T_n}{\partial \lambda} \right\}, \quad (12)$$

where  $r'$  is the geocentric radius of an arbitrary point inside the lithosphere. The stress tensor coefficients  $\{K_n^i; i = 1, 2, \dots, 5\}$  in Eqs. (7)–(12) are given by

$$K_n^1 = A_n [2n + \alpha_n (3 + n)] t^n + B_n [-2(n + 1) + \bar{\alpha}_n (2 - n)] t^n, \quad (13)$$

$$K_n^2 = A_n (n + \alpha_n) (n + 1) t^{-n-1} - B_n n [\bar{\alpha}_n - (n + 1)] t^n + \frac{C_n}{R^2} (n - 1) n t^{1-n} + \frac{D_n}{R^2} (n + 1) (n + 2) t^{n+2}, \quad (14)$$

$$K_n^3 = A_n (n + \alpha_n) (n + 1) t^{-n-1} - B_n n [\bar{\alpha}_n - (n + 1)] t^n + \frac{C_n}{R^2} n t^{1-n} - \frac{D_n}{R^2} (n + 1) t^{n+2}, \quad (15)$$

$$K_n^4 = A_n (n + 1) t^{-n-1} - B_n n t^n + \frac{C_n}{R^2} (n - 1) t^{1-n} - \frac{D_n}{R^2} (n + 2) t^{n+2}, \quad (16)$$

$$K_n^5 = A_n (n + 1) t^{-n-1} - B_n n t^n + \frac{C_n}{R^2} t^{1-n} - \frac{D_n}{R^2} t^{n+2}, \quad (17)$$

where the parameters  $\alpha_n$ ,  $\bar{\alpha}_n$ , and  $t$  read

$$\alpha_n = -2 \frac{n\chi + (3n + 1)\mu}{(n + 3)\chi + (n + 5)\mu}, \quad (18)$$

$$\bar{\alpha}_n = 2 \frac{(n + 1)\chi + (3n + 2)\mu}{(2 - n)\chi + (4 - n)\mu}, \quad (19)$$

$$t = \left( \frac{R}{r'} \right). \quad (20)$$

It is worth mentioning that Liu (1983) presented similar formulae for the stress tensor coefficients by assuming that  $\chi = \mu$ .

## 2.4. Estimation Model

The stress tensor coefficients  $\{K_n^i; i = 1, 2, \dots, 5\}$  in Eqs. (13)–(17) depend on the parameters  $A_n$ ,  $B_n$ ,  $C_n$ , and  $D_n$ , which define mechanical properties of the lithosphere. Since these parameters are not defined mathematically, their estimation is carried out by solving the system of observation equations

$$\mathbf{A}\mathbf{x} = \mathbf{l}, \quad (21)$$

where the design matrix  $\mathbf{A}$ , the vector of observations  $\mathbf{l}$ , and the vector of unknown parameters  $\mathbf{x}$  are defined by

$$\mathbf{A} = R^{-1} \begin{pmatrix} s^n H_1 & s^{-(n+1)} H_2 & s^{n-2} \frac{H_3}{R^2} & s^{-(n+3)} \frac{H_4}{R^2} \\ H_1 & H_2 & \frac{H_3}{R^2} & \frac{H_4}{R^2} \\ s^n G_1 & s^{-(n+1)} G_2 & s^{n-2} \frac{G_3}{R^2} & s^{-(n+3)} \frac{G_4}{R^2} \\ G_1 & G_2 & \frac{G_3}{R^2} & \frac{G_4}{R^2} \end{pmatrix}, \quad (22)$$

$$\mathbf{l} = \begin{pmatrix} 0 \\ 0 \\ F \\ 0 \end{pmatrix}, \quad (23)$$

$$\mathbf{x} = \begin{pmatrix} A_n \\ B_n \\ C_n \\ D_n \end{pmatrix}. \quad (24)$$

The force-generating function  $F$  in the observation vector  $\mathbf{I}$  (Eq. 23) was defined by Runcorn (1967) in the following form

$$F = \frac{Mg}{4\pi(R-D)^2} \frac{2n+1}{n+1} s^{-(n+1)}. \quad (25)$$

Moreover, the design matrix in Eq. (22) is formed by the parameters:

$$H_1 = \mu(2n + \alpha_n) + 2n(\chi + \mu) + \alpha_n[(n+3)\chi + (n+2)\mu], \quad (26)$$

$$H_2 = -\mu[-2(n+1) + \bar{\alpha}_n](n+1) - 2(n+1)(\chi + \mu) + \bar{\alpha}_n[(n-2)\chi + (1-n)\mu], \quad (27)$$

$$H_3 = 2\mu(n-1)n, \quad (28)$$

$$H_4 = 2\mu(n+2)(n+1), \quad (29)$$

$$G_1 = \mu(2n + \alpha_n), \quad (30)$$

$$G_2 = \mu(-2(n+1) + \bar{\alpha}_n), \quad (31)$$

$$G_3 = 2\mu(n-1), \quad (32)$$

$$G_4 = -2\mu(n+2). \quad (33)$$

### 3. Numerical Studies

Theoretical definitions from Sect. 2 were applied here to compute globally the lithospheric stress at the Moho interface. In order to better understand the propagation of stresses through the lithosphere, we first investigated the depth-dependence of stress tensor coefficients.

#### 3.1. Kernel Behaviour

To illustrate the sensitivity of the stress tensor coefficients  $\{K_n^i; i = 1, 2, \dots, 5\}$  with changing depth, we adopted a uniform lithospheric model of a constant thickness 100 km. A signal degree of the stress tensor coefficients (at the spectral window between the degrees from 13 to 180) at depths 10, 30, and 70 km (below sea level) are shown in Fig. 1. It is worth mentioning here that this choice of depths closely corresponds with some principal characteristics of global Moho undulations, with a typical

thickness of the oceanic crust of about 10 km, an average thickness of the continental crust of about 30–35 km, and the maximum Moho deepening of about 70 km under the Himalayan–Tibetan orogen. Moreover, the long-wavelength contribution below the spherical harmonic degree of 13 was subtracted in order to reduce the stress signature of sub-lithospheric convection pattern (cf. also Liu 1977).

For certain degrees of the stress tensor coefficients  $\{K_n^i; i = 1, 2, \dots, 5\}$  defined in Eqs. (13)–(17), the system of observation equations in Eq. (21) becomes singular, because of their spectral behaviour. One example can be given for  $s = 1$ . In this case, the design matrix (in Eq. 22) is singular, because the first two rows are identical as well as the last two rows. In fact, the non-singularity of the design matrix holds only for  $s > 1$  (cf. Eq. 22). A detailed inspection also revealed that the degree signal increases unboundedly for  $n$  or  $n - 2$ , while rapidly decreases for  $-(n + 1)$  and  $-(n + 3)$ . Such spectral behaviour causes that some elements of the design matrix become very large at higher degrees, while others very small. In addition, the diagonal elements, comprising terms  $s^n$  and  $s^{n-2}$ , grow unlimitedly with  $n$ , thus causing that the design matrix becomes ill-conditioned. For this reason, we used Moore–Penrose’s pseudo-inverse technique (cf. Moore 1920; Bjerhammar 1951; Penrose 1955), instead of applying a regular matrix inversion.

In absolute sense, the stress tensor coefficient  $K_n^1$  increases up to the degree of about 15–20 (depending on the depth), and then monotonously decreases (cf. Fig. 1). Moreover, the magnitude of  $K_n^1$  increases (again in absolute sense) when computed at larger depths. For  $K_n^2$ , the situation is opposite. In this case, the signal of  $K_n^2$  decreases with depth. The coefficients  $K_n^1$  and  $K_n^2$  are used to compute the stress tensor component  $\sigma_{zz}$  according to Eq. (9). For the elasticity parameters:  $\chi = 0.71 \times 10^{12}$  and  $\mu = 0.56 \times 10^{12}$ , the contribution of the  $K_n^2$ —term on the right-hand side of Eq. (9) is larger than the corresponding contribution of the  $K_n^1$ —term. Hence, the stress tensor component  $\sigma_{zz}$  magnifies (in absolute sense) with an increasing depth, as it is clear from the depth-dependent behaviour of  $K_n^2$  (see Fig. 1).

As also seen in Fig. 1, among the stress tensor coefficients  $K_n^1$ ,  $K_n^3$ , and  $K_n^5$ , used in Eqs. (7) and (8),

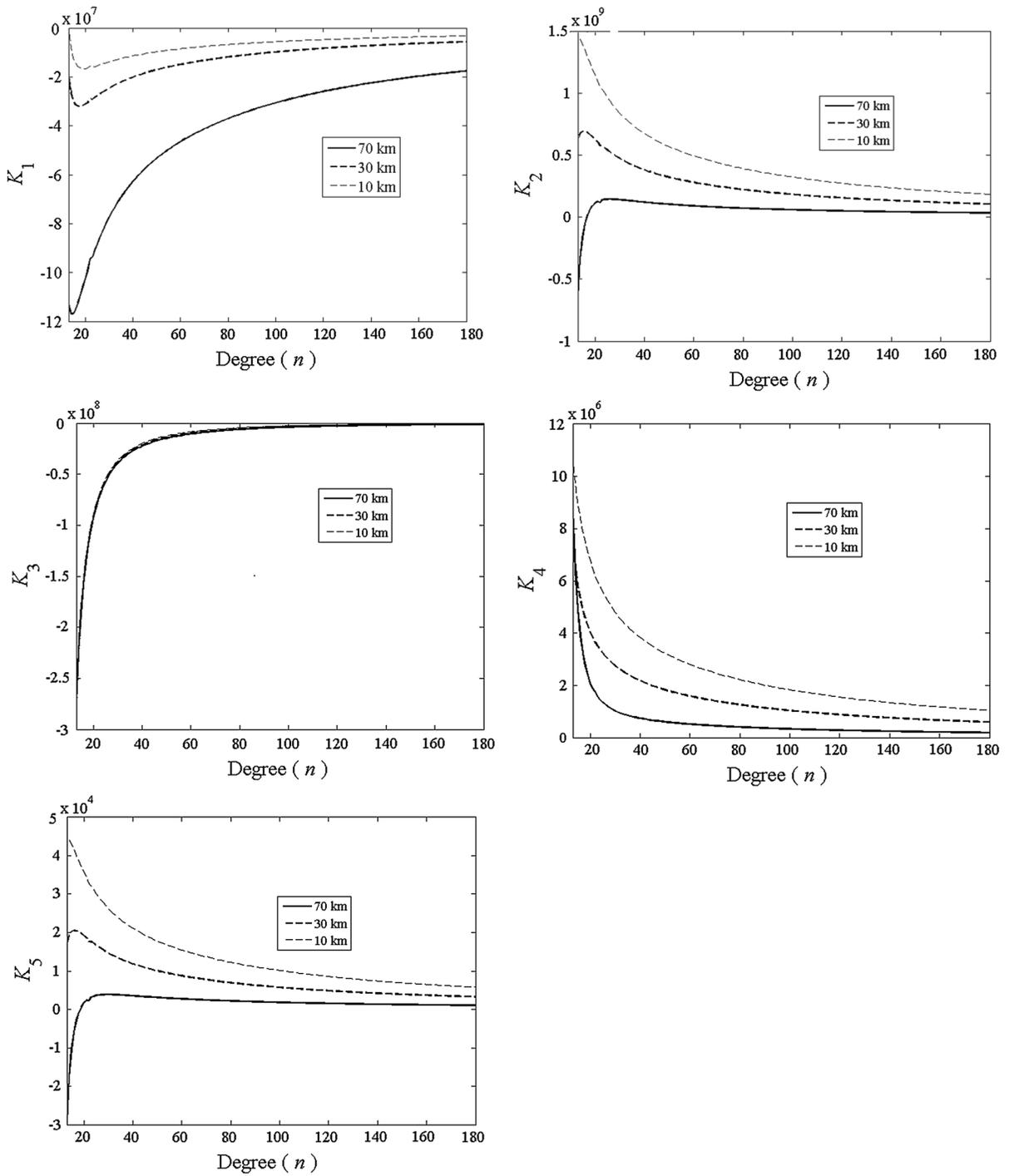


Figure 1

Signal degree of the stress tensor coefficients  $\{K_n^i; i = 1, 2, \dots, 5\}$  at the spectral window between the spherical harmonic degrees from 13 to 180

the contribution of  $K_n^5$  is the smallest. Moreover, the coefficients  $K_n^1$  and  $K_n^3$  are negative, and  $K_n^3$  does not show any significant dependence on depth, but when added to  $K_n^1$ , their combined contribution becomes more depth-sensitive. As seen in Eq. (10), the component  $\sigma_{xy}$  is functionally related only with the coefficient  $K_n^5$ . Since the contribution of  $K_n^5$  is the smallest, the component  $\sigma_{xy}$  is also the smallest among the stress tensor components. In Eqs. (11) and (12) we observe that the stress tensor components  $\sigma_{xz}$  and  $\sigma_{yz}$  involve the combination  $K_n^4 - K_n^5 + K_n^3$ . Among these coefficients,  $K_n^3$  has the strongest signal which dominates the total signal of the combination.

### 3.2. Data Acquisition

The computation of the lithospheric stresses at the Moho interface from the gravity data requires information about the lithospheric thickness and the Moho geometry. In our study, we used the lithospheric-thickness model presented by Conrad and Lithgow-Bertelloni (2006).

They adopted a half-space cooling model (Turcotte and Schubert 1982) according to which the oceanic lithospheric thickness increases proportionally with the square-root of its age. Since they used data of the ocean-floor age from Müller et al. (1997) to compute the oceanic lithospheric thickness; we updated their result based on using more recent data from Müller et al. (2008). Despite some substantial deviations have been documented, especially under oceanic subduction zones (cf. Conrad and Lithgow-Bertelloni 2006), a half-space cooling model provides a reasonable lithospheric-thickness estimate for most of oceanic regions. For the continental lithosphere, they computed the characteristic thickness by following the method of Gung et al. (2003), who employed the maximum depth for which the seismic velocity anomaly, as determined using Ritsema et al. (2004) seismic tomography model S20RTSb, is consistently greater than 2%, while imposing 100 km as the minimum continental and maximum oceanic characteristic thickness.

The global map of the lithospheric thickness is shown in Fig. 2. The lithospheric thickness varies from 5 to 270 km. A thin lithosphere along mid-oceanic ridges increases with the ocean-floor age. A

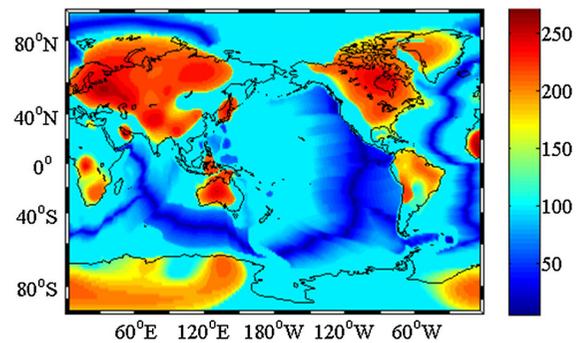


Figure 2  
Lithospheric thickness (km). Black lines show shorelines

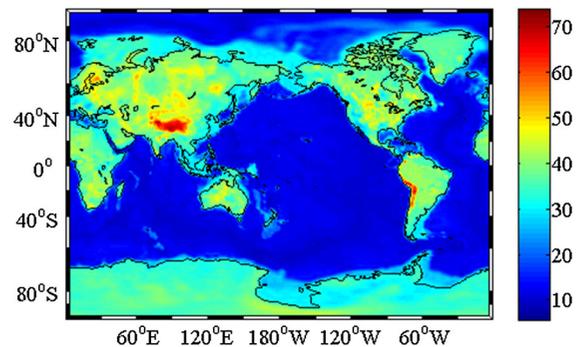


Figure 3  
Moho depth (km)

much more complex continental lithosphere is characterized by the largest lithospheric deepening under cratons. A significant lithospheric deepening due to the lithospheric subduction is also detected under some orogens (such as Himalaya, Tibet, Central Asian Orogenic Belt, and Andes). It is worth mentioning here that the continental lithosphere may feature even larger variations in thickness, including continental roots that may penetrate to depths as much as 400 km beneath cratonic shields (e.g., Jordan 1975; Ritsema et al. 2004), and are likely cold and highly viscous (e.g., Rudnick et al. 1998).

The Moho depth, taken from the CRUST1.0 seismic crustal model (Laske et al. 2013), is shown in Fig. 3. A maximum Moho deepening to about 70 km is detected under orogens of Himalaya, Tibet, and central Andes. The most pronounced feature in the global Moho pattern is a contrast along continental margins between a thick continental crust and a much thinner oceanic crust.

### 3.3. Stress Tensor Components

The stress tensor components at the Moho interface were computed according to Eqs. (7)–(12) on a  $1 \times 1$  arc-deg global grid from spherical harmonics of the disturbing potential  $T_n$ , with a spectral resolution up to the degree of 180. As mentioned above, the long-wavelength harmonics below the degree of 13 were subtracted from the investigated stress spectrum. The spherical harmonics  $T_n$  were generated from the GOCO-05S coefficients (Mayer-Gürr et al. 2015). The computation was realized by applying the semi-vectorisation algorithm (Eshagh and Abdollahzadeh 2011) in order to take into consideration spatial variations in lithospheric and crustal thickness. The normal and shear stress components are shown in Figs. 4 and 5, respectively, and their statistical summary is given in Table 1.

The most pronounced features in maps of the horizontal normal stress components  $\sigma_{xx}$  and  $\sigma_{yy}$  (Fig. 4a, b) are active convergent tectonic margins of oceanic subductions as well as continent-to-continent tectonic plate collisions. Positive values (of the stress intensity) on the side of subducted lithosphere (oceanic trenches and continental basins) are coupled by negative values on the side of back-arc rifting and orogens. As seen in Fig. 4c, oceanic subductions and continental collisions are again clearly manifested in map of the vertical normal stress component  $\sigma_{zz}$ . In overall, the vertical component has a spatial pattern similar to that seen in horizontal components, but of opposite sign. The maximum shear stresses (see Fig. 5) are again detected along oceanic subductions and continental collisions.

### 3.4. Total Stress Intensity

We further used the stress tensor components to compute the total stress intensity, individually for the horizontal  $S_{hh}$ , vertical  $S_{vv}$ , and (mixed) horizontal–vertical  $S_{hv}$  components according to the following expressions

$$S_{hh} = \sqrt{(\sigma_{xx}^2 + \sigma_{yy}^2)^2 - 4\sigma_{xy}^2}, \quad (34)$$

$$S_{vv} = |\sigma_{zz}|, \quad (35)$$

$$S_{hv} = \sqrt{\sigma_{xz}^2 + \sigma_{yz}^2}. \quad (36)$$

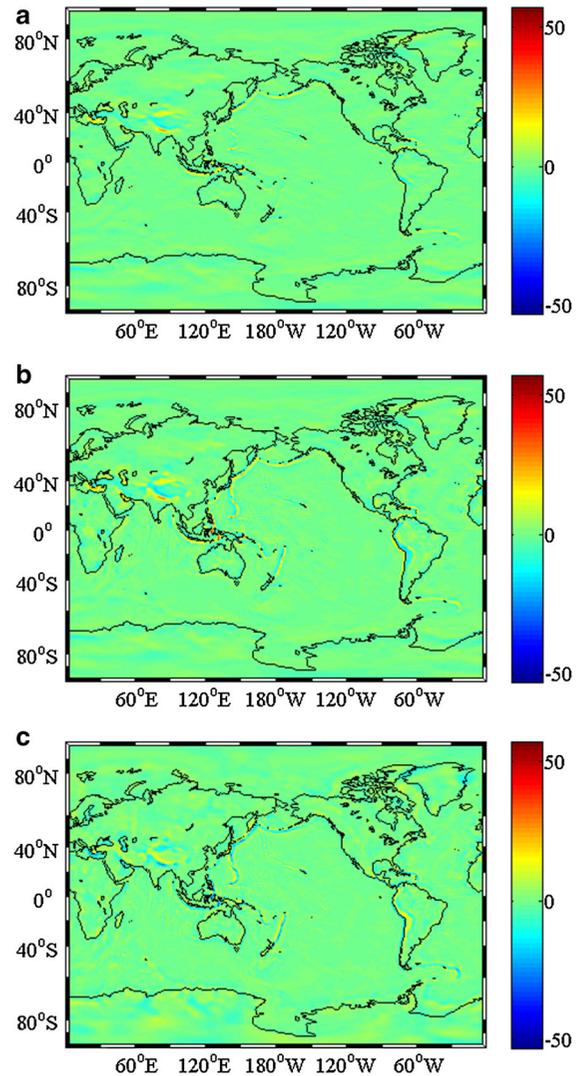


Figure 4  
Normal stress components: **a**  $\sigma_{xx}$ , **b**  $\sigma_{yy}$ , and **c**  $\sigma_{zz}$  (MPa)

The results are plotted in Fig. 6, and their statistical summary is given in Table 2.

The maximum total stress intensity is detected along oceanic subductions in western Pacific (Kermadec-Tonga, Ryukyu, Mariana, Japan, Kuril, and Aleutian trenches). A slightly less pronounced stress intensity is also seen along oceanic subductions in Atlantic (Puerto Rico and Pacific–Antarctic trenches) and eastern Pacific (Peru–Chile trench). Along continental collisions, large stress intensity is seen between the African and Eurasian plates (Hellenic trench) and between the Indian plate and Tibetan

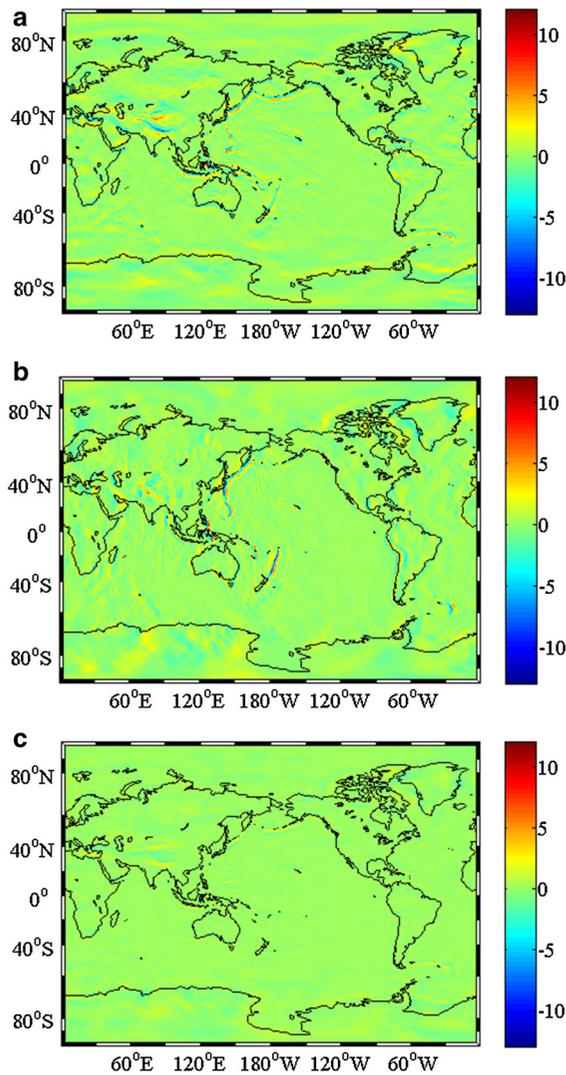


Figure 5  
Shear stress components: **a**  $\sigma_{xz}$ , **b**  $\sigma_{yz}$ , and **c**  $\sigma_{xy}$  (MPa)

block. The intra-plate stresses are clearly manifested along Hawaii volcanic islands, and along some active thrust fault systems in central Eurasia. We could also see a similar spatial pattern in horizontal and vertical stress intensity, but the vertical stress intensity is about two times larger than the horizontal one (cf. Table 2). It is worth mentioning that the intensity of vertical normal stress is modified by a Moho deepening to about 30% under Himalaya, Tibet, and Andes, because of its dependence on depth (cf. Figure 1), while along oceanic subductions such modification is minor due to a relatively small Moho variations.

#### 4. Discussion and Concluding Remarks

Our results (Fig. 6) agree with findings of Tenzer and Eshagh (2015) and Eshagh and Tenzer (2015) that the lithospheric stresses are mainly manifested along active tectonic margins of oceanic subductions and along continent-to-continent tectonic plate collisions. The maximum stresses are induced by subduction slab pull forces on the side of subducted lithosphere. These stresses are coupled by slightly less pronounced stresses on the side of overriding lithospheric plate of which origin could be explained by trench suction. Our results also showed the presence of (intra-plate) lithospheric loading stresses along Hawaii islands. On the other hand, the stresses due to ridge push force along divergent tectonic plate boundaries are not clearly manifested.

Table 1

Statistics of the lithospheric stress intensity

Stress tensor component	Max (MPa)	Mean (MPa)	Min (MPa)	STD (MPa)
$\sigma_{xx}$	39.1	0.0	-23.4	2.2
$\sigma_{yy}$	57.3	0.0	-36.1	3.0
$\sigma_{zz}$	34.9	0.0	-53.7	3.0
$\sigma_{xz}$	12.2	0.0	-13.6	0.8
$\sigma_{yz}$	12.7	0.0	-11.4	0.8
$\sigma_{xy}$	4.7	0.0	-2.8	0.3

The STD denotes a standard deviation

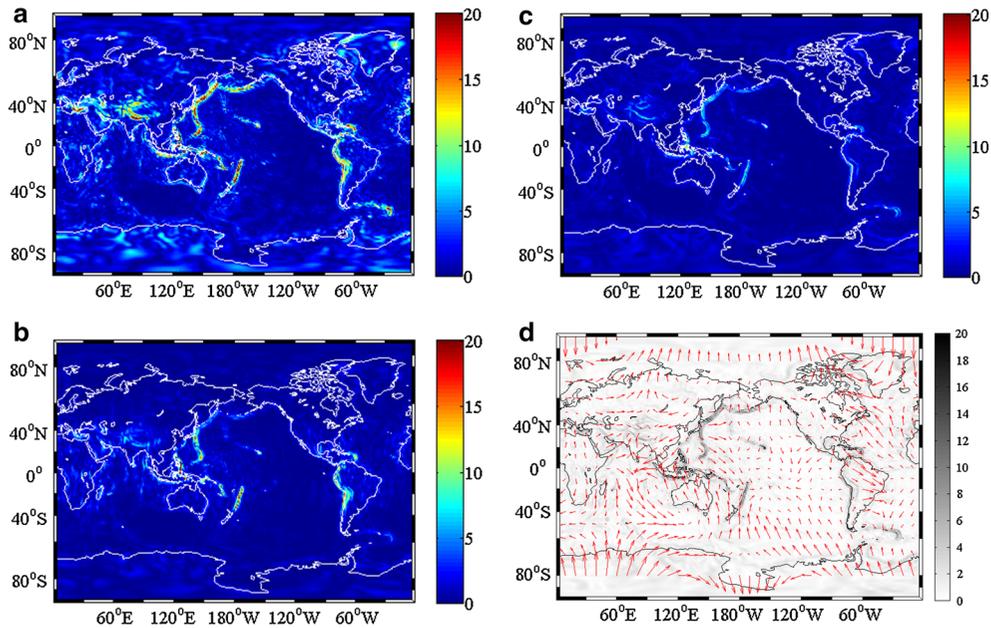


Figure 6

Total lithospheric stress intensity: **a** horizontal  $S_{hh}$ , **b** vertical  $S_{vv}$ , **c** horizontal–vertical  $S_{hv}$ , and **d** stress vectors on the background on the horizontal–vertical stress from spectral degrees 2 to 180. Note that for a better illustration of locations with a maximum stress intensity (for  $S_{hh}$  and  $S_{vv}$ ) we limited the *colour scale* in such a way that all values exceeding 20 MPa have the *same colour*

Table 2

*Statistics of the total horizontal  $S_{hh}$ , vertical  $S_{vv}$ , and horizontal–vertical  $S_{hv}$  lithospheric stress intensity*

Total stress component	Max (MPa)	Mean (MPa)	Min (MPa)	STD (MPa)
$S_{hh}$	28.9	0.8	0.0	1.3
$S_{vv}$	53.7	1.8	0.0	2.4
$S_{hv}$	15.1	0.7	0.0	0.8

A more detailed inspection of the horizontal normal stresses (Fig. 4a, b) revealed the presence of extensional tectonism of continental basins (Gangetic and Tarim basins). Positive values of the horizontal normal stresses over continental basins are coupled by negative values along Himalaya and the northern Tibet, indicating the compressional tectonism of these orogenic formations. Our results thus confirmed findings from the study conducted in central Eurasia by Tenzer et al. (2017) based on applying Runcorn's theory. They demonstrated that the convergent pattern of horizontal shear stress vectors agrees with the compressional tectonism of orogens, while their divergent orientation indicates the existence of

extensional tectonism of continental basins. Here we also shown that a similar pattern of the normal horizontal stresses is detected along oceanic subductions with positive values on the side of subducted oceanic lithosphere, coupled by negative values on the side of overriding lithospheric plate (i.e., back-arc rifts or orogens).

The horizontal compressional tectonism of orogens (characterised by negative values of the normal horizontal stresses) is responsible for a lithospheric thickening, which should be manifested by an apparent tensional vertical stresses. Our results confirmed this, showing negative values of the normal vertical stresses. In case of continental basins, we

detected again the same coupling effect in the horizontal and vertical normal stress components, given by their opposite signs. The same pattern was also found for oceanic subductions and back-arc rifts.

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