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Deontic Logic and Legal Rules

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Introduction: The Concept of Deontic Logic

Deontic logic is a branch of logic that studies normative concepts such as obligation (“ought”), permission (“may”), prohibition (“may not”), and other related notions (“optional,” “good,” “bad,” “claim,” “power,” “liberty,” “immunity,” “super-erogatory,” “blameworthy,” “praiseworthy,” etc.). It investigates the logical features of these concepts, as well as the logical relations among propositions that contain them as their essential constituents. Deontic logic is also concerned with the study of norms, normative systems, and different forms of normative reasoning. That is why it is considered to be particularly relevant for investigating the logical aspects of law, ethics, and other fields in which this kind of reasoning plays a prominent role (see Hilpinnen 1981, 2002; McNamara 2006; Von Wright 1951, 1999).

The origin of the term “deontic” can be traced back to the Greek word “δέον” (gen. δέοντος), which may be translated as “duly,” “as it should be,” and “that which is binding.” Deontic logic is often treated as a branch of modal logic. This is due to the fact that the logical behavior of deontic

concepts (“deontic modalities” – obligatory, permitted, forbidden) is found to be similar to the behavior of the so-called alethic modalities (necessary, possible, impossible), which are explored in the framework of basic modal logic (see Von Wright 1951). However, the laws that govern alethic modalities are not identical to those of deontic modalities. For example, while if something is necessary, it must also be (true), the fact that something is obligatory does not imply that it will, indeed, be brought about. In fact, people often fail to comply with the normative requirements, and that introduces a need for a concept of sanction for breaking the deontic obligations. Thus, although it is closely connected with modal logic, deontic logic is a field with its own theoretical and practical interest.

Historical Development of Deontic Logic

The analogy between alethic and deontic modalities has been investigated since medieval times, especially by the logicians of the fourteenth century (see Knuuttila 1981). In the seventeenth century, the young Leibniz, in his essay on natural law, also utilized this analogy to define what he called “legal modalities” (*iuris modalia*). He observed that the legal concepts of permitted (*licitum*) and obligatory (*debitum*) can be defined through corresponding alethic concepts of possibility and necessity. Thus, according to him, the

permitted is that which is possible for a good person to do, and the obligatory is that which is necessary for a good person to do (see Hilpinen 2001, 159–160).

From the nineteenth century onward, different aspects of the study of normative phenomena have been treated in the work of prominent philosophers and jurists. Jeremy Bentham developed his “science of morality” under the name “deontology.” Alexius Meinong explored the possibility of defining normative concepts from the perspective of actions of agents and their praiseworthiness or blameworthiness. Wesley Newcomb Hohfeld attempted disambiguation of the term “rights” by exploring the opposition and correlation of the concepts of right, duty, privilege, power, immunity, liability, etc. A more formal approach to deontic concepts was developed by Ernst Mally in his work *Grundgesetze des Sollens: Elemente der Logik des Willens* (1926). In fact, Mally constructed the first formal system of deontic logic, conceived of as a basis for the exact system of pure ethics that he intended to build.

However, in the 1930s and 1940s, critical objections started to be raised concerning the very possibility of logical treatment of norms and related deontic entities. Thus, for example, Mally’s system of deontic logic was subjected to harsh criticism by Karl Menger. Additionally, Jørgen Jørgensen (1937) convincingly problematized the applicability of the standard conception of logical truth and validity to imperatives and other normative sentences (for the detailed history of the development of deontic logic, see Hilpinen and McNamara 2013).

The decisive impetus, however, for the constitution of deontic logic as “a full-fledged branch of symbolic logic” (McNamara 2006, 199) came around the middle of the twentieth century. In 1951, Georg Henrik von Wright published his seminal paper “Deontic Logic,” which marked the beginning of a new era in the study of deontic logic. Von Wright stayed preoccupied with the field of deontic logic through all his intellectual life, greatly contributing to its development with several important publications: *Norm and Action: A Logical Enquiry* (1963), *An Essay in Deontic*

Logic and the General Theory of Action (1968), *Deontic Logic: A Personal View* (1999), etc. Apart from von Wright, the problems of deontic logic have also inspired the work of many other logicians, philosophers, and legal theorists of the twentieth and twenty-first centuries, like Georges (Jerzy) Kalinowski, Stig Kanger, Alan Ross Anderson, Nicholas Rescher, Héctor-Neri Castañeda, Lennart Åqvist, Carlos Eduardo Alchourrón, Eugenio Bulygin, Jan Woleński, Kazimierz Opalek, Zygmunt Ziembiński, Jerzy Wróblewski, Jaakko Hintikka, Dagfinn Føllesdal, Risto Hilpinen, Paul McNamara, Simo Knuutila, Patrice Bailhache, Arend Soeteman, Henry Prakken, Giovanni Sartor, Marek Sergot, Guido Governatori, Pablo E. Navarro, Jorge L. Rodríguez, and many others.

In recent decades, research in deontic logic has been closely connected with the field of Artificial Intelligence and Law, especially in relation to the formalization of fundamental legal concepts, the building of formal models of legal reasoning and argumentation, and expert systems in law. These new trends make deontic logic a particularly diversified and vibrant field of contemporary logical investigations.

The Formal Structure of Deontic Logic: Syntax and Semantics

Von Wright’s system of 1951 represents a basis for what is nowadays known as Standard Deontic Logic (SDL). It is built in an axiomatic manner: starting from a number of primitive, undefined notions and a number of axioms, the theorems of deontic logic are derived by a set of transformation rules. These axioms and theorems are supposed to reflect the laws that govern our intuitive notions of obligation and permission, although not all of these laws are intuitively obvious.

SDL is a monadic system, which means that the deontic concepts (called “deontic operators”) apply to single, elementary propositions in order to build compound, molecular propositions (the presentation of the fundamentals of SDL below is based, in its main lines, on McNamara 2006). It

uses the language of classical propositional logic in which there is an infinite set of propositional variables (symbolized by small Latin letters p, q, r , etc.). The basic logical operators (connectives) of SDL are negation (\sim) and implication (\rightarrow), interpreted as a material conditional (other logical operators – conjunction (“and,” $\&$) disjunction (“or,” \vee), equivalence (“if and only if,” \equiv) can be defined through the basic operators of the system – negation and implication).

A new element, added to the language of propositional logic, is the deontic operator O , obligatory. Thus, Op reads “ p is obligatory.” Other deontic operators can be defined with the help of the operator O and negation. Thus, for P (permitted), we have the equivalence $Pp \equiv \sim O \sim p$ (informally: if p is permitted, then its negation is not obligatory and vice versa), and for F (forbidden), the equivalence $Fp \equiv O \sim p$ (informally: if p is forbidden, then its negation is obligatory and vice versa).

Besides the logical truths of propositional logic, the new, deontic axioms of the system are the following:

- (a) $O(p \rightarrow q) \rightarrow (Op \rightarrow Oq)$, which means that if it is obligatory that p implies q , then obligatory p implies obligatory q (in other words: “if a material conditional is obligatory, and its antecedent is obligatory, then so is its consequent”).
- (b) $Op \rightarrow \sim O \sim p$ – if p is obligatory, then its negation is not obligatory.

From the transformation rules, the system uses the *modus ponens* rule:

If $\vdash p$ and $\vdash p \rightarrow q$, then $\vdash q$, meaning that if a material conditional and its antecedent are theorems, then the consequent is also a theorem, and the rule

If $\vdash p$, then $\vdash Op$, meaning that if p is a theorem of the system, then the claim that p is obligatory is also a theorem.

For the formal system SDL, it is possible to construct a standard, Kripke-style semantics, based on three fundamental elements:

- A set of possible worlds, W
- A function of interpretation, I , which assigns a truth value to each proposition in each possible world
- A relation R that connects the possible worlds (Rij means that the world i is related to the world j by the relation R).

The relation Rij holds if and only if j is a world where everything that is obligatory in i holds (i.e., in j there are no violations of the obligations that hold in i). The worlds which are so related to i are called “ i -acceptable worlds.” The relation R is “serial,” which means that for every world i , there is at least one i -acceptable world. In a given world, propositions are either true or false. The basic idea of this semantics is that “the normative status of a proposition from the standpoint of a world i can be assessed by looking at how that proposition fairs at the i -acceptable worlds” (McNamara 2006, 211). Thus, iff p holds in all the i -acceptable worlds, it is obligatory; iff p holds in some such worlds, it is permissible; iff p holds in no such world, it is impermissible; etc. A formula is considered to be valid when it must be true at any world in any such model of worlds related in a serial way (McNamara 2006, 212).

The formal construction of deontic logic in its basic form makes it possible to formulate important and interesting deontic theorems, some of which express the basic “deontic truths” that we intuitively capture. In his 1951 essay, von Wright mentions some of them; for example, if something is obligatory, then it is also permitted; doing the permitted can never commit us to do the forbidden; if something commits us to the forbidden, then that thing is also forbidden; it is logically impossible to be obliged to choose between forbidden alternatives; etc.

However, the first attempts to construct deontic logic as a formal system revealed some

fundamental dilemmas in its foundations, including the question of the types of entities to which deontic operators are supposed to apply – names of act types (smoking, murder, adultery, etc.) or names of sentences, like in classical propositional logic. Other posed dilemmas are: Can deontic operators be iterated, i.e., should the expressions of the type OOp (“it is obligatory that it is obligatory that p ”) be part of the formal deontic system? How to formally express the phenomenon of the violation of an obligation? Although the system of SDL successfully resolved some of these questions by interpreting the variables and operators as applying to propositions (and thus enhanced the original von Wright’s system, in which they applied to names of acts), it still generates a considerable number of theoretical puzzles and paradoxes.

Controversies and Challenges of Deontic Logic: Ambiguities of Normative Discourse

The puzzles and paradoxes related to the system of SDL are numerous and heterogeneous (see, e.g., Hansen 2006; Hilpinen and McNamara 2013; Navarro and Rodríguez 2014). Some of the most widely known include:

- “Ross’ paradox” (named after Alf Ross, who first discovered it), related to the formula $Op \rightarrow O(p \vee q)$; informally, if some state of affairs p is obligatory, then that state of affairs or any other state of affairs is also obligatory; for example, if somebody is obliged to mail a letter, then he/she is obliged to mail the letter or to burn it, which sounds counterintuitive.
- “Prior’s paradox” (named after A.N. Prior), related to the formula $O \sim p \rightarrow O(p \rightarrow q)$; informally, the doing of what is forbidden commits us to the doing of anything whatsoever, e.g., the forbidden act of stealing commits us to committing adultery, which sounds counterintuitive.
- “Good Samaritan paradox” related to the formula $O(p \ \& \ q) \rightarrow Oq$; for example, if helping the robbed man is obligatory, it follows that his

being robbed is likewise obligatory, which sounds counterintuitive.

- “Free choice permission paradox,” related to the formula $P(p \vee q) \rightarrow Pp \vee Pq$; informally, if a disjunction of two states of affairs is permitted, then each of them is permitted. Although this formula sounds plausible, it cannot be derived in the system of SDL.

The abovementioned paradoxes are but a small part of a long list of paradoxes and other conceptual challenges related to the system of SDL. However, it seems that the most general objection not only to this system, but to the very perspective of deontic logic, is contained in the so-called Jørgensen’s dilemma (Jørgensen 1937), which in von Wright’s formulation reads as: “Since norms are usually thought to lack truth value, how can logical relations such as contradiction and entailment (logical consequence) obtain between norms?” (Von Wright 1991, 266).

This problem contributed to the revealing of the fundamental ambiguity that lies in the conceptual basis of deontic logic: the ambiguity between the *prescriptive* and the *descriptive* aspect of deontic sentences. Interpreted in a prescriptive way, deontic sentences express norms. In a descriptive interpretation, however, “they express (true of false) propositions to the effects that certain norms exist” (Von Wright 1991, 265). The formulation of this ambiguity inspired the proposal to devise, besides a logic of norms, a separate “logic of norm propositions,” as a means of overcoming the problems generated by the ambiguity (see Alchourrón and Bulygin 1989).

Apart from this ambiguity, other conceptual confusions in our deontic intuitions that were identified by the formal-logical approach concern the notions of permission and obligation. It turned out that they are not homogeneous, but inherently heterogeneous. Consequently, it is important to distinguish between different kinds of permissions/obligations (e.g., between the so-called “strong” and “weak” permissions/obligations) in order to avoid conceptual confusions in normative discourse (see Von Wright 1968, Hansson 2013; Hansen et al. 2007; Soeteman 1989; for a criticism

of the concept of “permissive norm”, see Opalek and Woleński 1991).

Other, more generalized ways to overcome paradoxes and challenges of standard deontic logic include (but are not reduced to):

1. Strengthening the SDL with new axioms
2. Reducing deontic logic to alethic modal by introducing a specific deontic constant stating that all relevant normative demands are met
3. Building a so-called dyadic calculus of deontic logic in which the deontic operators apply not to propositions, but to corresponding pairs of propositions, built by the symbol “/” (p/q is interpreted as “ p is permitted, given that q ”)
4. Building a stronger connection of deontic logic with logic of action and temporal logic as its theoretical prerequisites (see McNamara 2006; Von Wright 1963, 1968).

However, although these approaches contributed to the resolving some of the puzzles of deontic logic, many of them are “still alive and kicking” (Hansen 2006).

Deontic Logic, Legal Systems, and Legal Rules

Law is one the most important normative phenomena of human society, both by its internal complexity and its deep influence on the life of communities and individuals. As a logic of norms and normative systems, deontic logic is arguably highly relevant for the analysis, representation, and better understanding of conceptual, logical, and inferential structure of law and legal reasoning (see Alchourrón and Bulygin 1971; Soeteman 1989). Indeed, the theoretical analyses related to deontic logic treated many important questions that concern some of the fundamental concepts of law: What is a norm? What types of norm are there? Which factors determine the existence and the validity of a certain norm? Are permissive norms genuine norms or only imperative norms deserve that name? What is the relation between norms and rules that express them? What types of legal rules are there? What is the logical structure of legal rules, which allow exceptions and which

can be changed or overridden in the course of time? Which are the factors that, in some situations, can block the actual application of applicable legal rule(s) to a particular case? What are the most important features of legal systems as corpora of norms? Which are the peculiarities of consistency and completeness as features of legal systems compared to the same features of axiomatic systems in logic? How to deal with normative gaps and normative conflicts in legal systems? How to establish prioritization and hierarchies of principles in legal reasoning?

In an attempt to answer these and other related questions, research on deontic logic contributed to the clarification of the specific ontological status and ontological effects of legal rules as vital parts of the “institutional reality” of human society. As Hage (2005) emphasizes, they have the capacity to impose specific structures on the (legal) world, be it in a form of legal meaning conventions (definitions), competence-conferring rules, or deontic legal rules in a narrower sense. The rules of the last group characteristically attach deontic facts to the facts that satisfy their conditions, or in cases where they lack conditions, they directly create deontic facts (Hage 2005, 194–195).

Another important contribution of contemporary application of the perspective of (deontic) logic to the study of law is the distinction of two kinds of study related to legal rules: reasoning *with* legal rules and reasoning *about* legal rules (see Prakken and Sartor 2015). The first kind includes phenomena related to the representation of legal rules and corpora of legal regulations. Here, the emphasis is put on legal conclusions that deductively follow from these corpora. The second kind includes phenomena related to the reasoning about legal rules and their application in novel circumstances, in situations that need interpretation or order to establish the facts of the case (Prakken and Sartor 2015, 1). These analyses show that there are significant logical specificities in reasoning that includes legal rules compared to the reasoning with ordinary sentences. To wit, legal rules should be applied in order to generate deontic consequences, and a great deal of legal reasoning is concerned precisely with the conditions of applicability of

certain rules and the factors that determine their actual application.

However, despite its important results, there are still different opinions concerning the real capacity “of conceiving deontic logic as a suitable account of the logical behavior of (sentences expressing legal norms),” because of “the ontologic, semantic, and epistemic features of legal norms” and rules that express them (Mazzarese 1991, 374). Also, in the field of AI and Law, there is some “uncertainty concerning the potential role of deontic logic in legal knowledge representation,” based on the belief that “a good deal can be achieved [...] in the absence of explicit representation of the deontic notions” (Jones 1990, 237).

Conclusion

The question of assessing the real degree to which deontic logic contributes to the study of legal rules and other legal phenomena is still open to discussion. However, as the history of deontic logic has shown, the fundamentals of deontic systems often allow for the possibility of their further sophistication and upgrading of their expressive power. Some of the contemporary research is taking precisely that direction of extending deontic logic with new concepts (like groups of actors, authorities/enactment, authority hierarchy/applicability, etc.) in order to accommodate the peculiarity of legal rules and reasoning with them (Royakkers 1998). Their eventual success may soften the explicit or implicit skepticism concerning the theoretical merit of deontic logic, by reaffirming its role as “a study of conditions that must be satisfied in a rational norm-giving activity” (Von Wright 1991, 266) and a tool for enhancing the criteria of normative rationality.

Cross-References

- ▶ Brute Facts, “Institutional Facts” and Law
- ▶ Descriptive and Normative Statements
- ▶ Is and Ought Distinction in Legal Philosophy

- ▶ Law as a Normative System
- ▶ Legal and Moral Norms
- ▶ Legal Rules as ‘Ought’ Statements
- ▶ “Rules of Law in a Descriptive Sense” (Kelsen)
- ▶ The Concept of Norm

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