MASS AND ORBIT ESTIMATION OF PLANET X VIA A FAMILY OF COMETS

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(Received 8 October, 1990)

Abstract. The crucial assumption of this paper is, that the observed clustering of aphelion distances of intermediate-period comets in the 70–90 AU range is due to the influence of a tenth planet, called Planet X. We contribute to the search for Planet X a new and extended evaluation of a family of comets assumed to be Planet X's family of comets.

By averaging the aphelion distances of comets that belong to a transplutonic family of comets, we get Planet X's semi-major axis $a_X = (83.0 \pm 5.3)$ AU. The comets' orbits also yield the upper limit of the planet's orbital eccentricity $e_X \simeq 0.019$. If this planet played an important part in 'sending' quasi-periodic comet showers to the inner solar system, we can calculate its orbital inclination $i_X = 46\overset{\circ}{.}1 \pm 3\overset{\circ}{.}6$. By distributing all planets' masses into the heliocentric, torus-like zones, in which they were formed, we get the density distribution of the primordial solar nebula. Extrapolating this distribution we find the mass of the planet $M_X = (5.1\,^{+3.6}_{-2.4})\,M_{\text{Earth}}$. A few plausible assumptions (e.g. Uranus and Neptune perturbations being caused by Planet X) lead to Planet X's actual location with declination and ecliptic longitude being $\delta = 57^\circ \pm 17^\circ$ and $\lambda = 54^\circ \pm 34^\circ$, respectively (1989.5 position). In addition, we give Planet X's apparent brightness dependent on its unknown albedo. All those properties and predictions are more or less in agreement with earlier work on Planet X.

1. Introduction

The interest in Planet X – or, as sometimes called, Transpluto – has increased in recent years:

Advanced orbit calculations allow predictions of the actual location of Planet X by evaluating Uranus's and Neptune's positions supposing that Planet X perturbed these planets (e.g. Harrington, 1988). New research on the stability of the planetary system lead to a maximum mass and a minimum heliocentric distance of Planet X in order to exclude major perturbances of Pluto and Neptune (Jackson *et al.*, 1988). In the course of the IRAS sky survey, many small infrared sources were discovered, one of which may be a still unknown planet. Artificial spacecrafts are reaching the transneptunian region. The fact, that Pioneer 10 has not been disturbed by any unknown object, leads to an upper limit of Planet X's mass (Anderson and Standish, 1986). By assuming, that Planet X is the source of a periodic flux of short period comets, one can calculate orbital parameters of the planet (e.g. Matese and Whitmire, 1986).

Schütte (1949, 1965) found a transplutonic family of comets consisting of eleven comets with aphelion distances between 73.1 AU and 102.5 AU. He postulated that this family of comets is 'Planet X's family of comets' and obtained the planet's semi-major axis by averaging the comets' aphelion distances: (88.4 ± 9.5) AU.

Earth, Moon, and Planets **54**: 193–202, 1991.

TABLE I

The transplutonic family of comets

Comet number	Roman numeral designation	Name	Aphelion distance Q in AU
0	1937 II	Wilk	64.87
1	1905 III	Giacobini	73.1
2	1857 IV	Peters	75.52
3	1855 II	Donati	79.26
4	1932 X	Donnell-Forbes	80.77
5	1885 III	Brooks	83.73
6	1840 IV	Bremikov	85.43
7	1932 V	Peltier-Whipple	86.75
8	1979 X	Bradfield	87.71
9	1932 I	Houghton-Ensor	88.76
10	1874 IV	Coggia	89.12
11	1941 II	Friend-Reese-Honda	99.37
12	1931 III	Nagata	99.63
13	1955 III	Mrkos	99.93
14	1979 IX	Meier	105.43
15	1964 VIII	Ikeya	106.2

With an improved data set we repeated and extended this work. We present new estimates of mass, brightness, and orbital parameters, which are of particular interest in order to make predictions that could lead to the discovery of Planet X.

2. Method and Data

During close encounters, comets and planets interact due to their gravitational attraction. Since a planet's mass is by far larger than a comet's mass, the comet's orbit will be changed. In the course of a close encounter of a comet with a planet, the planet can actually 'catch' the comet, altering the comet's aphelion distance to become almost equal to the planet's heliocentric distance. Jupiter is known to have caught approximately 70 comets. These comets are members of the 'Jupiter-family of comets'. Saturn, Uranus, and Neptune have families of comets, too.

Therefore, comets with almost the same aphelion distances are candidates for a family of comets caught by one single planet. If Planet X exists, it may have caught a few comets, too. By looking for such a family of comets, we may find its approximate heliocentric distance. To a first approach, the semi-major axis of Planet X is equal to the average aphelion distance of the comets.

In Table I, we list all known comets with aphelion distances between 60 AU and 107 AU. Aphelion distances Q are calculated with perihelion distances q and eccentricities e, $Q = q(1 + e)/(1 - e)$. Roman numeral designations, names, q, and e follow the nomenclature and listing of Marsden (1982). Comets 1, 2, 4, 5, 7, 9, 10, 11, 12, 13, and 15 are the comets Schütte listed in 1965, when comet Ikeya's (comet 15) aphelion distance was thought to be 93.0 AU, but this distance has been revised to 106.2 AU. Therefore, a gap appears between comets 10 and 11 spanning 10.2 AU. Hence, comet 10 is the outermost comet of this family. Comets 11 to 16 are not part of this transplutonic family. The family's innermost comet is comet 1, because there is a gap of 8.2 AU between comets 0 and 1. Those two gaps appear as very large as the largest gap between two neighbouring comets in the remaining family is 3.7 AU.

Though comet aphelion vectors in the 70–90 AU range seem to be randomly distributed, it could be possible that one single object, e.g. Planet X, has intersected the original orbits of all ten comets of the family in order to alter their orbits. However, this is possible only if that object circles the Sun in a highly eccentric and inclined orbit. If that is the case, comets 1 to 10 constitute a family of comets, Planet X's family of comets.

3. Orbital Parameters of Planet X

3.1. HELIOCENTRIC DISTANCE

We obtain Planet X's semi-major axis a_X by averaging the aphelion distances of the ten comets of Planet X's family (see Table I). The result is

$$a_X = (83.0 \pm 5.3)\ \text{AU}\,. \tag{1}$$

N.B.: Our value is in good agreement with the so-called Titius-Bode law. According to this 'law', the distance of the n-th planet is given by $a_n = (0.4 + 0.3 \times 2^n)$ AU. Inserting $n = 8$ yields $a_8 = 77.2$ AU. Some authors (e.g. Neuhäuser and Feitzinger, 1986) did point to the fact, that the distance of Planet X Schütte got (Schütte: $a_X = 88.4$ AU) does agree with the Titius–Bode law.

3.2. ORBITAL ECCENTRICITY

The aphelion distances Q_k ($k = 1, 2, 3, \ldots, 10$) of the ten comets also yield the planet's eccentricity assumed to be equal to the maximal difference between the comets' aphelion distances and the planet's semi-major axis, i.e.,

$$e_X = \frac{\operatorname*{Max}_k\{|Q_k - a_X|\}}{a_X}\,. \tag{2}$$

With the maximal value of $|Q_k - a_X|$ being 9.9 AU, we find that

$$e_X = 0.1193 \pm 0.0076\,. \tag{3}$$

Of the known planets, only Mercury and Pluto have larger eccentricities. Using Equations (1) and (3), we get for Planet X a perihelion distance of $q_X = (73.1 \pm 4.9)$ AU, an aphelion distance of $Q_X = (92.9 \pm 6.0)$ AU, and a period of (756 ± 72) years.

It must be mentioned here that, by this method, we can calculate the planet's eccentricity only to a first approach. The same method applied on the Jupiter family of comets yields an eccentricity of 0.13, whereas the real value is 0.048. Our method of calculating the eccentricity just gives an upper limit.

3.3. Inclination

To assess the planet's orbital inclination, we make the following assumption: There are impacts on the Earth (Alvarez and Muller, 1984) due to comet showers, which seem to peak every 39 Myrs (Matese and Withmire, 1986). If Planet X is the source of this periodicity, we can calculate its inclination. The peaks may be due to interactions of Planet X with a disk of comets or planetesimals lying beyond Neptune's orbit. Planet X's perihelion and aphelion points precess, and cause the peaks in comet showers when the planet penetrates most deeply into the disk. We assume a perihelion precession period of $T = 60$ Myrs (Matese and Whitmire, 1986).

Kozai (1959) developed from a general potential expression a formula for the precession rate, connecting semi-major axis a, eccentricity e, inclination i, and precession period T in a unique relation. Neglecting octopole and higher terms, the following equation of Kozai

$$a = \left[\frac{3}{2} T \left(\sum_j m_j a_j^2\right)(1 - e^2)^{-2} \left(\left|1 - \frac{5}{4} \sin^2 i\right|\right)\right]^{2/7} , \tag{4}$$

with planet masses m_j (in units of mass of sun) and semi-major axes a_j (in AU), where we treat the outer planets as rings with radii equal to their semi-major axes (see Matese and Whitmire, 1986), leads to

$$i_X = \arcsin \left[\frac{4}{5} - \frac{8}{15}\, a_X^{7/2} (1 - e_X^2)^2 \left(T \sum_j m_j a_j^2\right)^{-1}\right]^{1/2} . \tag{5}$$

Since m_j and a_j are known for all planets ($j = 1, 2, 3, \ldots, 9$ for Mercury, Venus, ..., Neptune, and Pluto; $j = X$ for Planet X), we obtain with the above results for a_X and e_X the value of

$$i_X = 46\overset{\circ}{.}1 \pm 3\overset{\circ}{.}6 . \tag{6}$$

Though no known planet has such an high-inclination orbit, some planetoids do have highly inclined orbits, e.g. Pallas 34°, Hidalgo 43°, Betulia 52°. High-inclination orbits are no rare exceptions in our planetary system.

4. Mass of Planet X

To calculate Planet X's mass, we use the present day picture of the formation of our solar system. Neighbouring planetesimals competed for the material from which they were formed. Every planetesimal/planet had a feeding-zone in which it alone picked up matter. In this heliocentric, torus-like zone, no other major object could exist. Planetesimals too close together either coalesce or cease to

TABLE II

The planet' feeding-zones

Planet (or group)	Semi-major axis a/AU	Orbital ecc. e	Zones boundaries Q/AU	q/AU	Mass M/M$_E$	Density log(ρ/M$_E$ AU^{-3})
Inner planets	0.909	–	–	–	–	−0.988
Jupiter	5.203	0.048	4.953	5.453	318.00	0.271
Saturn	9.539	0.056	9.005	10.073	95.22	−1.108
Uranus	19.182	0.047	18.280	20.084	14.55	−2.770
Neptune	30.057	0.009	29.786	30.328	17.23	−2.553

grow. The size of a feeding-zone depends on the orbital eccentricity. If a_j is the semi-major axis of the j-th planet and e_j its present eccentricity – supposed to be equal to the eccentricity during the planet's formation – , the inner and outer boundaries of the feeding-zone are given by the planet's perihelion and, respectively, aphelion distances:

$$q_j = a_j \times (1 - e_j) \text{ and } Q_j = a_j \times (1 + e_j)\,. \tag{7a\&b}$$

The feeding-zone boundaries of the Jovian planets are listed in Table II. The planetary mass M_j distributed into the appropriate feeding-zone gives every feeding-zone's density

$$\rho_j = M_j \frac{3}{4\pi} (Q_j - q_j)^{-1}\,. \tag{8}$$

Because of the small eccentricities and masses of the inner planets, we give only one single value for the density in the region where Mercury, Venus, Earth, and Mars were formed; the semi-major axis given and the value for log ρ are averaged. Pluto is left out because of its small mass and its unknown origin (satellite of Neptune?).

The densities of the five successive feeding-zones as given in Table II give, to a first approach, the radial density distribution within the primordial disk around the protosun. We extrapolate to Planet X's feeding-zone to find mass M_X of Planet X, boundaries and volume of this feeding-zone are obtained using Equations (1), (3), and (7a&b).

Since accepted theories on the formation of the solar system indicate that the density in the primordial solar nebula decreased exponentially with distance, we connect the five points in the log ρ_j versus a_j plot with a straight line (least mean-square fit). For Planet X, three feeding-zones are evaluated:

Maximum mass $M_X^{\max}$ will result for smallest distance – i.e., highest density, ($a_X = 77.7$ AU) and largest eccentricity ($e_X = 0.1269$) – i.e.,

$$\text{'maximum' feeding-zone: } q_X^{\max} = 67.8 \text{ AU and } Q_X^{\max} = 87.6 \text{ AU}\,. \tag{9a}$$

Analogously, minimum and medium mass and, hence, boundaries of minimum and medium feeding-zones result, to be noted in equations (9b&c), which we do not give explicitly in this article, since they are trivial.

With the help of our log ρ_j versus a_j plot (not included in this article, since it is trivial, too) we determine for those three zones the following densities (with M_E being the mass of Earth):

$$\rho^{\mathrm{max}} = 5.821 \times 10^{-6}\, M_E\, \mathrm{AU}^{-3}\,, \tag{10a}$$

$$\rho^{\mathrm{min}} = 1.412 \times 10^{-6}\, M_E\, \mathrm{AU}^{-3}\,, \tag{10b}$$

$$\rho^{\mathrm{med}} = 2.985 \times 10^{-6}\, M_E\, \mathrm{AU}^{-3}\,. \tag{10c}$$

Equations (8), (9a,b,c), and (10a,b,c) give the mass estimate of Planet X as

$$M_X = (5.1\,^{+3.6}_{-2.4})\, M_E\,. \tag{11}$$

This mass is smaller than the masses of all planets known to have a family of comets. A planet with small orbital velocity and small mass is less likely to be able to capture comets than Jovian planets are. But M_X is significantly larger than cometary masses; hence, the crucial gravitational effect of capture is not impossible.

5. Actual Location and Brightness of Planet X

Since all Uranus positions obtained since 1830 cannot be fit with one single ephemeris, but all (post-discovery) data for Neptune positions can be fit (Seidelmann *et al.*, 1986), it is possible that an unknown body such as Planet X has 'shaped' the current orbit of Uranus (Harrington, 1986), but not Neptune's orbit; or, at least, Uranus has been perturbed much more than Neptune (Anderson and Standish, 1986).

If we assume that Planet X caused the inconsistencies in the Uranus data, these inconsistencies indicate that Planet X may have made its last close approach to Uranus in the 1750's (Harrington, 1986). In those years, the separation between Uranus and Neptune was a maximum. If Planet X made a very close approach to Uranus during those years, it would have perturbed Uranus much more than Neptune. We conclude, that Planet X may have crossed the ecliptic plane in 1750 (to be close to Uranus) being in heliocentric conjunction with Uranus (to be very close to Uranus). At that time, the ecliptic longitudes λ of Uranus and, thus, Planet X, too, were $\lambda = 300°$. We also assume, that Planet X passed its perihelion point in 1750, so that the separation between Planet X and Uranus was minimal and, hence, the perturbation maximal.

Since a semi-major axis $a_X = (83.0 \pm 5.3)$ AU corresponds to an orbital period of (756 ± 72) years, the last aphelion passage of Planet X would have taken place in 1939 (± 72).

Due to its high-inclination orbit, Planet X was, at that time, either at $\delta = (i_X +$ angle of inclination of Earth's orbital plane) $= 46\overset{\circ}{.}1 \pm 3\overset{\circ}{.}6 + 23\overset{\circ}{.}5 = 69\overset{\circ}{.}6 \pm 3\overset{\circ}{.}6$ (during northern summer, i.e. in 1939.5) or at $\delta \simeq -46.^\circ 1 \pm 3.^\circ 6 + 23.^\circ 5 = -22.^\circ 6 \pm 3.^\circ 6$ (in 1939.5, southern winter) depending on whether Planet X was/is closing its ascending or descending node, respectively, in this century.

But Lowell and Tombaugh, who were intensively searching for a new planet in the first half of this century, surveyed only those parts of the (northern) sky, which are close to the ecliptic plane (see e.g. Hoyt, 1980) and, hence, could not detect Planet X.

As outlined above, we assume that Planet X passed both the ecliptic plane and its perihelion point in 1750. To obtain the actual location of this planet, we also have to take into account its semi-major axis, eccentricity, inclination, and orbital period. Considering the two possible cases (descending or ascending node in the 1700's), we get the following positions, one of which should be the actual location of Planet X in 1989.5: ($\delta = 57^\circ \pm 17^\circ$ and $\lambda = 54^\circ \pm 34^\circ$) or ($\delta = -11^\circ \pm 17^\circ$ and $\lambda = 234^\circ \pm 34^\circ$). Due to the high uncertainty of orbital parameters and since our assumptions on perihelion and ecliptic plane passages are unproven, our prediction is highly speculative.

Harrington (1988) analyzed perturbations in the orbits of Uranus and Neptune produced by an unknown Planet X including all position data available through 1982. He chose a large number of value sets for mass and position vector of Planet X and checked, whether the computed residuals of Uranus and Neptune were in agreement with the observed ones. He varied the ecliptic longitude from 1 to 24 h, the ecliptic latitude from -45° to $+45^\circ$, and the mass from 3 to $5\,M_E$. 183 test cases fit with the Uranus data, no case fit with the Neptune data. The 1988 heliocentric position of Planet X according to his best fit solutions cluster in two regions: Right ascension $3h < \alpha < 7h$ and declination $-10^\circ < \delta < 50^\circ$ and right ascension $14h < \alpha < 21h$ and declination $-70^\circ < \delta < -10^\circ$. The two possible positions we predicted lie in corners of those clusters. In a similar way, using opposition normal points of residuals of Uranus, Powell (1989) evaluated the perturbation of Uranus by Planet X. According to his very best fit, the present location of Planet X is $\alpha = 10.6\,\mathrm{h}$ and $\delta = 16^\circ$. The brightness of a planet depends, among other properties, on its radius, i.e. on its mass and density. Since only the mass of Planet X is known, we have to make assumptions on the density. The masses of all known Jovian planets are larger than the assumed mass of Planet X ($M_X \simeq 5.1\,M_E$) and the masses of all known terrestrial planets are lower. Hence, we cannot decide whether Planet X is of terrestrial or Jovian nature. Therefore, we distinguish two cases. We use the average density D of the known terrestrial (respectively Jovian) planets as Planet X's density, i.e.:

$$D_X^{\mathrm{terr}} = 1.454 \times 10^{-11}\,M_E\,\mathrm{km}^{-3} \text{ and } D_X^{\mathrm{Jov}} = 3.533 \times 10^{-12}\,M_E\,\mathrm{km}^{-3}.$$

If Δ is the distance between Planet X and the Earth (at 1989 opposition: $\Delta = (1.3 \pm 0.2) \times 10^{10}\,\mathrm{km}$), a_X the heliocentric distance of Planet X ($a_X = (87.7 \pm 15.1)$ AU in 1989), Λ_X and D_X the albedo and density, respectively, of

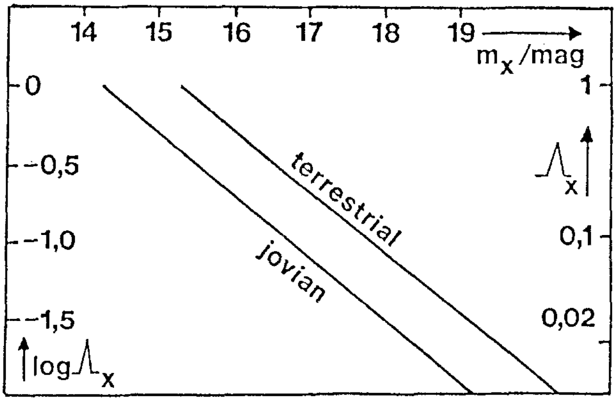

Fig. 1. Brightness of Planet X. The brightness of Planet X depending on its albedo Λ_X for the two cases (Planet X being either terrestrial or Jovian), as calculated with Equation (12). See text for details.

the planet, and $m_\odot = -26^{\mathrm{m}}.74$ the sun's apparent brightness, then the apparent opposition brightness of Planet X, m_X, in 1989 is given by

$$m_X = m_\odot + 5 \log \left[\frac{\Delta a_X}{\Lambda_X^{1/2}} \left(\frac{3M_X}{4\pi D_X} \right)^{-1/3} \right]. \tag{12}$$

Since the albedo is unknown, we present Planet X's brightness as a function of its albedo, $m_X = m_X(\Lambda_X)$, for both cases (Planet X as a terrestrial or Jovian planet). As Figure 1 shows, the brightness of Planet X lies somewhere between 14^{m} and 20^{m}, or, if one assumes Planet X's albedo to be similar to the albedo of cool objects in the outer solar system (e.g. cometary nuclei, Uranus' rings), i.e. $\Lambda_X = 0.02$, between 18^{m} and 20^{m}. Thus, this planet should be detectable in the course of a careful search if its diameter is not too small.

The assumed density of Planet X (D_X^{ter} and D_X^{Jov}, see above) and M_X yield the diameter of the planet to be $(8.7 \pm 1.7) \times 10^3$ km, respectively $(12.8 \pm 1.3) \times 10^3$ km if Planet X is Jovian. I.e., as seen from Earth, its apparent opposition diameter in 1989.5 would be approximately $0''.14 \pm 0''.05$ ($0''.20 \pm 0''.05$ respectively).

6. Discussion

Rawlins and Hammerton (1973) took into account Lalande's pre-discovery observation of Neptune in May 1795 and all post-discovery observations of both Uranus and Neptune to find position and mass of a new planet that would fit with those data. They found that the mass of a planet less distant than 100 AU should be limited to a few M_E.

Jackson and Killen (1988) recently analysed the stability of Pluto's orbit. So far, Pluto's (and Charon's) orbit(s) seem(s) to be stable, they survived in a 3:2 resonance with Neptune. If an unknown planet would disturb their orbit, close encounters could become possible leading to an unstable orbit of Pluto. Jackson *et al.* looked at four different cases: A Planet X with ($M_X = 0.1\,M_E$, $a_X = 48.3$ AU, $e_X = 0$, $i_X = 8\overset{\circ}{.}79$) or ($M_X = 1.0\,M_E$, $a_X = 75.5$ AU, $e_X = 0.265$, $i_X = 15\overset{\circ}{.}37$) has almost no influence on Pluto and Neptune, but a planet with $M_X = 5\,M_E$, $e_X = 0.3$, $i_X = 45°$ and either $a_X = 52.5$ AU or $a_X = 62.5$ AU would disturb Pluto and Neptune and leads to close approaches. Pluto's orbit would not be stable. They conclude that an unknown body with a mass of 5 M_E must be more distant than 75 AU.

Anderson and Standish (1986) evaluated the path of spacecraft Pioneer 10, apparently undisturbed by any unknown object. If there is a Planet X (or a transplutonian/-neptunian belt of comets/planetesimals), its mass has to be less than 5 M_E

Matese and Whitmire (1986) analyzed orbital parameters of Planet X assuming that Planet X is the source of the periodic flux of short period comets as briefly discussed in Chapter 3. From the comet shower life time and the angular thickness of the disk around β Pic – believed to be similar to the primordial solar nebula around the protosun – they deduced Planet X's eccentricity to be $0.1 \leqslant e_X \leqslant 1 - 0.1 i_X$. They picked the value $e_X = 0.3$. A detailed discussion of Equation (4) (with $T = 30$ Myrs) leads to limits for i_X, $25° \leqslant i_X \leqslant 63\overset{\circ}{.}4$, and for a_X, $50 \text{ AU} \leqslant a_X \leqslant 100$ AU. For $e_X = 0.3$, they found $i_X = 45°$ and $a_X = 80$ AU.

The contribution of Planet X to the net cratering rate yielded Planet X's mass $M_X \geqslant 1\,M_E$. Evaluating Shoemaker's and Wolfe's (1986) work on the interaction of a $10\,M_E$-Planet X with 500 planetesimals (with $a_X \simeq 90$ AU, $e_X \simeq 0.1$, and $i_X \simeq 5°$), Matese and Whitmire found the upper limit of M_X to be 5 M_E. Harrington and Van Flandern (1979) evaluated a close encounter of Planet X with the primordial Neptune satellite system. In this scenario, Planet X ejected Pluto and Charon leaving Triton in its retrograde orbit and Nereid in its highly eccentric orbit. Hence, Planet X's mass would have to be between 2 and 5 M_E, its distance between 50 and 100 AU and its eccentricity $\leqslant 0.6$.

Acknowledgements

It is an honour to thank Professor Schmidt-Kaler of Ruhr University Bochum for helpful suggestions during research, and Dr. Harrington for critical comments and remarks during formulation of our results.

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