Christopher Glass

The Pythagopod

In 1967 lecture at Yale Architecture School Anne Tyng discussed integrating the five Pythagorean solids into a single shape and suggested the shape as an architectural solid. Christopher Glass aims to sphere the cube in the manner of Buckminster Fuller, but with reference not only to the engineering models he uses but to the cultural models of the Pythagorean proportions as well. The author has developed computer models of the resulting plan at least two scales: the original glass house and a smaller hermitage pod.

In the fall of 1966 or 1967 I was at the Yale School of Architecture and heard a presentation by Anne Tyng, Louis Kahn's associate, about the five Platonic solids and the ways they could be made to interrelate. The lecture has stayed with me over the years. At the time Buckminster Fuller had just built the American Pavilion at Expo 67 in Montreal, Steve Baer was working with "Zomes" at Drop City in Colorado, and, in general, traditional architecture had been under assault from Archigram and Ant Farm and other radical modernists. Kahn had designed his proposal for Philadelphia City Hall as a tetrahedral space frame. Traditional building seemed obsolete.

Having been raised in the shadow of a Gothic cathedral (Washington, D.C.), I had always had reservations about modern architecture's ability to relate to its human users. The geodesic dome seemed to epitomize the problem. Since the geometry of the dome is merely an attempt to follow the skin of a sphere with straight struts, it is and appears to be a purely abstract engineering solution. The other image that had appeal was the Lunar Lander, a piece of engineering devoid of overt aesthetic appeal but having a significance derived from its completeness in its environment - "a man's home is his capsule". How, I asked, could such a capsule or dome be seen to be a continuation of classical design without explicitly incorporating classical decorative detail? The Platonic solids, especially the cube and the dodecahedron, seemed to offer an alternative that connected to classical proportioning systems and to the idea that geometry could relate to human scale.

Over the years I played with the shapes at various scales, designing a "drafting pod" module and a small meditation house - neither taken further than the drawing board. Then a Japanese magazine invited Philip Johnson to judge a contest for a new Glass House, and I used the contest as an occasion to design a one-bedroom house plan on the order of Johnson's in the module. The contest entry sank without a trace, but I have put the ideas together for this article.

A Platonic solid is a convex polyhedron whose every face is the same regular polygon (and, to be precise, such that the same number of faces surround each vertex). There are only five; three of them have equilateral triangles as faces, one has squares, and one has regular pentagons (Figure 1).

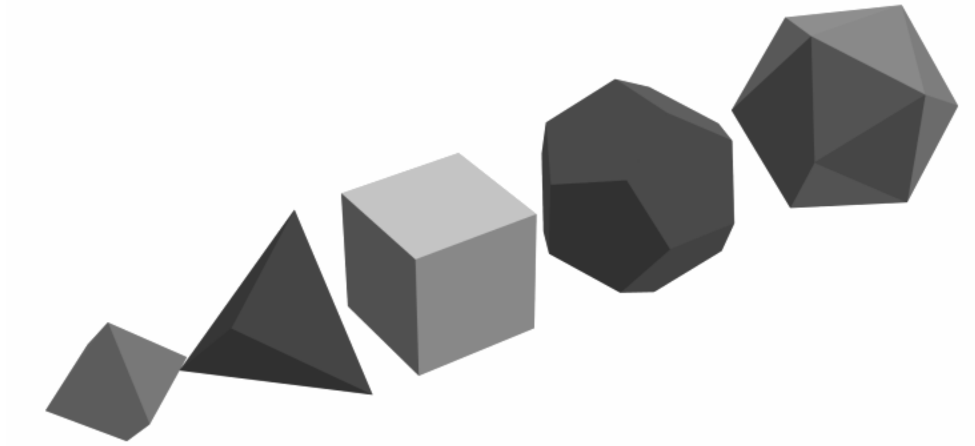

Figure 1. The Platonic solids.

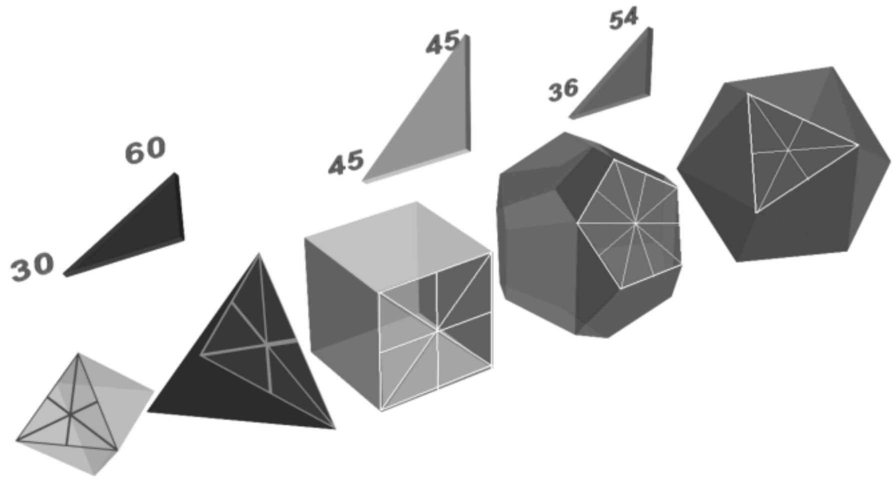

Figure 2. The construction of the Platonic solids from right isosceles triangles.

They are called Platonic because they are described in Plato's dialogue (actually more of a lecture) with the astronomer Timaeus, in which four of the five solids are described in detail, and the other is presumably referred to (but then rather conspicuously ignored). Timaeus, with no interruptions from Socrates, describes the tetrahedron, octahedron and icosahedron (the three made out of triangles) and the cube (made out of squares) as the geometrical building blocks of matter. He then says "there was yet a fifth combination which God used in the delineation of the universe with figures of animals" [*Timaeus* 55c]. This short shrift is intriguing. It suggests that the astronomer is not interested in the dodecahedron because it has to do with life rather than the mechanics of inorganic matter, but, as it appears, also because it does not fit into the scheme he is proposing, whereby one form of matter can shift into another. For Timaeus, the elementary polygons ("elementary particles") are not the equilateral triangle and the square: they are the right triangle whose short side is half its hypotenuse (the kind draftsmen refer to as a 30/60 triangle), and the isosceles triangle, specifically (but not explicitly) a right isosceles triangle.[1] He generates the equilateral triangle from not two but six 30/60 triangles, and the square from not two but four right isosceles triangles. Another reason for his ignoring the dodecahedron may be that its pentagonal face is not constructible from the two elementary particles, though by analogy it could be constructed of either isosceles (but not right) or right (but not isosceles) triangles (Figure 2).

Timaeus goes on to assign by "probability" the four shapes to the four basic elements, using reasoning that strikes us as fanciful but presumably impressed his contemporaries as rational and therefore likely. The smallest of the shapes, using the fewest parts and having the sharpest external angles, is the tetrahedron. It must, he states, therefore correspond to fire, the most active and literally pyrotechnical of the elements. The octahedron, resembling the tetrahedron but more complex, must be air. He says the cube is the most stable of the shapes (which is not true, as Fuller would point out) and therefore must constitute earth, and that leaves the rounder, drop-like icosahedron, which must be water. He then spends a long time explaining how these combine in increasingly unlikely ways to produce the appearances we call the physical world.

This is the same dialogue in which Timaeus tells the story of Atlantis, and there is a suggestion from this juxtaposition that both the stories have the quality of useful but preposterous legends that will do as provisional explanations of the world as we experience it.[2] It is significant that Socrates never asks any questions, probing or otherwise, as he does of his other talk-show guests. It's as if it is not worth the effort, since this is so obviously fanciful.

Timaeus never really discusses relating the shapes mathematically. He describes only the characters of the elements and their interactions. On the other hand, Euclid, in the culminating Book 13 of his *Elements*, does relate the shapes mathematically. For example, in Proposition 18 (the final proposition of the *Elements*), he compares the side lengths of the Platonic solids when all five are circumscribed by the same sphere. Euclid does not, however, try to connect them in any way so as to share common edges or vertices. The first time I saw them put together after Ann Tyng's talk was in the curious book that Dover has kept in print called *The Geometry of Art and Life*, by Matila Costiescu Ghyka [1946]. Ghyka states that the idea for linking them proportionally was mentioned by Campanus of Novara[3] and developed by Johannes Kepler, who believed the proportional relations between the spheres generated by the solids were proportional to the orbits of the (then known) five planets. The only illustration I have seen of Kepler's ideas show the more complex ones on the inside, with radically differing sizes. Ghyka presents a "modern version of the Keplerian interlocking of the five regular solids" [Ghyka 1946: 43-44]. My construction is a modified version of Ghyka's.

Essential to Ghyka's construction are the relationships between pairs of solids. Of the five solids, two pairs of them are "duals", in which the centers of the faces of one form the vertices of the other. This is true of the cube and octahedron, and of the dodecahedron and icosahedron (the tetrahedron is its own dual). However, for the purposes of constructing a Fuller-like space frame it is not very helpful to have vertices at the centers of faces. It is much easier and structurally stronger to connect vertices at edges, forming new vertices of more complex shapes.

Fortunately, dualities apply to edges as well: dual solids have the same number of edges. For example, the dodecahedron has twelve pentagonal faces, but each of the face's edges is common to two pentagons, so it has 12 * 5 / 2 = 30 edges; the icosahedron has twenty triangular sides, and so by the same reasoning it has 20 * 3 / 2 = 30 edges. The other essential relationships that can be found among the solids are those of the diagonals of the faces: one diagonal from each of the six square faces of a cube form the edges of a tetrahedron. The midpoints of those diagonals, which are the midpoints of the edges of the tetrahedron, form the vertices of the octahedron.

Since the diagonal of a unit square is the square root of 2, the edges of the cube and its inscribed tetrahedron are in the ratio $1:\sqrt{2}$. The octahedron's edges can easily be seen to be half those of the tetrahedron. So among these three shapes there is a pretty simple relationship. (Scale is irrelevant when considering ratios of quantities, but for the sake of definiteness take the edge length of the cube to be 1.)

The mathematics behind this is that ϕ is the unique positive number that satisfies $\phi - 1 = 1/\phi$. The fun starts with the dodecahedron. As the tetrahedron is formed by diagonals of the Pythagopod's cube, so is the cube formed by diagonals of the dodecahedron. And the ratio of the length of the diagonal of a pentagon to its side is the Golden Section, commonly denoted by ϕ. Therefore the ratio of the side of the dodecahedron to the side of the largest inscribed cube – one that connects pairs of vertices on each face of the dodecahedron in a regular rhythm of alternation – is , about 1:1.618, which by the "magic" of ϕ is equal to $\phi - 1$, about .618.

All of this brings me to the starting point of the construction of my Pythagopod. Each of the faces of a cube can been envisioned as the base of a shape like a hipped roof. The "ridge" of that roof is one of the edges of the dodecahedron, elevated above the surface of the cube far enough that the four "hips" of that roof are all the same length as the ridge. Each face of the cube has the same roof structure, but each face is rotated 90 degrees from the adjoining ones. It turns out that the distance between ridges on opposite sides is ϕ. So the easy way to construct the dodecahedron is to construct three rectangles (the rectangles with ellipses cut out of them in Figure 3) whose sides are ϕ and $1/\phi$, and connect the corners of the rectangles to the vertices of the cube.

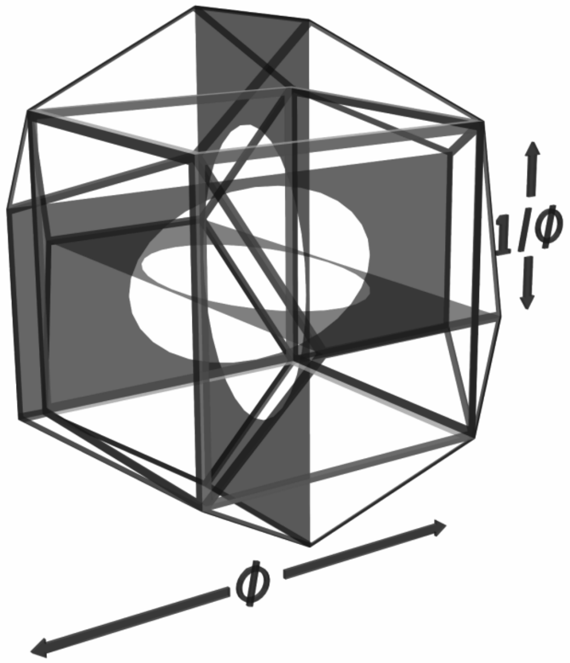

Figure 3. The easy way to construct the dodecahedron is to construct three rectangles whose sides are ϕ and $1/\phi$, placed mutually perpendicular to each other, and connect the corners of the rectangles to the vertices of the cube.

For the icosahedron the construction is simpler, but the relationship is more complex. The scale of the icosahedron is chosen so that its edges and those of the dodecahedron intersect at their midpoint, made possible in part by the fact that each solid has the same number of edges (30), as detailed above. In fact, when the midpoints of the edges are made to intersect, they turn out to be at right angles to each other. And since opposite edges of the icosahedron are the same distance apart as opposite edges of the dodecahedron, they are ϕ apart in relation to the cube in the dodecahedron.

When such an icosahedron is constructed, its edge length will turn out to be 1 – that is, the same as the inscribed cube. Thus it can be constructed analogously to the dodecahedron. First, generate three rectangles whose sides are 1 and ϕ. Then place these within the cube, as was done for the dodecahedron, but ignore the cube and connect the twelve vertices, in order to form the triangular faces of the icosahedron (Figure 4).

So far, then, a figure containing all five Platonic solids is obtained, with side lengths related to each other either by $\sqrt{2}$ or by ϕ (Figure 5). Again, the tetrahedron is comprised of the diagonals of the cube, the octahedron is the dual of the cube, and so the vertices of the octahedron are the midpoints of the edges of the tetrahedron.

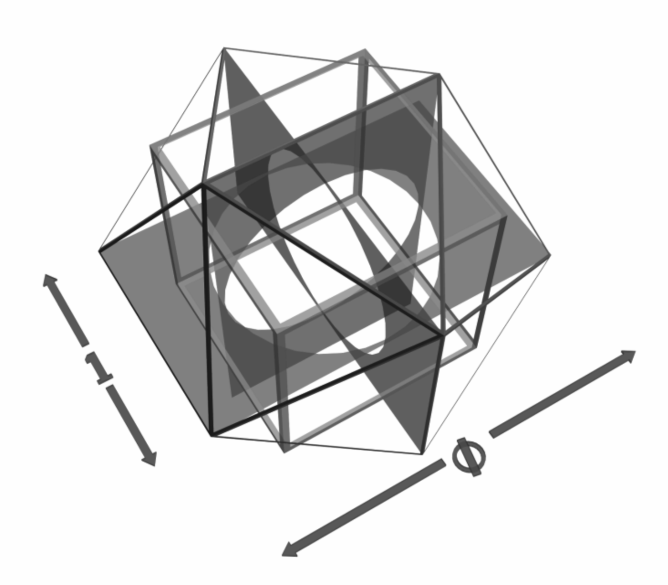

Figure 4. The icosahedron can be constructed analogously to the dodecahedron. First generate three rectangles whose sides are 1 and ϕ . Then place these, mutually perpendicular and centered within the cube, as was done for the dodecahedron, then ignore the cube and connect the twelve vertices, in order to form the triangular faces of the icosahedron.

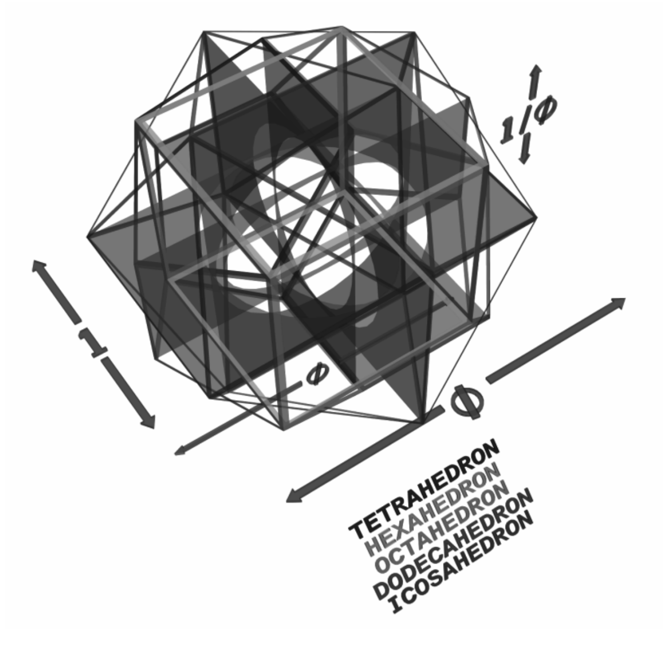

Figure 5. A figure containing all five Platonic solids, with side lengths related to each other by either √2 or by ϕ.

The final step is to create the "skin" of the shape, which is given by the outermost extent of the faces of the dodecahedron and the icosahedron (Figure 6). The additional edges necessary are obtained simply by connecting the intersections of the two solids' edges. The result is a surface consisting of 120 triangles: 60 equilateral triangles, each of whose sides is half that of the icosahedron, and 60 isosceles triangles, whose long bases are sides of the 60 equilateral triangles, and whose short legs are each half the length of the dodecahedron's edges. Thus, the edges of the skin share the same ϕ proportions as the edges of their "parents", the dodecahedron and the icosahedron (recall, the ratio of the edge of the cube to the edge of the dodecahedron is ϕ, and the edge of the icosahedron is the same as the edge of the cube). What I like about this figure is that, unlike a geodesic dome, it has two strut lengths and two triangular faces, and that the struts are related by the Golden Section. I also like the way the smaller isosceles triangles can be regarded as the solids, and the equilateral triangles as voids or windows, thus making a distinction of material and function that is absent in the dome.

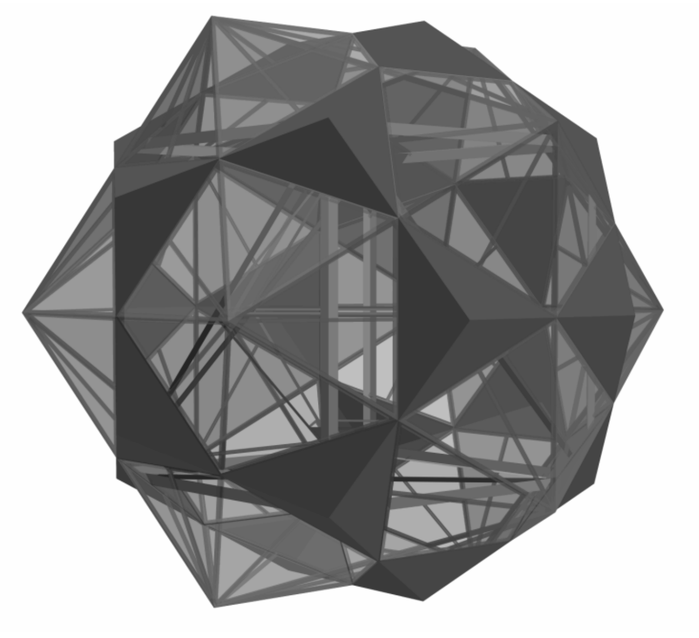

Figure 6. The creation of the "skin" of the shape, which is given by the outermost extent of the faces of the dodecahedron and the icosahedron.

In fact, returning to the *Timaeus*, one can assign "functions" to the five frames which are analogous to the five elements. Looking at the shape as it evolves into the house plan, I have made the small interior octahedron into a laser light sculpture - it is in the way if it is a solid frame. So the octahedron represents fire. Timaeus had assigned fire to what he (or his translator) called the pyramid – presumably the tetrahedron. My tetrahedron becomes the diagonal stiffener for the cube, since, contrary to *Timaeus* but following Fuller, the tetrahedron is the most stable of the shapes. It would be pushing the analogy to call it air or fire, but calling it and the cube it inhabits the earth would work, since this structural cube is the skeleton of the functional structure.

Surrounding the cube is the skin. The windows, being of glass – which is technically a liquid, are the analog of water, which Timaeus assigned to the icosahedron. And the dodecahedron is the shape that makes the transition and introduces the Golden Section. It is the "quintessence", the fifth element uniting them all, the element Timaeus dismisses as the shape of the animal world, but which we might think of as the basis for organic life. In the house it should be thought of as metal. Timaeus regarded metal as the result of the operation of fire on earth, causing it to flow like water, so it is a good physical analog to the immaterial fifth element (Figure 7).

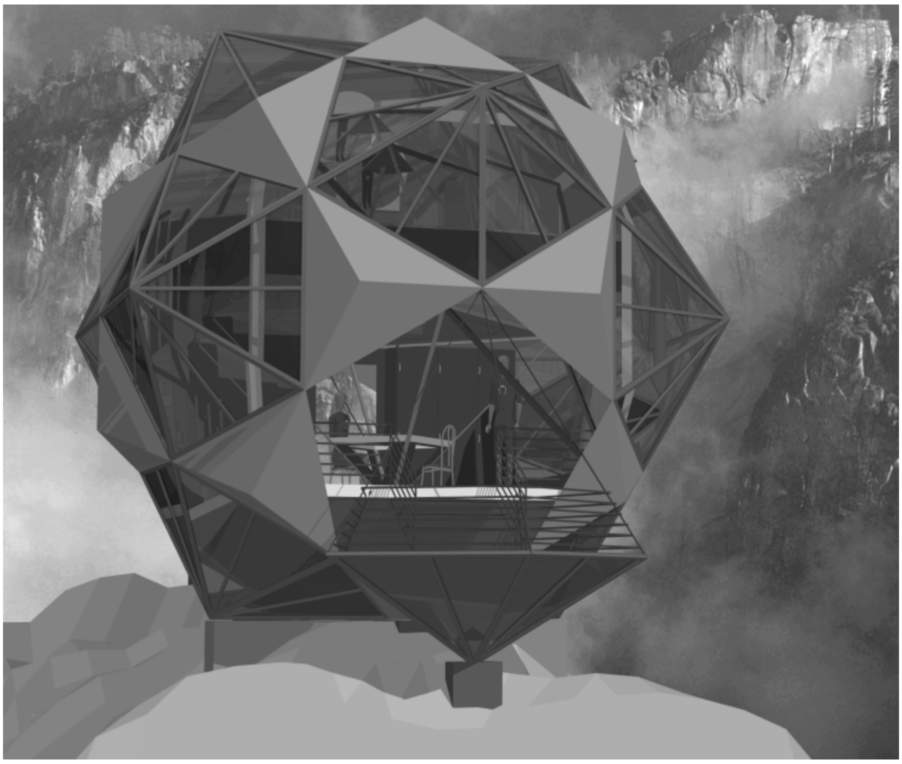

Figure 7. The "skin" of Pythagopod.

The design that follows from this is basically just a cube house. It is axially oriented, with the entrances being on the sides that have the horizontal edges, and the "exedras" of stair and fireplace (or television, depending on your theology) on the sides that have the vertical edges. I account for the asymmetry of the tetrahedron (in relation to the cube's axes) by choosing the handedness that works with access to the stair and the bathtub.

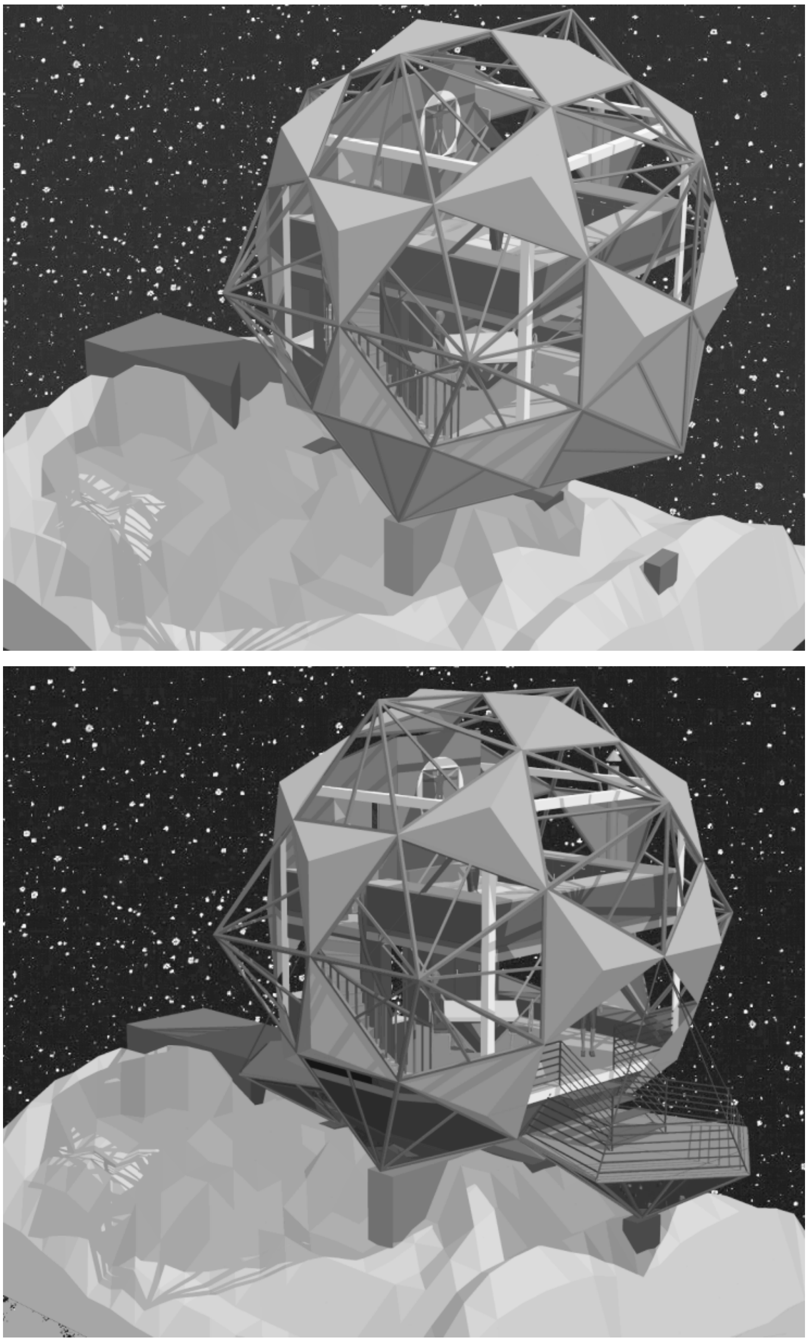

Figure 8. Views of the Pythagopod.

In the center of the downstairs is a square table whose inverted pyramid base follows the edges of the octahedron. Upstairs is the square bed, above which is a pyramidal ceiling light which is the top point of the octahedron, thus forming a virtual Masonic pyramid like the one on the Great Seal of the United States. The bedroom is open to the spaces below on three sides; the bathroom is tucked into the area over the entrance. The horizontal edges of the octahedron are flush with the ceiling plane below, but there is a glazed slot in the floor that allows the light to appear above as well. The diagonal edges of the octahedron penetrate walls, cabinets, and floors by passing through tiny tubes.

The base of the structure is a cross consisting of one edge of the dodecahedron and one of the icosahedron. They are, of course, in ϕ proportion.

A final word about the entrances is in order. I mentioned earlier the imagery of the Lunar Lander (technically the Lunar Excursion Module or LEM). In a similar way, the entrances of the Pythagopod are designed as hatches that lower to open and rise to close. The lifting cables are shown in the model, and when the hatches lift, the stairs and railing fold along with the hatch planes. So when secured, the shape is complete. When open, it is vulnerable (Figure 8).

For illustration, "snapshots" have been generated from the 3D model (Figure 9).

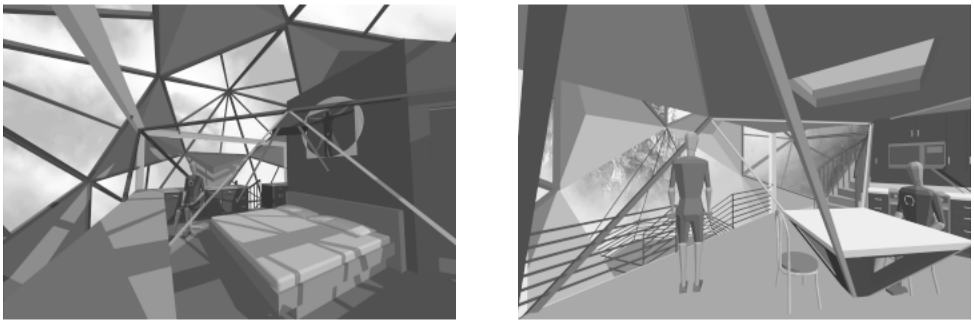

Figure. 9. "Snapshots" of the Pythagopod generated from the 3-D computer model.

As I stated at the outset, this exercise was an opportunity to explore the relationships among the five solids, and to play the game of fitting human functions into an abstract geometry. Whether the proportions generated by the presence of in the geometry make the form beautiful I leave to the beholder to decide.

As Plato has Timaeus suggest,

A man may sometimes set aside meditations about eternal things, and for recreation turn to consider the truths of generation, which are probable only; he will thus gain a pleasure not to be repented of, and secure for himself, while he lives, a wise and moderate pastime. Let us grant ourselves this indulgence…

Acknowledgment

The author acknowledges his debt to Buckminster Fuller, who is, after all, the inspiration for this project, even if not in the way he might have chosen.

Notes

1. It is interesting that these two triangles were the basis for drafting tools until the invention of the adjustable or protractor triangle. When I started out in the fifties, we all had to have a T-square and two triangles, and we had to learn how to draw other angles using combinations of those two.
2. Alternatively, of course, it could be a hint that the lost superior culture of Atlantis was the source of the knowledge of the physical world which Timaeus proceeds to explain.
3. His reference is in the following footnote: "Campanus of Novara states in a subtle verbal antithesis that the Golden Section (*proportionem habentem medium duo que extrema*) brings together the five regular bodies in a logical way (*rationabiliter*) but by a symphony ruled by an irrational (geometrical) proportion (*irrationali symphonia*)" [Ghyka: 43-44].

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About the Author

Christopher Glass is an architect with a one-person practice in coastal Maine. He attended Saint Albans School in Washington D.C., studied philosophy at Haverford College and architecture at Yale. He teaches an introductory architecture studio at Bowdoin College and is trying to cut back on professional work to spend more time playing with toys like the Pythagopod.