Abstract

This chapter describes a history of the kinematics and dynamics in robotics. First, the progress of four basic computation schemes, namely, forward kinematics, inverse kinematics, forward dynamics, inverse dynamics, and identification of mass properties are briefly reviewed from the viewpoint of both theory and algorithm. Then, some particular issues for anthropomorphous systems are summarized. Mobile robots are commonly modeled as a floating-base kinematic chain, which was first adopted in space robotics. It explains the relationship between the net external forces and the total momentum of the system and shows an importance of the contact dynamics, which is also addressed in the following section, in order to exploit the structure-varying nature. Some representative ground references that are helpful for dealing with contact constraint in a context of control are introduced. Reduced-order models to abstract the dominant dynamics of the system in an intuitive and comprehensive manner are also presented with the techniques for reciprocal mapping of motion and input with the full-scale model. Finally, possible future directions are discussed.

Keywords

Forward kinematics · Inverse kinematics · Forward dynamics · Inverse dynamics · Differential inverse kinematics · Floating-base system · Structure-varying kinematic chain · Reduced-order model
1 Introduction

Humanoid robots are artifacts that are expected to act intelligently as if it were humans. Although it is technically hard to define intelligence, we certainly recognize it in ourselves when we reasonably and robustly orchestrate behaviors by purposefully articulating our bodies under the severe natural laws. To consider the kinematics and dynamics is the basis for anything in the humanoid robotics since it provides the way to describe robot behaviors mathematically, to know what behaviors are feasible or infeasible in the real world, and to embody intelligence in one’s thoughts.

It seems that people all over the world have had attempts to build humanlike machines since ante-Christum. One may still see many sketches and drafts of ancestors of humanoid robots by Leonardo da Vinci (1452–1519, Italy), Pierre Jaquet-Droz (1721–1790, France), and Yorinao Hosokawa (1730?–1796, Japan), for example. As we can imagine that they were carefully designed based on the modern mathematics in those days, which covers methods to represent anthropomorphic mechanisms, they are not only the roots of actual robots but also that of theory of humanoid motion computation.

Though the study of kinematics and dynamics has a long history as the above tells, this chapter focuses on the advancement of theory and algorithm in the contemporary robotics and overviews issues that are particularly related to humanoid robots.

2 Robot Kinematics and Dynamics Revisit

Four basic computations, namely, forward kinematics, inverse kinematics, forward dynamics, and inverse dynamics, have been studied as fundamental problems concerning with the robot motion.

The forward kinematics means to map motion of joints to that of effectors. From the beginning of robotics in 1940~1950s, vectors and matrices have been broadly used for it as they can conveniently relate the motion of joints to that of the entire system in Cartesian space based on the chained transformation. A minimal notation invented by Denavit and Hartenberg [1] (DH representation) often works. It is no longer a problem as long as the mechanism does not have closed-loop structures.
In contrast to the above, the inverse kinematics, in which the desired motion of the effectors is converted to the (hopefully equivalent) motion of joints, is a difficult problem. In the 1960s, it was proven to be analytically unsolvable in general due to the fact that the polynomial equation in higher than fourth order does not have the solution formula. Accordingly, numerical methods should be used in many cases. In 1969, Whitney [2] pointed out a favorable fact that the instantaneous relationship between the velocity of the effector and that of joints is represented by a linear equation and showed later an efficient computation method of the coefficient matrix of the equation, which is also known as the manipulator Jacobian matrix [3]. This led to an idea to apply Newton-Raphson method or its variation to the inverse kinematics and also to directly control the velocity of a manipulator in online (differential inverse kinematics/resolved motion rate control). Redundancy and singularity have been the central issues of discussion in the latter problem and actively studied in the 1980s [4–7].

In terms of the dynamics, the equation of motion represented in the vector-matrix form was introduced in the 1960s by Uicker [8] and Kahn [9]. In the 1970s, an efficient recursive computation of the inverse dynamics was delivered by Stepanenko and Vukobratović [10] and Orin et al. [11], where the chained kinematics was again exploited to compute the required driving force from the desired acceleration. In 1980, it was followed by recursive Newton-Euler formulation by Luh et al. [12] and recursive Lagrangian formulation by Hollerbach [13], both of which are now acknowledged.

Comparing with the other three, the first solution to the forward dynamics, in which the acceleration of the robot is predicted from the applied driving force, was presented later. In 1982, Walker and Orin [14] proposed several algorithms, which were with $O(n^3)$ for $n$-DOF robots. Though Featherstone [15] invented a recursive $O(n)$ algorithm in 1983, it took longer time for computation than the above $O(n^3)$ methods in the case of robots with less than nine joints, while most of industrial robots have six joints. The situation changed in the 1990s due to two events. One is that the parallel computing technology has become easily accessible, and the algorithm has been improved to be $O(\log n)$ [16, 17]. The other is that humanoid robots with tens of joints have been brought into the reality and the algorithm with less computation complexity made sense.

In addition to the above four computation schemes, a new aspect of the equation of motion was found by Mayeda et al. [18] and Atkeson et al. [19] in the 1980s. The actuation forces are related with the mass properties, namely, a set of mass, the center of mass multiplied by mass, and the moment of inertia of links, in a linear form where the coefficient matrix (regressor matrix) includes velocities and accelerations. It means that the mass properties of a robot can be identified from a collection of motion loci and associated actuation forces through the linear least square regression.
3 Kinematics and Dynamics of Floating-Base System

When Vukobratović and Stepanenko [20] studied a full-scale anthropomorphic kinematic chain in 1973, they already noticed that it was different from ordinary manipulators at the point that it lacked mechanical connection to the environment and the base link of the chain floated to the air. In other words, the base link is connected to the inertial frame via fictitious 6-DOF unactuated joints that yield free translation and rotation. Such floating-base systems are necessarily underactuated.

This is not a particular property for humanoid robots but a common structure for several types of robots – satellite-type manipulators might be the representative ones, which were discussed in the 1980s [21, 22]. The dynamics of the floating base in the gravity-free space is ruled by the momentum conservation law.

In 1997, Yoshida et al. [23] summarized some common characteristics in this class, where the dynamics coupling between the base motion and limbs was the central issue of discussions. Fujimoto and Kawamura [24] also modeled a humanoid robot as a floating-base system in 1998 and focused on the partial equations corresponding to the absent actuation forces to the base link. They imply a simple fact that the total linear and angular momenta are no longer conserved under the presence of the external forces. More importantly, the rate of them changes only due to the net external forces and is not affected by the internal forces including actuation forces. When the system is in contact with the environment, the external force is caused by the reaction of the internal force. In this sense, the external force can be regarded as the indirect actuation force.

It is largely different from the satellite robots that legged robots including humanoid robots move in the gravitational field and are pushed to the ground. Since the contact with the ground is not permanent as explained in the next section, they are always exposed to risks of falling (note that falling is still ill-defined). The key issue to prevent it is how to achieve an effective exchange from the (direct) actuation forces to external forces, which includes how to choose supporting points in the environment. This idea had become a basis of motion planning and control.

4 Contact Dynamics and Ground References

The way how the humanoid robot interacts with the environment frequently changes during a motion, an alternation of the single-support and the double-support during a walk, a typical walking cycle with heel-strike and toe-off, transitions between standing-up and sitting-down postures, leaning on a wall or a handrail, etc. Nakanamura and Yamane [25] modeled a humanoid robot as a structure-varying kinematic chain that enables such discontinuous transformations of topology collocating with the environment.

The structural changes are triggered when contacts occur or vanish. While the theory of contact dynamics was developed in much earlier era by Coulomb (1736–1806, France), Hertz (1857–1894, Germany), and other giants, an idea to formulate
the model of frictional contact of rigid bodies as the linear complementarity problem was proposed in 1984 by Lötstedt [26], It figures out that consistent combinations of contact forces and velocities of contact points compose a complex set represented by a complementarity condition, which comes down to a linear complementarity condition, provided the kinetic friction coefficient coincides with the maximum friction coefficient and the friction cone is approximated by a pyramidal cone. Many following works [24, 27, 28] have been made based on this approach in order to develop techniques to simulate multibody systems.

In the context of motion planning and control, Hirukawa et al. [29] proposed an idea to check if the supposed motion is physically feasible based on the contact wrench sum (CWS), namely, the set of net wrenches of any possible contact forces with respect to the given allocation of contact points under the constraints of unilaterality and friction limit. If the desired net wrench that is equivalent with the desired acceleration is within CWS, there exist some combinations of contact forces that produce it, so that the real robot can move as desired. This can be seen by solving a linear programming problem.

While the above method is systematic, the computation cost always concerns in practice. It is also a problem how the preferable motion should be synthesized if the originally desired motion is physically infeasible. From this viewpoint, some useful ground references have been proposed. They focus on the unilaterality constraint and do not deal with the friction limit and, thus, are incomplete as a substitute of CWS. However, they are intuitively associated with the whole-body motion.

In 1968, McGhee and Frank [30] explained that a legged robot does not fall as long as the ground projection of the center of mass (GCOM) is within the convex hull of all the contact points, which are supposed to be on an identical planar ground (the base of support/the supporting region). The stability margin was defined as the distance from the boundary of the base of support to GCOM. Though the above is not strict since the effect of inertial force is omitted, it approximately gives a good measure of stability for quasi-static motions.

In 1972, Vukobratović and Stepanenko [31] defined the zero-moment point (ZMP), which is another interpretation of the center of pressure (COP). They discussed the condition under which one foot detaches off the ground while the other remains contact and concluded that it is possible if ZMP is within the sole of the foot to be a pivot. Note that the friction limit does not concern this. One of the benefits of considering ZMP is that this criterion can be checked with respect to the “supposed” motion – if the condition is discussed based on the conventional COP, which one may know only with respect to the “resulted” motion, this is not possible. Another is that the condition is geometrically represented as a relationship between a point and a two-dimensional convex region, so that it is easy to handle. A strong assumption to this point available is that all the contact points are again supposed to be on an identical plane.

ZMP is determined from the inertial movement of the whole body. If the rate of change of the angular momentum about the center of mass (COM) is negligible, it is an intersection of the ground and a line that passes COM and is parallel to the total external force as Mitobe et al. [32] pointed out in 2000. Popovic et al. [33] named
that approximated ZMP the centroidal momentum pivot (CMP), which allows much easier understandings.

5 Reduced-Order Models and Motion Resolution

When tackling the problem of controller design, it is often more preferable to set up an intuitive and comprehensive model that represents dominant behavior of the system than to use the strict full-body dynamics model. It has been naturally done by many works [30, 34, 35] from 1960 to the 1970s in the field of locomotion control. This is not only a qualitative discussion. As Fujimoto et al. [24] pointed out, the partial equation of motion corresponding to the floating base link implies the total linear and angular momenta changes only due to the net external forces, and the effect of internal forces is concealed. The total linear momentum is equivalent with the velocity of COM multiplied by the total mass. Thus, it is reasonable to abstract the translational movement of COM and the rotational movements of body parts about COM. This idea was proposed by Miyazaki and Arimoto [36] in 1980.

Reduced-order dynamics based on the COM movement has been much more evolved in the 1990s. Kajita et al. [37] proposed an idea to strictly linearize the two-dimensional dynamics of COM in sagittal plane by artificially constraining it on a spatial line (not necessarily a horizontal line as misunderstood by several publications) and named the dynamics the linearized inverted pendulum mode (LIPM). It was enhanced to three-dimensional model by Hara et al. [38]. As noted above, Mitobe et al. [32] developed a model that represents the relationship between COM and ZMP (CMP), with which the robot motion and the contact constraints can be dealt with rather easily. Nagasaka et al. [39] and Sugihara et al. [40] explained that the model is analogous to an inverted pendulum supported by a massless movable cart.

A reciprocal conversion between the reduced motion/input and the full-scale motion/input is required in order to exploit the above reduced-order models. Favorably, the instantaneous relationship between the total linear and angular momenta and the velocities of joints is represented by a linear equation as well as that between the effector and joints of a manipulator. Hence, techniques around the differential inverse kinematics developed for manipulators are available. This is utilized in control frameworks by Tamiya et al. [41] in 1998 and Sugihara et al. [40] in 2001. The mapping matrix from the velocities of joints to the linear momentum is essentially the same with COM Jacobian matrix, which was proposed by Boulic et al. [42] in 1994. On the other hand, the mapping matrix to the angular momentum was proposed by Sugihara [40] and named angular momentum Jacobian matrix in 2001, which is inappropriate since the angular momentum is not a derivative of a certain amount. Orin et al. [43] renamed the above matrices the centroidal momentum matrix. Kajita et al. [44] proposed an efficient recursive $O(n)$ algorithm to compute it in 2003.

The whole-body motion of a humanoid robot is orchestrated not only from the desired total momentum but also from other many physical and task-oriented
constraints. The structure-varying property suggests that any body part can be an effector at any instance, so that the task space frequently changes – it can be redundant in some occasions or overconstrained in others. Thus, a robust and scalable algorithm for the inverse kinematics is crucial. Recently [45, 46] some sophisticated techniques have been developed.

6 Future Directions and Open Problems

The kinematics and dynamics are fundamental issues in robotics. However, they are possibly to be evolved more in several directions.

It is noted in the last of Sect. 2 that the forward kinematics is no longer a problem as long as the mechanism does not have closed-loop structures. Advanced CAD softwares encourage more complex mechanical design, and closed-loop structures are now commonly used in various robots including humanoid robots. It is known that the forward kinematics of the parallel kinematic chain is more difficult than the inverse kinematics in general since motions of actuated joints and passive ones are coupled in a complicated manner. A sophisticated unification of forward and inverse kinematics for such complex mechanism is to be demanded.

Another big change in this field is that some torque-controlled humanoid robots have appeared. They are potentially tough against and responsive to perturbations. Thus, the importance of the inverse dynamics computation is increasing in order to exploit the hardware. Since the issue of computation cost has become relatively less severe, thanks to powerful computers that are available today, a computation scheme in which the inequality constraints on both the contact forces and actuation forces are strictly taken into account has become preferred [47,48], where efforts are made to improve the robustness against observation noises on the robot state. The author is personally interested in a question which is more promising to use such a straightforward scheme or an art to avoid detailed representation of dynamics.

Finally, it is worth noting that robotics contributes more to the human understanding, which is one of the motivations to study humanoid robots. Techniques to deal with a detailed model of kinematics and dynamics are directly applicable to a realistic human model, which is created based on modern anatomical studies. It is now possible to replicate and estimate behaviors of a complex musculoskeletal model in a computer [49] and to identify a human’s mass properties without dissection based on the floating-base dynamics [50]. The author expects that such a coevolution of human science and robotics continues.

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