UNSOLVED PROBLEMS

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Let A be the Hopf algebra $k[x]$, k a field, with x primitive. The Hopf algebra dual A^0 is the space of linearly recursive sequences (B.Peterson and E.J.Taft, Aequationes Math. 20 (1980), 1-17). We gave an algorithm for diagonalizing a linearly recursive sequence in terms of the finite-dimensional subcoalgebra it generates. The product is that of divided-power series. Now let $W_1 = \mathrm{Der}\, k[x]$, the Witt algebra. (Take characteristic $k \neq 2$). The Lie algebra W_1 has basis $e_i = x^{i+1}\frac{d}{dx}$ for $i \geq -1$, with $[e_i, e_j] = (j-i)e_{i+j}$. W.Michaelis has recently shown that W_1 is a Lie bialgebra, with Lie coalgebra structure given by $\delta(e_n) = n(e_n \wedge e_{-1}) + (n+1)(e_0 \wedge e_{n-1})$, where $a \wedge b = a \otimes b - b \otimes a$ in $W_1 \otimes W_1$. This seems to be related to the fact that $e_0 \wedge e_{-1}$ satisfies the classical Yang-Baxter property in $W_1 \otimes W_1 \subseteq U(W_1) \otimes U(W_1)$. W_1 is a locally finite Lie coalgebra. Now W_1^0, the Lie coalgebra dual to the Lie algebra W_1, is also a Lie subalgebra of the convolution Lie algebra W_1^*, and W_1^0 is a Lie bialgebra. W.Nichols has recently shown that W_1^0 is the space of linearly recursive sequences.

Problem 1. *Is there an algorithm to compute $\delta(f)$ in $W_1^0 \otimes W_1^0$ for f in W_1^0? This is easy if $f = e_n^*$ in the dual basis, i.e., for the finite sequences. W_1^0 is not locally finite as a Lie coalgebra, so this may not be analogous to Δf in $A^0 \otimes A^0$.*

Problem 2. *Is there any relation between the (commutative, cocommutative) Hopf algebra A^0 and the Lie bialgebra W_1^0 (both of which are the space of linearly recursive sequences)? Since everything starts from $k[x]$, there should be some relation between the two structures.*

Problem 3. *Is there a quantum deformation of any of these structures - A, A^0, W_1 and W_1^0?*