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# Correction and Remark to "ON STABLE AND UNIFORM RANK-2 VECTOR BUNDLES ON $P^2$ IN CHARACTERISTIC p"[1)]

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Correction: Lemma 3.3 of the paper is incorrect: the degree of inseparability of r in general does not divide d(E). An example is due to Hartshorne (private communication): Take a suitable elementary transformation of a Frobenius pull-back of a stable bundle of generic splitting type (0,-1) on $P^2$. Hence Theorem 3.1 has to be modified. The proof apart from Lemma 3.3 gives:

Theorem 3.1: Let E be a stable rank-2 vector bundle on $P^2$ of generic splitting type (0,-d) and $d>0$. If $r: P(T_p 2) \longrightarrow P(E)$ denotes the canonical rational map associated to E, then r is finite and $d \leq$ deg insep r.

Theorem 3.1 may be considered a generalisation of the theorem of Grauert-Mülich to arbitrary characteristics, since in characteristic 0 it just says $d \leq 1$ (which is Grauert and Mülich's theorem) and moreover the estimate is best possible (for the Frobenius pullbacks of the twisted tangent bundles one has equality).

Remark: To get in characteristic $p>0$ an estimate for d in terms of the Chern classes of E proceed as follows: -deg r can be interpreted as the first Chern class of $q_* p^* E$. Compute the second Chern class of $p^* E$ (suggested by Barth)

1) H. Lange: On stable and uniform rank-2 vector bundles on $P^2$ in characteristic p, manuscripta math. 29, 11 - 28 (1979)

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to get an estimate for deg r. The following inequality is best possible as the bundles $F^{n*}T_{P^2}$ show:

<u>Corollary</u>: <u>Let</u> E <u>be a stable rank-2 vector bundle on</u> $P^2$ <u>of generic splitting type</u> (a,b) <u>and</u> $\Delta := c_1(E)^2 - 4c_2(E)$. <u>Then</u>

$$|a - b| \leq \sqrt{\frac{-\Delta}{3}}$$

Since the restriction of a semistable rank-2 vector bundle on $P^n$ to a general plane is semistable, this result is correct also for rank-2 bundles on $P^n$.

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