

# Problems

1. (**Lee P.Y., University of Singapore**) Let the space of Henstock integrable functions on  $[a, b]$  be called the Denjoy space  $D$ . It is known that  $D$  can be normed by

$$\|f\| = \sup\{|\int_a^x f(t)dt| : a \leq x \leq b\}.$$

Given a set  $X$  in the Denjoy space  $D$  with the given norm, find conditions such that  $X$  is compact, or compact in some other sense. In other words, characterize compact sets in  $D$ .

2. (**Lee P.Y., University of Singapore**) We conjecture that if  $f_n$  for  $n = 1, 2, 3, \dots$ , and  $f$  are all Henstock integrable on  $[a, b]$ ,  $f_n(x) \rightarrow f(x)$  pointwise almost everywhere as  $n \rightarrow \infty$  and

$$\int_a^b f_n(x)g(x)dx \rightarrow \int_a^b f(x)g(x)dx \text{ as } n \rightarrow \infty$$

whenever  $g$  is of bounded variation on  $[a, b]$ , then the primitives  $F_n$  of  $f_n$  are ACG\* uniformly in  $n$ .

3. (**V.A. Skvortsov, Moscow State University**) Prove (or disprove) that the Perron integral, defined for an abstract derivative base by means of its major and minor functions continuous with respect to this base, is equivalent to the integral for the same base but defined without the precondition of continuity of major and minor functions. If the answer is negative, describe the class of bases for which this equivalence holds true.
4. (**S. James Taylor, University of Virginia**) Does there exist a non-countable set  $A \subset \mathbb{R}$  such that  $\forall B \subset \mathbb{R}$  with Lebesgue measure zero,  $\Lambda(A \times B) = 0$ ? ( $\Lambda$  denotes Hausdorff linear measure.) Is this true for Pfeffer 'small' subsets or 'slight' subsets? [It is known that, for each  $B$  with  $|B| = 0$ , there is a perfect set  $A$ , depending on  $B$ , such that  $\Lambda(A \times B) = 0$ .]