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Erratum

A Differential Geometric Criterion for Moishezon Spaces

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Moishezon Spaces

The purpose of this note is to correct an error in reference [1]. All of our notation and definitions will be consistent with that reference.

We recall the statement of Proposition 1b:

Proposition 1b. *Let $\mathscr{L}$ be a pseudonegative sheaf over a compact analytic space X. Let (S, U, h) be pseudonegative data for $\mathscr{L}$. If* $\operatorname{codim} S \geqq 2$, *then* $H^1(X, \mathscr{L}) = 0$ *and* $H^1(X, \mathscr{L}^* \otimes K(X)) = 0$.

Takeo Ohsawa has pointed out that our proof of this result is incorrect, because the harmonic projection operator is not support-preserving. At present, we have only the following result with which to replace Proposition 1b:

Proposition. *Let X be a compact analytic space. There exists a pseudonegative sheaf $\mathscr{L}$ over X such that $H^1(X, \mathscr{L}) = 0$.*

We will see in the proof that $\mathscr{L}$ can always be taken to be a sheaf of the type constructed in Proposition 3.

Proof. Let $\pi: Y \to X$ be a projective desingularization of X, and let E be the exceptional set in Y. If M is any negative line bundle over Y, then by Proposition 3, we know that $\pi_*(M \otimes \mathcal{O}(-E))$ is a pseudonegative sheaf on X. But let's assume that, in fact, M is so negative that $H^1(Y, M \otimes \mathcal{O}(-E)) = 0$. [For example, this will be true if $M \otimes \mathcal{O}(-E)$ is negative, by Kodaira's Vanishing Theorem.]

Let $\mathscr{L} = \pi_*(M \otimes \mathcal{O}(-E))$. We will show that $H^1(X, \mathscr{L}) = 0$. Let $\{U_i\}$ be a locally finite open cover of X, and let $\{\alpha_{ij}\}$ be a 1-cocycle with respect to this cover. We want to show that there exists a 0-cocycle $\{\beta_i\}$ with respect to our open cover such that $\alpha_{ij} = \beta_j - \beta_i$. But $\{\pi^{-1} U_i\}$ is a locally finite open cover of Y, and by definition of π_*, the cocycle $\{\alpha_{ij}\}$ gives rise in canonical fashion to a cocycle (which we denote $\{\alpha'_{ij}\}$) on $\{\pi^{-1} U_i\}$. Furthermore, since (as is well-known) the natural map from $H^1(\{\pi^{-1} U_i\}, M \otimes \mathcal{O}(-E))$ to $H^1(Y, M \otimes \mathcal{O}(-E))$ is one-to-one, there exists a

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cocycle $\{\beta'_i\}$ such that $\alpha'_{ij}=\beta'_j-\beta'_i$. But now we use the definition of π_* once more: $\{\beta'_i\}$ gives rise canonically to $\{\beta_i\}$, a 0-cocycle relative to $\{U_i\}$ which satisfies $\alpha_{ij}=\beta_j-\beta_i$.

References

1. Frankel, R.: A differential geometric criterion for Moishezon spaces. Math. Ann. **241**, 107–112 (1979)

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